An Introduction to Eigencentrality

Nate Iverson

The University of Toledo

Toledo, Ohio

Graphs

A simple undirected graph $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges** such that E has the following properties:

- Antireflexive: $(x, x) \notin E$ for all $x \in V$.
- Symmetric: If $(x, y) \in E$ then $(y, x) \in E$.

Graphs

A simple undirected graph $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges** such that E has the following properties:

- Antireflexive: $(x, x) \notin E$ for all $x \in V$.
- Symmetric: If $(x, y) \in E$ then $(y, x) \in E$.

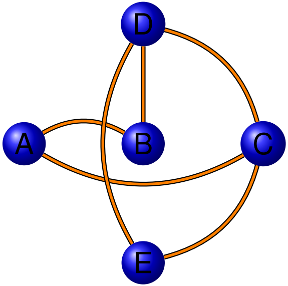
In practice we think of them as pictures:

Graphs

A simple undirected graph $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges** such that E has the following properties:

- Antireflexive: $(x, x) \notin E$ for all $x \in V$.
- Symmetric: If $(x, y) \in E$ then $(y, x) \in E$.

In practice we think of them as pictures:

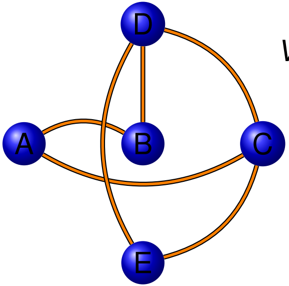


Graphs

A simple undirected graph $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges** such that E has the following properties:

- Antireflexive: $(x, x) \notin E$ for all $x \in V$.
- Symmetric: If $(x, y) \in E$ then $(y, x) \in E$.

In practice we think of them as pictures:



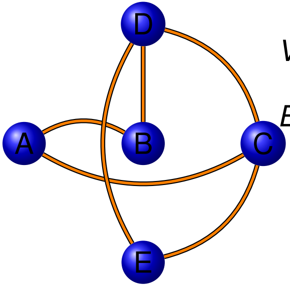
$$V = \{A, B, C, D, E\}$$

Graphs

A simple undirected graph $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges** such that E has the following properties:

- Antireflexive: $(x, x) \notin E$ for all $x \in V$.
- Symmetric: If $(x, y) \in E$ then $(y, x) \in E$.

In practice we think of them as pictures:



$$V = \{A, B, C, D, E\}$$

$$E = \{(A, B), (A, C), (B, D), (C, D), (C, E), (D, E), (B, A), (C, A), (D, B), (D, C), (E, C), (E, D)\}$$

Vertex-Vertex Adjacency Matrix

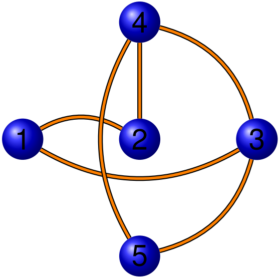
For a graph $\Gamma = (V, E)$ with n vertices we can relabel the vertices to be $V = \{1, 2, \ldots, n\}$. An **adjacency matrix** A for Γ is defined in the following way:

$$(A)_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{if } (i, j) \notin E \end{cases}$$

Vertex-Vertex Adjacency Matrix

For a graph $\Gamma = (V, E)$ with n vertices we can relabel the vertices to be $V = \{1, 2, \ldots, n\}$. An **adjacency matrix** A for Γ is defined in the following way:

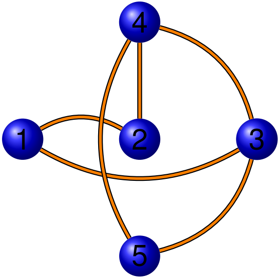
$$(A)_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases}$$



Vertex-Vertex Adjacency Matrix

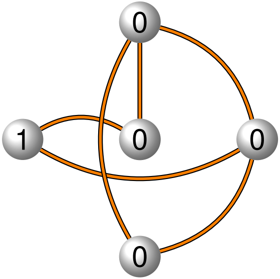
For a graph $\Gamma = (V, E)$ with n vertices we can relabel the vertices to be $V = \{1, 2, \ldots, n\}$. An **adjacency matrix** A for Γ is defined in the following way:

$$(A)_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{if } (i,j) \notin E \end{cases}$$



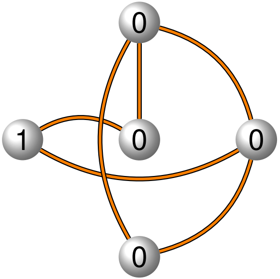
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Multiplication By an Adjacency Matrix

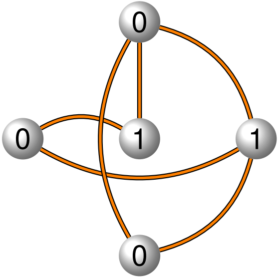


$$\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Multiplication By an Adjacency Matrix

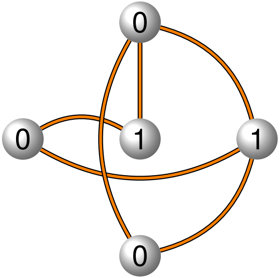


$$\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

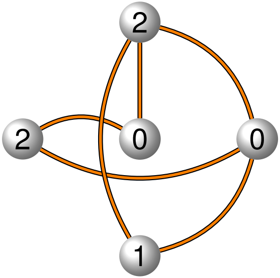


$$A\mathbf{x}_0 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Multiplication By an Adjacency Matrix

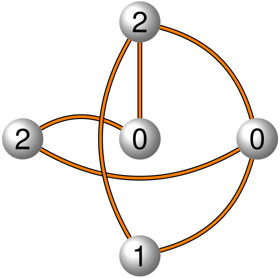


$$A\mathbf{x}_0 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

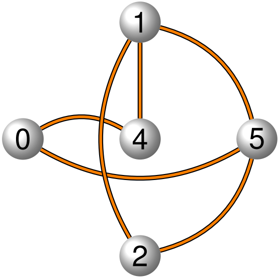


$$A^2\mathbf{x}_0 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

Multiplication By an Adjacency Matrix

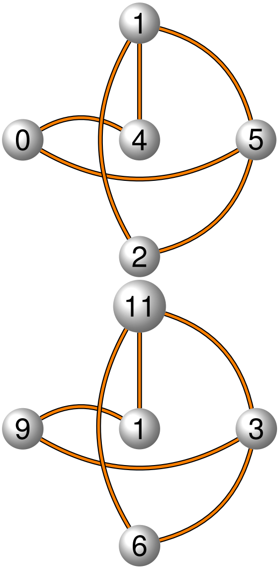


$$A^2\mathbf{x}_0 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$



$$A^3\mathbf{x}_0 = \begin{bmatrix} 0 \\ 4 \\ 5 \\ 1 \\ 2 \end{bmatrix}$$

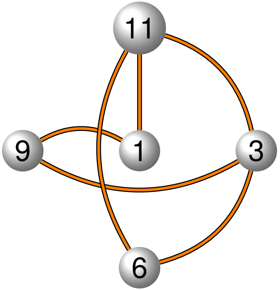
Multiplication By an Adjacency Matrix



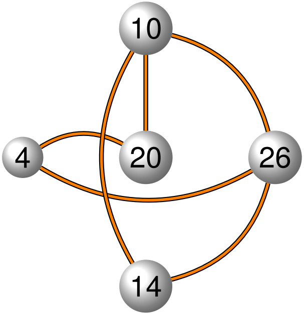
$$A^3\mathbf{x}_0 = \begin{bmatrix} 0 \\ 4 \\ 5 \\ 1 \\ 2 \end{bmatrix}$$

$$A^4\mathbf{x}_0 = \begin{bmatrix} 9 \\ 1 \\ 3 \\ 11 \\ 6 \end{bmatrix}$$

Multiplication By an Adjacency Matrix

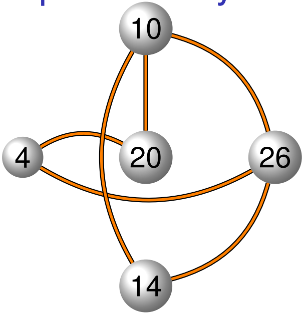


$$A^4\mathbf{x}_0 = \begin{bmatrix} 9 \\ 1 \\ 3 \\ 11 \\ 6 \end{bmatrix}$$



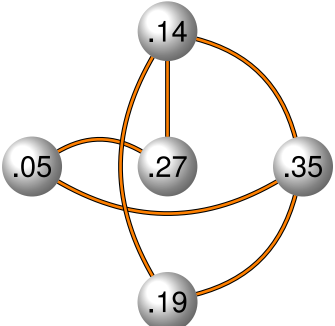
$$A^5\mathbf{x}_0 = \begin{bmatrix} 4 \\ 20 \\ 26 \\ 10 \\ 14 \end{bmatrix}$$

Multiplication By an Adjacency Matrix



$$A^5\mathbf{x}_0 = \begin{bmatrix} 4 \\ 20 \\ 26 \\ 10 \\ 14 \end{bmatrix}$$

Things start getting large so lets scale them to add up to 1



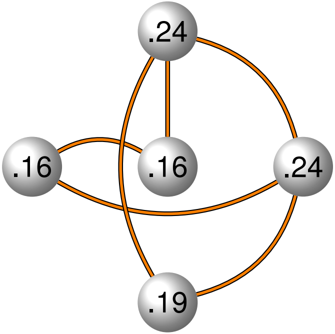
$$\frac{A^5\mathbf{x}_0}{\|A^5\mathbf{x}_0\|_1} \approx \begin{bmatrix} 0.05 \\ 0.27 \\ 0.35 \\ 0.14 \\ 0.19 \end{bmatrix}$$

Multiplication By an Adjacency Matrix

Applying the following procedure repeatedly

$$\mathbf{x}_{n+1} = \frac{A\mathbf{x}_n}{\|A\mathbf{x}_n\|_1}$$

will sometimes result in a stable value $\mathbf{x}_{n+1} \to \mathbf{x}$ For this particular graph and $\mathbf{x}_0$ we get the following:



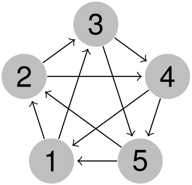
$$\mathbf{x}_n \to \mathbf{x} = \begin{bmatrix} 0.16 \\ 0.16 \\ 0.24 \\ 0.24 \\ 0.19 \end{bmatrix}$$

Directed Graphs

A directed graph (or **digraph**) $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges.** We think of each edge (i, j) as being a directed edge (arrow) based at i pointing to j.

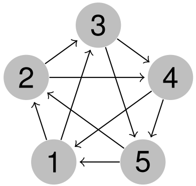
Directed Graphs

A directed graph (or **digraph**) $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges.** We think of each edge (i, j) as being a directed edge (arrow) based at i pointing to j.



Directed Graphs

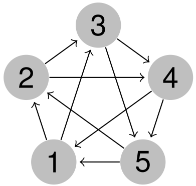
A directed graph (or **digraph**) $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges.** We think of each edge (i, j) as being a directed edge (arrow) based at i pointing to j.



Note: A simple undirected graph $\Gamma = (V, E)$ is also a digraph. You can think of splitting each undirected edge between i and j into two directed edges (i, j) and (j, i).

Directed Graphs

A directed graph (or **digraph**) $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges**. We think of each edge (i, j) as being a directed edge (arrow) based at i pointing to j.

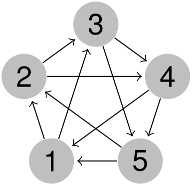


Note: A simple undirected graph $\Gamma = (V, E)$ is also a digraph. You can think of splitting each undirected edge between i and j into two directed edges (i, j) and (j, i).

You can also assign each directed edge (i, j) a weight $w_{ij} > 0$.

Directed Graphs

A directed graph (or **digraph**) $\Gamma = (V, E)$ where V is a set called the **vertices** and $E \subseteq V^2$ is called the **edges.** We think of each edge (i, j) as being a directed edge (arrow) based at i pointing to j.



Note: A simple undirected graph $\Gamma = (V, E)$ is also a digraph. You can think of splitting each undirected edge between i and j into two directed edges (i, j) and (j, i).

You can also assign each directed edge (i, j) a weight $w_{ij} > 0$. Then define adjacency matrix

$$(A)_{ij} = \begin{cases} w_{ij} & \text{if } (i, j) \in E \\ 0 & \text{if } (i, j) \notin E \end{cases}$$

Definitions

1. If real square matrix A has eigenvalues $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_r|$ then λ_1 is a **dominant eigenvalue.**

Definitions

1. If real square matrix A has eigenvalues $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_r|$ then λ_1 is a **dominant eigenvalue.**
2. matrix A is **irreducible** if for every two indices i and j there is a $k \in \mathbb{N}$ such that $(A^k)_{ij} > 0$.

Definitions

1. If real square matrix A has eigenvalues $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_r|$ then λ_1 is a **dominant eigenvalue.**
2. matrix A is **irreducible** if for every two indices i and j there is a $k \in \mathbb{N}$ such that $(A^k)_{ij} > 0$.
3. A graph Γ is **strongly connected** if for every to vertices i, j there is a directed path from i to j.

Definitions

1. If real square matrix A has eigenvalues $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_r|$ then λ_1 is a **dominant eigenvalue.**
2. matrix A is **irreducible** if for every two indices i and j there is a $k \in \mathbb{N}$ such that $(A^k)_{ij} > 0$.
3. A graph Γ is **strongly connected** if for every to vertices i, j there is a directed path from i to j. **Note:** if Γ is strongly connected iff its adjacency matrix A is *irreducible*.

Definitions

1. If real square matrix A has eigenvalues $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_r|$ then λ_1 is a **dominant eigenvalue.**
2. matrix A is **irreducible** if for every two indices i and j there is a $k \in \mathbb{N}$ such that $(A^k)_{ij} > 0$.
3. A graph Γ is **strongly connected** if for every to vertices i, j there is a directed path from i to j. **Note:** if Γ is strongly connected iff its adjacency matrix A is *irreducible*.
4. The gcd of all the lengths of closed directed paths at i is called the **period of index** i. It can be computed by $p(i) = \text{gcd}\{k \mid (A^k)_{ii} > 0\}$.

Definitions

1. If real square matrix A has eigenvalues $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_r|$ then λ_1 is a **dominant eigenvalue.**
2. matrix A is **irreducible** if for every two indices i and j there is a $k \in \mathbb{N}$ such that $(A^k)_{ij} > 0$.
3. A graph Γ is **strongly connected** if for every to vertices i, j there is a directed path from i to j. **Note:** if Γ is strongly connected iff its adjacency matrix A is *irreducible*.
4. The gcd of all the lengths of closed directed paths at i is called the **period of index** i. It can be computed by $p(i) = \gcd\{k \mid (A^k)_{ii} > 0\}$. If Γ is strongly connected then these periods are independent of the base index and are simply called the **period of** A.

Definitions

1. If real square matrix A has eigenvalues $|\lambda_1| > |\lambda_2| \geq \cdots \geq |\lambda_r|$ then λ_1 is a **dominant eigenvalue.**
2. matrix A is **irreducible** if for every two indices i and j there is a $k \in \mathbb{N}$ such that $(A^k)_{ij} > 0$.
3. A graph Γ is **strongly connected** if for every to vertices i, j there is a directed path from i to j. **Note:** if Γ is strongly connected iff its adjacency matrix A is *irreducible*.
4. The gcd of all the lengths of closed directed paths at i is called the **period of index** i. It can be computed by $p(i) = \gcd\{k \mid (A^k)_{ii} > 0\}$. If Γ is strongly connected then these periods are independent of the base index and are simply called the **period of** A.
5. if the period of A is 1 we call A **aperiodic**.

The Crucial Theorem

Theorem (The Perron-Frobenius theorem)

[1] Let A be an $n \times n$ *real valued nonnegative matrix. If A is irreducible then it has an eigenvalue* $\lambda > 0$ *such that:*

The Crucial Theorem

Theorem (The Perron-Frobenius theorem)

[1] Let A be an $n \times n$ real valued nonnegative matrix. If A is irreducible then it has an eigenvalue $\lambda > 0$ such that:

1. *λ is a simple root of the characteristic polynomial.*

The Crucial Theorem

Theorem (The Perron-Frobenius theorem)

[1] Let A be an $n \times n$ real valued nonnegative matrix. If A is irreducible then it has an eigenvalue $\lambda > 0$ such that:

1. *λ is a simple root of the characteristic polynomial.*
2. *λ has strictly positive left eigenvector **l** and right eigenvector **r**.*

The Crucial Theorem

Theorem (The Perron-Frobenius theorem)

[1] Let A be an $n \times n$ real valued nonnegative matrix. If A is irreducible then it has an eigenvalue $\lambda > 0$ such that:

1. *λ is a simple root of the characteristic polynomial.*
2. *λ has strictly positive left eigenvector **l** and right eigenvector **r**.*
3. *The eigenvectors of λ are unique up to a scalar.*

The Crucial Theorem

Theorem (The Perron-Frobenius theorem)

[1] Let A be an $n \times n$ real valued nonnegative matrix. If A is irreducible then it has an eigenvalue $\lambda > 0$ such that:

1. *λ is a simple root of the characteristic polynomial.*
2. *λ has strictly positive left eigenvector **l** and right eigenvector **r**.*
3. *The eigenvectors of λ are unique up to a scalar.*
4. *Any eigenvalue μ of A has $|\mu| \leq \lambda$*

The Crucial Theorem

Theorem (The Perron-Frobenius theorem)

[1] Let A be an $n \times n$ real valued nonnegative matrix. If A is irreducible then it has an eigenvalue $\lambda > 0$ such that:

1. *λ is a simple root of the characteristic polynomial.*
2. *λ has strictly positive left eigenvector **l** and right eigenvector **r**.*
3. *The eigenvectors of λ are unique up to a scalar.*
4. *Any eigenvalue μ of A has $|\mu| \leq \lambda$*
5. *If $0 \leq B \leq A$ and β is an eigenvalue of B then $|\beta| \leq \lambda$ and equality occurs iff A=B.*

The Crucial Theorem

Theorem (The Perron-Frobenius theorem)

[1] Let A be an $n \times n$ real valued nonnegative matrix. If A is irreducible then it has an eigenvalue $\lambda > 0$ such that:

1. *λ is a simple root of the characteristic polynomial.*
2. *λ has strictly positive left eigenvector **l** and right eigenvector **r**.*
3. *The eigenvectors of λ are unique up to a scalar.*
4. *Any eigenvalue μ of A has $|\mu| \leq \lambda$*
5. *If $0 \leq B \leq A$ and β is an eigenvalue of B then $|\beta| \leq \lambda$ and equality occurs iff A=B.*
6. *If A has period p then $\mu = \lambda e^{2\pi ki/p}$ are the p eigenvalues with $|\mu| = \lambda$*

The Crucial Theorem

Theorem (The Perron-Frobenius theorem)

[1] Let A be an $n \times n$ real valued nonnegative matrix. If A is irreducible then it has an eigenvalue $\lambda > 0$ such that:

1. *λ is a simple root of the characteristic polynomial.*
2. *λ has strictly positive left eigenvector **l** and right eigenvector **r**.*
3. *The eigenvectors of λ are unique up to a scalar.*
4. *Any eigenvalue μ of A has $|\mu| \leq \lambda$*
5. *If $0 \leq B \leq A$ and β is an eigenvalue of B then $|\beta| \leq \lambda$ and equality occurs iff A=B.*
6. *If A has period p then $\mu = \lambda e^{2\pi ki/p}$ are the p eigenvalues with $|\mu| = \lambda$*
7. *If A is aperiodic and $\mathbf{lr} = 1$ then*

$$\lim_{k \to \infty} A^k / \lambda^k = \mathbf{rl}$$

The Power Method

Let A be the $n \times n$ adjacency matrix of a connected simple undirected graph Γ.

The Power Method

Let A be the $n \times n$ adjacency matrix of a connected simple undirected graph Γ.

- A has a dominant eigenvalue λ_1 with right and left eigenvectors $\mathbf{r}, \mathbf{r}^T \geq \mathbf{0}$ scaled so that $\mathbf{r}^T\mathbf{r} = 1$ (Perron-Frobenius)

The Power Method

Let A be the $n \times n$ adjacency matrix of a connected simple undirected graph Γ.

- A has a dominant eigenvalue λ_1 with right and left eigenvectors $\mathbf{r}, \mathbf{r}^T \geq \mathbf{0}$ scaled so that $\mathbf{r}^T\mathbf{r} = 1$ (Perron-Frobenius)
- A is diagonizable (Spectral Theorem $A^T = A$) so the period of A is either 1 or 2

The Power Method

Let A be the $n \times n$ adjacency matrix of a connected simple undirected graph Γ.

- A has a dominant eigenvalue λ_1 with right and left eigenvectors $\mathbf{r}, \mathbf{r}^T \geq \mathbf{0}$ scaled so that $\mathbf{r}^T\mathbf{r} = 1$ (Perron-Frobenius)
- A is diagonizable (Spectral Theorem $A^T = A$) so the period of A is either 1 or 2
- If Γ has a closed path of odd length then A is aperiodic.

The Power Method

Let A be the $n \times n$ adjacency matrix of a connected simple undirected graph Γ.

- A has a dominant eigenvalue λ_1 with right and left eigenvectors $\mathbf{r}, \mathbf{r}^T \geq \mathbf{0}$ scaled so that $\mathbf{r}^T\mathbf{r} = 1$ (Perron-Frobenius)
- A is diagonizable (Spectral Theorem $A^T = A$) so the period of A is either 1 or 2
- If Γ has a closed path of odd length then A is aperiodic.
- For $x_0 \in \mathbb{R}^n$

$$\lim_{k\to\infty} A^k/\lambda^k \mathbf{x}_0 = \mathbf{r}\mathbf{r}^T\mathbf{x}_0 = \text{proj}_{\mathbf{r}}(\mathbf{x}_0)$$

The Power Method

Let A be the $n \times n$ adjacency matrix of a connected simple undirected graph Γ.

- A has a dominant eigenvalue λ_1 with right and left eigenvectors $\mathbf{r}, \mathbf{r}^T \geq \mathbf{0}$ scaled so that $\mathbf{r}^T\mathbf{r} = 1$ (Perron-Frobenius)
- A is diagonizable (Spectral Theorem $A^T = A$) so the period of A is either 1 or 2
- If Γ has a closed path of odd length then A is aperiodic.
- For $x_0 \in \mathbb{R}^n$

$$\lim_{k\to\infty} A^k/\lambda^k \mathbf{x}_0 = \mathbf{r}\mathbf{r}^T\mathbf{x}_0 = \text{proj}_{\mathbf{r}}(\mathbf{x}_0)$$

- When $\text{proj}_{\mathbf{r}}(\mathbf{x}_0) \neq \mathbf{0}$ the power method $\mathbf{x}_{n+1} = \dfrac{A\mathbf{x}_n}{\|A\mathbf{x}_n\|}$ converges to $\dfrac{\mathbf{r}}{\|\mathbf{r}\|}$

Eigencentrality

Lemma

A connected simple undirected graph Γ with at least one odd length closed path has an aperiodic irreducible nonnegative valued adjacency matrix A.

Eigencentrality

Lemma

A connected simple undirected graph Γ with at least one odd length closed path has an aperiodic irreducible nonnegative valued adjacency matrix A.

Theorem (Eigencentrality)

*A simple undirected connected graph with at least one odd length closed path has a dominant eigenvector with a corresponding positive valued eigenvector. Once scaled this vector is the **eigencentrality** (or **eigenvector centrality**) ranking for the graph.*

Eigencentrality

Lemma

A connected simple undirected graph Γ with at least one odd length closed path has an aperiodic irreducible nonnegative valued adjacency matrix A.

Theorem (Eigencentrality)

*A simple undirected connected graph with at least one odd length closed path has a dominant eigenvector with a corresponding positive valued eigenvector. Once scaled this vector is the **eigencentrality** (or **eigenvector centrality**) ranking for the graph.*

Note: Larger coordinate entries represent the corresponding vertex in the graph being more central.

Graph theoretic conditions

Theorem

Let Γ be a directed graph with weighted nonnegative adjacency matrix A and the following:

1. *Γ is strongly connected*
2. *There are two closed paths at a single vertex with relatively prime length.*

Then λ is the Perron-Frobenius eigenvalue with left and right eigenvectors $\boldsymbol{l}$ and $\boldsymbol{r}$ and for any $\boldsymbol{x}_0 \in \mathbb{R}^n$:

$$\lim_{k\to\infty} (A/\lambda)^k \boldsymbol{x}_0 = a\boldsymbol{r}$$

with $a = 0$ iff $\boldsymbol{x}_0^T \in \boldsymbol{l}^\perp$

PageRank[2]

Let vertices be webpages.

PageRank[2]

Let vertices be webpages.
PageRank graph has two types of directed edges:

PageRank[2]

Let vertices be webpages.
PageRank graph has two types of directed edges:

1. Hyperlinks *weighted* (with probability α)

Where $0 < \alpha < 1$ is called the **damping**

PageRank[2]

Let vertices be webpages.
PageRank graph has two types of directed edges:

1. Hyperlinks *weighted* (with probability α)
2. Random jump to any other vertex (with probability $1 - \alpha$)

Where $0 < \alpha < 1$ is called the **damping**

PageRank[2]

Let vertices be webpages.
PageRank graph has two types of directed edges:

1. Hyperlinks *weighted* (with probability α)
2. Random jump to any other vertex (with probability $1 - \alpha$)

Where $0 < \alpha < 1$ is called the **damping**

- The adjacency matrix *A*, for PageRank, is related to the weighted adjacency matrix of hyperlinks *H* and the matrix *R* with all ones.

$$A = \alpha H + (1 - \alpha)R$$

PageRank[2]

Let vertices be webpages.
PageRank graph has two types of directed edges:

1. Hyperlinks *weighted* (with probability α)
2. Random jump to any other vertex (with probability $1 - \alpha$)

Where $0 < \alpha < 1$ is called the **damping**

- The adjacency matrix A, for PageRank, is related to the weighted adjacency matrix of hyperlinks H and the matrix R with all ones.

$$A = \alpha H + (1 - \alpha) R$$

- Since every vertex is connected to every other, the graph is strongly connected and if there are 3 webpages aperiodic.

PageRank[2]

Let vertices be webpages.
PageRank graph has two types of directed edges:

1. Hyperlinks *weighted* (with probability α)
2. Random jump to any other vertex (with probability $1 - \alpha$)

Where $0 < \alpha < 1$ is called the **damping**

- The adjacency matrix *A*, for PageRank, is related to the weighted adjacency matrix of hyperlinks *H* and the matrix *R* with all ones.

$$A = \alpha H + (1 - \alpha) R$$

- Since every vertex is connected to every other, the graph is strongly connected and if there are 3 webpages aperiodic.
- The PageRank is the resulting Perron-Frobenius right eigenvector (Appropriately scaled).

Sources

Bruce P. Kitchens.
Symbolic dynamics.
Universitext. Springer-Verlag, Berlin, 1998.
One-sided, two-sided and countable state Markov shifts.

Lawrence Page, Sergey Brin, Rajeev Motwani, and Terry Winograd.
The pagerank citation ranking: Bringing order to the web.
Technical Report 1999-66, Stanford InfoLab, November 1999.
Previous number = SIDL-WP-1999-0120.