MATH3860 - Elementary Differential Equations, Spring 2014

Quiz 5

April, 2 2014

Printed NAME

- You have 10 min to complete your quiz.
- Please show all your work neatly and indicate your final answers clearly If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem
- The quiz is closed book and notes **Calculators are not allowed**

GOOD LUCK :)

1 (10 points) Compute the solution of the following initial value problem

$$y'' + y' - 2y = e^t, \quad y(0) = 0, \quad y'(0) = 0$$

Associated homogeneous ODE $y'' + y' - 2y = 0$ (1)

General solution of (1): $r^2 + r - 2 = 0 \Rightarrow (r+1)(r+2) = 0$

$r_1 = 1, r_2 = -2$

$y_c = c_1 e^t + c_2 e^{-2t}$

Using undetermined coefficient method for finding a solution of $y'' + 2y' - 2y = e^t$:

$y_p = Ate^t$ since there is duplication!

Computing A: $y_p' = Ae^t(1+t)$; $y_p'' = Ae^t(1+t) + Ae^t$

substitute into the ODE:

$Ae^t(2+t) + Ae^t(1+t) - 2Ate^t = e^t \quad (e^t \neq 0)$

$A(2 + \cancel{t} + 1 + \cancel{t} - \cancel{2t}) = 1 \Rightarrow A = \frac{1}{3}$

G. solution of $y'' + 2y' - 2y = e^t$:

$y = c_1 e^t + c_2 e^{-2t} + \frac{1}{3} te^t$

Applying IC

$y(0) = c_1 + c_2 = 0$

$y' = c_1 e^t - 2c_2 e^{-2t} + \frac{e^t}{3}(1+t) \quad | \quad y'(0) = c_1 - 2c_2 + \frac{1}{3}$

$\longrightarrow$ tpp.

$$\begin{cases} c_1 + c_2 = 0 \\ c_1 - 2c_2 = -\frac{1}{3} \end{cases} \quad \begin{cases} c_1 = -\frac{1}{9} \\ c_2 = \frac{1}{9} \end{cases}$$

solution of IVP

$$y = \frac{1}{9} e^{-2t} - \frac{1}{9} e^{t} + \frac{1}{3} t e^{t}$$

$$y = \frac{1}{9} \left[e^{-2t} + e^{t}(3t - 1) \right]$$