2. **Extra Credit** (7 points)
Determine three numbers whose sum is 9 and whose sum of squares is minimum.

1

constrain $x+y+z=9 \Rightarrow g(x,y,z) = x+y+z-9 = 0$

Objective function. $x^2+y^2+z^2 = f(x,y,z)$

solve constraint for z (for example)

$z = 9 - x - y$

plug into $f(x,y,z) = x^2+y^2+(9-x-y)^2$

$f_x = 2x + 2(9-x-y)(-1) = -18+4x+2y$

$f_y = 2y + 2(9-x-y)(-1) = -18+4y+2x$

solve $f_x = 0 = f_y$

$$\begin{cases} -18+4x+2y=0 & (i) \\ -18+4y+2x=0 & (ii) \end{cases} \quad \begin{matrix} -18+4x+2y=0 \\ 18-6x=0 \end{matrix} \quad \Big\} \; x=3 \quad \Big\} \; y=3$$

multiply (i) by -2 and sum it to (ii)

$x=y=3$, then from (1) we get $z=3$

Point: $(x,y,z)=(3,3,3)$ only critical point since f, f_x, f_y continuous for all $\mathbb{R}$

2nd Derivative Test

$f_{xx} = 2$, $f_{yy}=2$, $f_{xy}=2$

Hessian $= f_{xx}f_{yy} - f_{xy}^2 = 4-2>0$

$f_{xx}>0$ and Hessian >0 $\Rightarrow$ $f(3,3,3)$ is a minimum

$f(3,3,3) = 27$ is the minimum; the numbers are: $x=y=z=3$

→ You could use also Lagrange method!!!

MATH2850 - Elementary Multivariable Calculus, Spring 2014
Quiz 6
March 11, 2014

Printed NAME:

- You have 15 min to complete your quiz.
- Please show all your work neatly and indicate your final answers clearly. If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem.
- The quiz is closed book and notes. **Calculators are not allowed.**

GOOD LUCK :)

1. (10 points) Using the Lagrange multipliers method determine the maximum value of the function

$$f(x,y) = x - y + z \quad (3)$$

subject to

$$x^2 + y^2 + z^2 = 1 \ (1) \quad \text{and} \quad z + x + y = 1. \ (2)$$

let $g(x,y,t) = x^2+y^2+z^2-1=0$ $\quad h(x,y,z) = z+x+y-1=0$

$\nabla f = \langle 1,-1,1\rangle$, $\nabla g = \langle 2x, 2y, 2z\rangle$, $\nabla h = \langle 1,1,1\rangle$

Lagrange multipliers satisfy: $\nabla f = \lambda_1 \nabla g + \lambda_2 \nabla h$

$$\Rightarrow \begin{cases} 1 = 2\lambda_1 x + \lambda_2 & (i) \\ -1 = 2\lambda_1 y + \lambda_2 & (ii) \\ 1 = 2\lambda_1 z + \lambda_2 & (iii) \end{cases}$$

from here follows that $\boxed{x = z}$ (4)

substitute (4) into (1) and (2) leads:

$$\begin{cases} 2x^2 + y^2 = 1 \\ 2x + y = 1 \end{cases} \quad \begin{cases} \text{---} \\ y = 1-2x \end{cases} \quad \begin{cases} \cancel{2(\ldots)^2 + \ldots} \\ 2x^2 + (1-2x)^2 = 1 \end{cases} \quad \begin{cases} 2x^2 + 1 - 4x + 4x^2 = 1 \\ y = 1-2x \end{cases}$$

$$\begin{cases} \overset{3}{\cancel{6}}x^2 - \overset{2}{\cancel{4}}x = 0 \end{cases} \quad \begin{cases} x = 0 \\ y = 1 \end{cases} \quad \text{or} \quad \begin{cases} x = \frac{2}{3} \\ y = -1/3 \end{cases}$$

$x = z$

$P_1(0, 1, 0)$ and $P_2\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$

$f(P_1) = -1$ $\quad \boxed{f(P_2) = \frac{2}{3} + \frac{1}{3} + \frac{2}{3} = \frac{5}{3}}$ ← maximum of f