Math 1730 Test #3 Review

Chapter 2 (2.7)

1. Given the demand function \( q = D(x) = \sqrt{459 - p} \), find
   a. The elasticity function, \( E(x) \).
   b. The elasticity at \( x = 96 \).
   c. Is demand elastic, is it inelastic, or does it have unit elasticity at \( x = 96 \)?
   d. Find the value of \( x \) for which total revenue is a maximum, assuming \( x \) is in dollars.

Chapter 3 (3.1, 3.2, 3.3, 3.4 and 3.5)

2. Find the derivatives of the following functions:
   a. \( f(x) = 7e^{5x} \)
   b. \( f(x) = x^3 + e^{-3x} \)
   c. \( f(x) = 2e^{x^3+2x} \)
   d. \( f(x) = 3x^3e^{2x} \)
   e. \( f(x) = \frac{e^{3x}}{x^4} \)

3. Find the derivatives of the following functions:
   a. \( f(x) = \ln(x^2 - 5x + 6) \)
   b. \( f(x) = x^2 \ln(3x) \)
   c. \( f(x) = \frac{\ln x}{x^4} \)

4. An initial deposit is made of $12,000 in an account paying 4% interest compounded continuously.
   a. How much will the account be worth in 6 years?
   b. How long will it take the account to double?

5. A painting that was worth $620 in 1952 is sold for $240,000 in 2012.
   a. Find the exponential growth rate, \( k \), to 3 decimal places.
   b. Find the exponential growth function, \( V(t) \).
   c. How much will the painting be worth in 2020, in dollars?
      (use the rounded off form of \( k \) from part a)
   d. In what year would you expect the value to reach $1,000,000?

6. Find the present value of a payment of $2300 that will be received 3 years from now, assuming that interest is 2.7% compounded continuously.

7. The population of a city has gone from 53,000 people in 1985 to 48,000 in 2012.
   a. Find a function \( P(t) \) that describes the population \( t \) years after 1985, assuming exponential decline. (round the rate, \( k \), to 5 decimal places).
   b. In what year would you expect the population to reach 45,000 people?
8. Find the derivatives of the following functions:
   a. \( f(x) = 4^x \)  
   b. \( f(x) = \log_9 x \)  
   c. \( f(x) = 7^{2x^2-5x+7} \)  
   d. \( f(x) = 5\log_3(x^4-7x) \)  
   e. \( f(x) = 3^x \cdot \log_2 x \)  
   f. \( f(x) = \log(x^3-x^2+5x) \)  

Chapter 4 (4.1, 4.2, and 4.3)

9. Find the following indefinite integrals:
   a. \( \int (15x^2 + 5x - 7) \, dx \)  
   b. \( \int \left(\frac{4}{x^2} - \frac{7}{x}\right) \, dx \)  
   c. \( \int (7e^{2x} + 7x^2) \, dx \)  
   d. \( \int (\sqrt{x^2} - \sqrt{x}) \, dx \)  

10. Find \( f \) such that \( f'(x) = 6x^2 + 8x - 4 \) and \( f(1) = 10 \). 

11. Approximate the area under the graph of \( f(x) = -x^2 + 4x + 14 \) over the interval \([-2, 6]\) using \( n = 4 \) and left endpoints of each subinterval. 

12. Evaluate the following definite integral: \( \int_{-2}^{6} (-x^2 + 4x + 14) \, dx \) 

13. A technology firm finds that the marginal profit, in dollars, from sale of \( x \) hard drives is given by \( P'(x) = 3x^{0.5} \). A customer orders 50 hard drives and later increases the order to 60. Find the extra profit resulting from the increase in order size. 

14. Find the area under the curve over the interval given:
   a. \( f(x) = 81 - x^2 \); \([-9, 9]\) 
   b. \( f(x) = e^{2x} \); \([0, 3]\)  
   (round answer to 3 decimal places)
Solutions:

1a. \( E(x) = \frac{(-p)^{\frac{1}{2}}(459-p)^{-\frac{1}{2}}(-1)}{(459-p)^{\frac{1}{2}}} \) or \( E(x) = \frac{p}{2(459-p)} \)

1b. \( \frac{16}{121} \) or 0.132

1c. Inelastic

1d. 306

2a. \( f'(x) = 35e^{5x} \)

2b. \( f'(x) = 3x^2 - 3e^{-3x} \)

2c. \( f'(x) = 2e^{x^3} + 2x(3x^2 + 2) \)

2d. \( 6x^3e^{2x} + 9x^2e^{2x} \)

2e. \( \frac{3x^4e^{3x} - 4x^3e^{3x}}{x^8} \) or \( \frac{3xe^{3x} - 4e^{3x}}{x^5} \)

3a. \( f'(x) = \frac{(2x - 5)}{x^2 - 5x + 6} \)

3b. \( f'(x) = x + 2x\ln(3x) \)

3c. \( f'(x) = \frac{x^3 - 4x^3\ln x}{x^8} \) or \( \frac{1 - 4\ln x}{x^5} \)

4a. $15,254.99$

4b. approximately 17 years

5a. 0.099

5b. \( V(t) = 620e^{0.099t} \)

5c. $520,070$

5d. 2027

6. $2121.05$

7a. \( P(t) = 53,000e^{-0.00367t} \)

7b. 2030

8a. \( f'(x) = (\ln 4) \cdot 4^x \)

8b. \( f'(x) = \frac{1}{(\ln 9)x} \)

8c. \( f'(x) = (\ln 7) \cdot 7^{2x^2 - 5x + 7}(4x - 5) \)

8d. \( f'(x) = \frac{5(4x^3 - 7)}{(\ln 3)(x^4 - 7)} \)

8e. \( f'(x) = 3^x \cdot \frac{1}{(\ln 2)x} + (\ln 3) \cdot 3^x \cdot \log_2 x \)

8f. \( f'(x) = \frac{3x^2 - 2x + 5}{(\ln 10)(x^3 - x^2 + 5x)} \)

9a. \( 5x^3 + \left(\frac{5}{2}\right)x^2 - 7x + C \)

9b. \( -4x^{-1} - 7(lnx) + C \)

9c. \( \left(\frac{7}{2}\right)e^{2x} + \left(\frac{7}{3}\right)x^3 + C \)

9d. \( \left(\frac{3}{5}\right)x^5 - \left(\frac{2}{3}\right)x^3 + C \)

10. \( f(x) = 2x^3 + 4x^2 - 4x + 8 \)

11. 96

12. \( \frac{304}{3} \) or 101 \( \frac{1}{3} \)

13. 222.41

14a. 972

14b. 201.214