

MATH3860 Elementary Differential Equations, Spring 2013
First Exam

February 18, 2013

NAME:

ID:

- You have 50 minutes to complete your exam.
- Please show all your work neatly and concisely, and indicate your final answers clearly. **If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem.**
- The exam is closed book and notes. Calculators are not allowed.

GOOD LUCK :)

Problem	Maximum	Score
1	4	
2	6	
3	8	
4	6	
5	6	
6	12	
7	9	
Total	51	

1. (4 points) In each part below, determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

(a) (2 points) $y''' + ty' + (\cos^2 y)t = t^3$.

(b) (2 points) $y' + t^2 = y$.

2. (6 points) Find a suitable integrating factor that could be used to solve the equation below.

$$\frac{1}{t^3} \frac{dy}{dt} + \frac{e^t}{t^2} y = \sin t \quad t > 0.$$

3. (8 points) The population p of bacteria in a certain culture grows according to the following differential equation

$$p' - 2p^2 = tp^2. \quad (1)$$

- (a) (5 points) Discuss the existence of singular solutions and determine the general solution of (1).

- (b) (3 points) Let $p(x)$ be the solution to the initial value problem consisting of (1) and the initial condition $p(0) = 1$. Determine $p(1)$.

4. (6 points) Consider the solution $y(t)$ of the initial value problem

$$\frac{dy}{dt} - \frac{2}{t}y = 2, \quad t > 0; \quad y(1) = 2.$$

What is $\lim_{t \rightarrow +\infty} y(t)$.

5. (6 points) Consider the initial value problem

$$\cos y \frac{dy}{dx} = \sin y, \quad y(1) = \frac{\pi}{2}$$

Can we guarantee that the initial value problem has a solution? Justify your answer.

6. (12 points)

- (a) (4 points) Consider the differential equation

$$(2\alpha y^2 - 2ye^{2x} + 4) + (2xy - e^{2x} - 3\alpha y^2)y' = 0.$$

Find the value α , if any, that would make this equation an exact equation.

- (b) (8 points) Given that the differential equation

$$(6x^2y^4 + y \sec^2(xy) + 2x) + (8x^3y^3 + x \sec^2(xy) - 1)y' = 0$$

is an exact equation, find the solution of the equation that also satisfies the initial value $y(0) = 6$. You may leave your answer in an implicit form.

7. (9 points) Consider the following second order linear differential equation

$$16y'' + 8y' + y = 0 \tag{2}$$

- (a) (4 points) Check if $y_1(t) = e^{-\frac{1}{4}t}$ solves equation (2).

- (b) (5 points) Determine the general solution of (2).

Scratch Paper

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