MATH3860 Elementary Differential Equations, Spring 2013
Second Exam, April 10, 2013

NAME: ID:

- You have 50 minutes to complete your exam.

- Please show all your work neatly and concisely, and indicate your final answers clearly. **If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem.**

- The exam is closed book and notes. Calculators are not allowed. The table of Laplace transforms is provided to you.

GOOD LUCK :)

Problem	Maximum	Score
1	5	
2	6	
3	15	
4	6	
5	5	
6	6	
7	10	
8	8	
Total	61	

1. (6 points) Given the differential equation

$$y'' + 6y' + 9y = 0, \tag{1}$$

  (a) (2 points) Determine the characteristic equation and two linearly independent solutions.

  (b) (3 points) Solve the initial value problem consisting of equation (1) and initial conditions: $y(0) = 1$, $y'(0) = 1$.

2. (6 points) Determine the real-valued general solution of the following differential equations:

$$y'' + 4y' + 8y = 0.$$

3. (15 points) Consider the differential equation

$$y'' - 4y' = 5x + 3xe^{4x} + \cos x \tag{2}$$

(a) (2 points) Write its associated homogeneous differential equation.

(b) (3 points) Determine the general solution of the differential equation written in (a).

(c) (4 points) Using undetermined coefficients method determine a solution of

$$y'' - 4y' = \cos x$$

(d) (3 points) Solve the initial value problem

$$y'' - 4y' = \cos x; \qquad y(0) = 0 \text{ and } y'(0) = -\frac{4}{17}.$$

(e) (6 points) Determine the *form* of the general solution of equation (2).

4. (6 points) Use variation of parameter method to determine a particular solution for the differential equation $y'' - 4y = 2e^{3t}$.

5. (5 points) Use the definition of Laplace transform to compute $F(s)$, where
$$f(t) = \begin{cases} 0 & 0 < t \leq 1 \\ 1 & t > 1. \end{cases}$$

6. (6 points) Compute the Laplace transforms of each of the following functions

   (a) (3 points) $2t^2 - t^2 e^t + 2 - e^{-t} + 2\cos(3t) + \sin(6t)$.

   (b) (3 points) $2e^{3t}t$, using s-translation property.

7. (10 points) Compute the inverse Laplace transform of each of the following:

   If you need to use partial fraction decomposition, compute the inverse without determining the values of the coefficients

   (a) (5 points) $\dfrac{2s+1}{s^2(s-1)}$.

   (b) (5 points) $\dfrac{5s+2}{s^2-2s+5}$.

8. (8 points) Solve the following initial value problem

$$y' - \int_0^t y(\tau)d\tau = 1, \qquad y(0) = -1$$

Scratch Paper

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