

MATH2850 - Elementary Multivariable Calculus, Spring 2014

Quiz 9 - Sol.

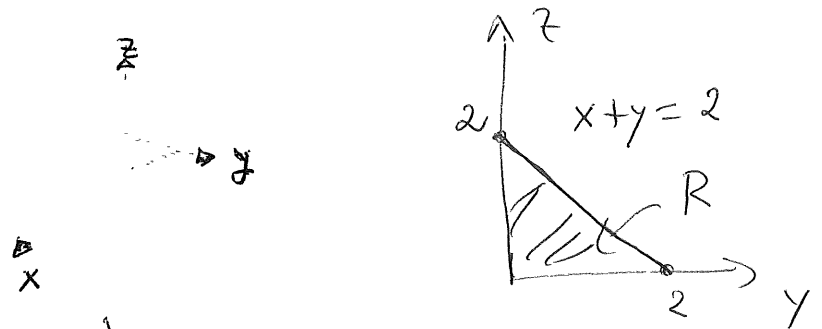
April 1, 2014

Printed NAME:

- You have 10 min to complete your quiz.
- Please show all your work neatly and indicate your final answers clearly. If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem.
- The quiz is closed book and notes. **Calculators are not allowed.**

GOOD LUCK :)

1. Compute the volume of the regions in the first octant bounded by the coordinate planes, the $y+z=2$, and the cylinder $x=4-y^2$ (see figure below.)



One way of solve it

$$\iint_R \int_{x=0}^{x=4-y^2} dx dA = \int_0^2 \int_0^{2-z} \int_0^{4-y^2} dx dy dz$$

$$= \int_0^2 \int_0^{2-z} (4-y^2 - 0) dy dz = \int_0^2 \left[4y - \frac{y^3}{3} \right]_0^{2-z} dz$$

$$= \int_0^2 \left[4(2-z) - \frac{1}{3}(2-z)^3 \right] dz = \left[-4 \frac{(2-z)^2}{2} + \frac{1}{12} (2-z)^4 \right]_0^2$$

$$= \left[-2(2-z)^2 + \frac{(2-z)^4}{12} \right]_0^2 = -2(0 - 2^2) + \frac{1}{12} (0 - 2^4)$$

$$= 8 - \frac{16}{12} = 8 - \frac{4}{3} = \frac{20}{3}$$