Practice Ex 1 set 2

Problem 1. 10pt Find the tangent line to the curve $r(t) = (t, \arctan(t), \cos \pi t)$ at the point $(1, \frac{\pi}{4}, -1)$

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Problem 2. 10pt Evaluate the following limits if they exist Otherwise prove that they do not exist

a. 5pt
$$\lim_{(x,y)\to(1,0)}\cos\left(\frac{x}{x-y}\right)$$

$$\left[\mathbf{b.5pt} \right] \lim_{(y,y)\to(0,0)} \frac{x^4}{x^4 + y^2}$$

Problem 3. 10pt Let $x^2 + xy^3 - z^4 - xz^3 = 1$ Find $\frac{\partial x}{\partial z}$ at (1,0,0)

Problem 4. 10pt Let $F(x) = \int_{\sin x}^{\cos x} \sqrt{x^2 + t^3} dt$. Find F'(0).

Problem 5. 10pt Let $F(x, y) = x^2 \cos y - y^2 \sin x$, $x = u + \cos v$, $y = u + \sin v$. Find $\frac{\partial^2 F}{\partial u \partial v} \Big|_{(u, v) = (1, 0)}$

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Problem 6. 10pt Let $f(x,y) = xy^2 + x^2 \cos y$ denote height of a hill and p = (1,1). Find the direction in which the height does not change at p and direction in which the height increases the most at the point p