(Math 2890) Review Problems II

Midterm II: March 31 at UH 1000 (Newton Lab)
You should practice using Maple to do the row reduction. Goto virtual lab: http://www.utoledo.edu/it/vlab/. Then Windows Basic Access Login .
Topics: 2.1-2.3, 2.8-2.9, 6.1-6.4 and the materials discussed in class.
Office hours before the midterm:
Monday (March 29) 12-2 pm Wednesday (March 31) 12-2 p.m and 4-5 p.m

1. (a) What is a subspace in R^n?
   (b) Is the set $\{(x, y, z)|x + y + z = 1\}$ a subspace?
   (c) Is the set $\{(x, y, z)|x - y - z = 0, x + y - z = 0\}$ a subspace?
   (d) What is a basis for a subspace?
   (e) What is the dimension of a subspace?
   (f) What is the column space of a matrix?
   (g) What is the null space of a matrix?
   (h) What is the subspace spanned by the vectors $v_1, v_2, \cdots, v_p$?

2. Find the inverses of the following matrices if they exist.
$$A = \begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 1 \\ -1 & 0 & -1 \end{bmatrix},\ C = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}.$$

3. (a) Let A be an 3×3 matrix. Suppose $A^3 + 2A^2 - 3A + 4I = 0$. Is A invertible? Express A^{-1} in terms of A if possible.
   (b) Suppose $A^3 = 0$. Is A invertible?

4. Find all values of a and b so that the subspace of $\mathbb{R}^4$ spanned by $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} b \\ 1 \\ -a \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ is two-dimensional.

5. Let $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \}$. You can assume that $\mathcal{B}$ is a basis for R^3

(a) Which vector x has the coordinate vector $[x]_B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

(b) Find the β-coordinate vector of $y = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$.

6. Let
$$M = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 1 & 2 & 5 & 1 \\ 1 & 3 & 7 & 2 \end{bmatrix}$$
(a) Find bases for $Col(M)$ and $Nul(M)$, and then state the dimensions of these subspaces.
(b) Express the third column vector A as a linear combination of the basis of $Col(M)$.

7. Find a basis for the subspace spanned by the following vectors $\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \}$.

What is the dimension of the subspace?

8. Determine which sets in the following are bases for $\mathbb{R}^2$ or $\mathbb{R}^3$. Justify your answer

(a) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \end{bmatrix}$. (b) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$. (c) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(d) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. (e) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

9. Find an orthogonal basis for the column space of the following matrices.

(a) $\begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -1 \\ 1 & 2 & 4 \end{bmatrix}$. (b) $\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$

10. (a) Let $W = Span\{u_1, u_2\}$ where $u_1 = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$. Find an orthogonal basis for W.

(b)Find the closest point to $y = \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix}$ in the subspace W.

(c) Find the distance between the point y and the subspace W.