

Review Problem for Midterm #1

Midterm I: Wednesday, September 22 in class

Topics: 1.1, 1.3 and 2.1-2.6 (except 2.3)

1. Let $f(x) = -2x^2 + 3x + 1$. Find an equation of the tangent line to the curve at $P(1, f(1))$.
2. Let $f(x) = \frac{2}{x^2+1}$. Find an equation of the tangent line to the curve at $P(1, f(1))$.
3. Let $f(x) = \frac{2}{\sqrt{x^2+3}}$. Find an equation of the tangent line to the curve at $P(1, f(1))$.
4. A piecewise defined function is given by

$$f(x) = \begin{cases} -x - 1, & x < -1 \\ x^2 - 1, & -1 \leq x < 2 \\ x + 2, & 2 \leq x \end{cases}$$

Determine if f is left continuous, right continuous or continuous at $x = -1$ or $x = 2$.

5. Classify the discontinuity of the following functions (removable, infinite, jump or oscillating discontinuity). Redefine the value of the function if it's removable.

(a) $f(x) = \frac{x-3}{x^2-4x+3}$

(b)

$$f(x) = \begin{cases} \frac{x+1}{x^2-1}, & x < -1 \\ \frac{x+1}{8}, & -1 \leq x \leq 1 \\ \frac{\sqrt{x}-1}{x^2-1}, & 1 < x \end{cases}$$

6. A piecewise defined function is given by

$$f(x) = \begin{cases} x - 1, & x < -1 \\ ax + b, & -1 \leq x < 1 \\ x^2, & 1 \leq x \end{cases}$$

- (a) Find the graph of $y = f(x)$ over the interval $(-\infty, -1) \cup [2, \infty)$.
- (b) Determine the value of a and b so that f is continuous everywhere. Also explain your answer geometrically.

7. Determine the following limits

(a) $\lim_{x \rightarrow 1} \frac{x-1}{x^2-3x+2}$

(b) $\lim_{x \rightarrow 2^+} \frac{x-4}{x^2-5x+6}$, $\lim_{x \rightarrow 2^-} \frac{x-4}{x^2-5x+6}$, $\lim_{x \rightarrow 2} \frac{x-4}{x^2-5x+6}$

(c) $\lim_{x \rightarrow \infty} \frac{-2x^2+x^6+1}{x^3-5x+6}$, $\lim_{x \rightarrow -\infty} \frac{-2x^2+x^6+1}{x^3-5x+6}$

(d) $\lim_{x \rightarrow \infty} \frac{-2x^6+x^2+1}{4x-5x+6x^6}$, $\lim_{x \rightarrow -\infty} \frac{-2x^6+x^2+1}{4x-5x+6x^6}$

(e) $\lim_{x \rightarrow \infty} \frac{-2x^6+x^2+1}{4x^3-5x^8+6}$, $\lim_{x \rightarrow -\infty} \frac{-2x^6+x^2+1}{4x^8-5x+6}$

(f) $\lim_{x \rightarrow \infty} x^2 - \frac{x^4+1}{x^2+1}$

(g) $\lim_{x \rightarrow \infty} \sqrt{x^2+1} - \sqrt{x^2-1}$

(h) $\lim_{x \rightarrow \infty} \sqrt{x^2+1} - \sqrt{x-1}$

8. Find the domain of the following functions and determine the vertical and horizontal asymptotes of the graph of the following functions.

(a) $f(x) = \frac{x-1}{x^2-3x+2}$

(b) $f(x) = \frac{x-1}{x^2-5x+6}$

(c) $f(x) = \frac{x^3-1}{x^2-5x+6}$

(d) $f(x) = \frac{-x^2+1}{x^2-5x+6}$