MATH3860 - Elementary Differential Equations, Spring 2014
Quiz 4
March, 10 2014

Printed NAME:

- You have 15 min to complete your quiz.

- Please show all your work neatly and indicate your final answers clearly. If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem.

- The quiz is closed book and notes. **Calculators are not allowed**.

GOOD LUCK :)

1. (10 points) If the Wronskian of f and g is $3e^{4t}$ and $f(t) = e^{2t}$, determine $g(t)$.
   Hint: Solve the ode resulting from computing the Wronskian.

$$W = \begin{vmatrix} e^{2t} & g \\ 2e^{2t} & g' \end{vmatrix} = g' e^{2t} - 2e^{2t}g = 3e^{4t} \Rightarrow g' - 2g = 3e^{2t}$$

integrating factor: e^{-2t}

$$\frac{d}{dt}\left(e^{-2t}g\right) = 3 \Rightarrow e^{-2t}g = 3t + c$$

$$e^{-2t}g' - 2e^{-2t}g = 3$$

$$g = 3te^{2t} + ce^{2t}$$

2. (5 points) Consider the differential equation

$$y'' - 2y' + 5y = 0.$$

(a) Write its characteristic equation.

$$R^2 - 2R + 5 = 0$$

(b) Give a set of fundamental solutions for the ODE.

$$R^2 - 2R + 5 = (R-1)^2 + 4 = 0 \Rightarrow R = 1 \pm 2i$$

linearly independent solutions:

$$y_1 = e^x \cos 2x \quad ; \quad y_2 = e^x \sin 2x$$

A set of fundamental solutions:

$$\{ e^x \cos 2x, \ e^x \sin 2x \}$$