(Math 2890) Review Problems II

Topics: 2.1-2.3, 2.8-2.9, 5.1-5.3, 6.1-6.3 and the materials discussed in class.

Office hours before the midterm: Monday (Nov 2) 2-3:30 pm
Wednesday (Nov 4) 11-12, 3-4 p.m and Friday (Nov 6) 10-12, 2-2:30 p.m

1. (a) What is a subspace in R^n?
(b) Is the set $\{(x, y, z)|x + y + z = 1\}$ a subspace?
(c) Is the set $\{(x, y, z)|x - y - z = 0, x + y - z = 0\}$ a subspace?
(d) What is a basis for a subspace?
(e) What is the dimension of a subspace?
(f) What is the column space of a matrix?
(g) What is the null space of a matrix?
(h) What is an eigenvalue of a matrix A?
(i) What is an eigenvector of a matrix A?
(j) What is the characteristic polynomial of a matrix A ?
(k) What is the subspace spanned by the vectors $v_1, v_2, \cdots, v_p$?

2. Find the inverses of the following matrices if they exist.
$$A = \begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 1 \\ -1 & 0 & -1 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}.$$

3. (a) Let A be an 3×3 matrix. Suppose $A^3 + 2A^2 - 3A + 4I = 0$. Is A invertible? Express A^{-1} in terms of A if possible.
(b) Suppose $A^3 = 0$. Is A invertible?

4. Find all values of a and b so that the subspace of $\mathbb{R}^4$ spanned by $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} b \\ 1 \\ -a \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ is two-dimensional.

5. Let $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \}$. You can assume that $\mathcal{B}$ is a basis for R^3

(a) Which vector x has the coordinate vector $[x]_B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

(b) Find the β-coordinate vector of $y = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$.

6. Let

$$M = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 1 & 2 & 5 & 1 \\ 1 & 3 & 7 & 2 \end{bmatrix}$$

(a) Find bases for $Col(M)$ and $Nul(M)$, and then state the dimensions of these subspaces.
(b) Express the third column vector A as a linear combination of the basis of $Col(M)$.

7. Find a basis for the subspace spanned by the following vectors $\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \}$.

What is the dimension of the subspace?

8. Determine which sets in the following are bases for $\mathbb{R}^2$ or $\mathbb{R}^3$. Justify your answer

(a) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \end{bmatrix}$. (b) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$. (c) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(d) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. (e) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

9. Let A be the matrix

$$A = \begin{bmatrix} -3 & -4 \\ -4 & 3 \end{bmatrix}.$$

Find a polynomial $f(A)$ in A such that $f(A) = 0$. Verify your answer.

10. Let A be the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

(a) Prove that $det(A - \lambda I) = -(\lambda - 1)^2(\lambda - 4)$.
(b) Find the eigenvalues and a basis of eigenvectors for A.
(c) Diagonalize the matrix A if possible.
(d) Find an expression for A^k. (e) Find an expression for the matrix exponential e^A.

11. Let B be the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find the characteristic equation of B.
(b) Find the eigenvalues and a basis of eigenvectors for B.
(c) Diagonalize the matrix B if possible.

12. Find an basis for $W^{\perp}$ for the following W. Verify your answer.

(a) $W = Span\{\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}\}$.

(b)$W = Span\{\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}\}$.

13. (a) Let $W = Span\{u_1, u_2\}$ where $u_1 = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$. Show that $\{u_1, u_2\}$ is an orthogonal basis for W.

(b)Find the closest point to $y = \begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix}$ in the subspace W.

(c) Find the distance between the point y and the subspace W.