

MATH2860 - Elementary Differential Equations, Spring 2014

Quiz 2 - solutions

Feb 10, 2014

Printed NAME:

- You have 15 min to complete your quiz.
- Please show all your work neatly and indicate your final answers clearly. If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem.
- The quiz is closed book and notes. Calculators are not allowed.

GOOD LUCK :)

- 1 Determine the behavior of the solution of the following ordinary differential equation when $t \rightarrow \infty$ by solving it first

$$y' + 2ty = 2te^{-t^2}$$

2. Solve the exact equation

$$9x^2 + y + 1 - (4y - x)y' = 0 \Rightarrow (9x^2 + y + 1)dx + (-4y + x)dy = 0$$

1) $\mu(t) = e^{\int 2t dt} = e^{t^2}$

$$\frac{d}{dt} (e^{t^2} y) = 2t e^{t^2} e^{t^2} = 2t e^{2t^2}$$

integrating both sides $e^{t^2} y = \int 2t e^{2t^2} dt + C \Rightarrow y = e^{-t^2} (t^2 + C)$

solution of the DE: $3x^3 + yx + x - 2y^2 = C$

(2) Look for $\psi(x,y)$ such that $\frac{\partial f}{\partial x} = 9x^2 + y + 1$ and $\frac{\partial f}{\partial y} = -4y + x$

integrating the first equality (a) with respect to x we have

$$\psi(x,y) = 3x^3 + yx + x + h(y) \quad (1)$$

now using (1) and the second equality (b) we obtain:

$$\frac{\partial}{\partial y} (3x^3 + yx + x + h(y)) = -4y + x \Rightarrow x + h'(y) = -4y + x$$

$$h(y) = -2y^2; \quad \psi(x,y) = 3x^3 + yx + x - 2y^2$$