MATH2850 - Elementary Multivariable Calculus, Spring 2014
Quiz 8
March 25, 2014

Printed NAME:

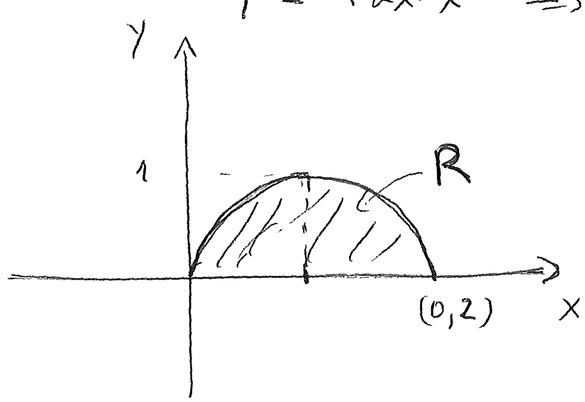
- You have 10 min to complete your quiz.
- Please show all your work neatly and indicate your final answers clearly. If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem.
- The quiz is closed book and notes. **Calculators are not allowed.**

GOOD LUCK :)

1. Evaluate the iterated integral by converting it into polar cordinates

$$(1) \quad \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx.$$

$y = \sqrt{2x-x^2} \Rightarrow y^2 + x^2 - 2x = 0, \; y > 0 \Rightarrow y^2 + (x-1)^2 = 1, \; y > 0$



polar coordinates

$x^2+y^2 = R^2, \quad x = R\cos\theta$

$y = R\sin\theta$

Therefore, $y = \sqrt{2x-x^2}$ becomes

$R^2\sin^2\theta = 2R\cos\theta - R^2\cos^2\theta$

$R^2(\sin^2\theta + \cos^2\theta) = 2R\cos\theta; \; R \neq 0$

$R = 2\cos\theta$

The half disk R in polar coordinates is given by

R: $0 \leq R \leq 2\cos\theta \; ; \; 0 \leq \theta \leq \pi/2$

and (1) is given by

$$\int_0^{\pi/2}\int_0^{2\cos\theta} R\,R\,dR\,d\theta = \int_0^{\pi/2} \frac{R^3}{3}\Big|_0^{2\cos\theta} = \frac{1}{3}\int_0^{\pi/2} 8\cos^3\theta\,d\theta$$

$$= \frac{8}{3}\int^{\pi/2} \cos\theta(1-\sin^2\theta)\,d\theta = \frac{8}{3}\int_0^{\pi/2} \cos\theta - \cos\theta\sin^2\theta \, d\theta =$$

$$= \frac{8}{3}\left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\pi/2} \therefore \frac{8}{3}\left(1 - \frac{1}{3}\right) - 0$$

$$= \frac{16}{9}$$