PART I MULTIPLE CHOICE QUESTIONS 64 POINTS

1. Find the indefinite integral $\int \cos^7 x \ \sin x \ dx$. Select the correct answer.

   (A) $\frac{1}{8}\cos^8 x + C$ (B) $-\frac{1}{8}\sin^8 x + C$ (C) $-\frac{1}{8}\cos^7 x + C$

   (D) $\frac{1}{8}\sin^8 x + C$ (E) $-\frac{1}{8}\cos^8 x + C$ (F) None of the above

2. Find the value of the integral $\int_0^{\pi/4} \sec^2 x \ dx$.

   (A) $1/2$ (B) $\pi/4$ (C) $3/4$ (D) $3\pi/4$

   (E) $\pi/2$ (F) $1/4$ (G) 1 (H) 0

   (I) None of the above

3. Find the value of the integral $\int_0^1 \frac{1}{1+x^2} \ dx$.

   (A) 1 (B) $3/4$ (C) $\pi/2$ (D) $3\pi/4$

   (E) $1/4$ (F) 0 (G) $\pi/4$ (H) $1/2$

   (I) None of the above

4. In the partial fraction decomposition $\frac{x}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$ find the value of A.

(A) $-3/4$ (B) 1 (C) -1 (D) $1/2$

(E) 2 (F) -2 (G) $3/4$ (H) $-1/2$

(I) None of the above

5. Find the slope of the tangent to the curve $x = t^3,\ y = t^4$ when $t = 3$.

(A) $9/4$ (B) $3/16$ (C) $4/9$ (D) $4/3$

(E) 3 (F) $3/4$ (G) $16/3$ (H) 4

(I) None of the above

6. Give an integral representing the length of the parametric curve $x = t^3,\ y = t^4,\ 0 \le t \le 1$.

(A) $\int_0^1 (t^3 + t^4)\ dt$ (B) $\int_0^1 \sqrt{t^3 + t^4}\ dt$ (C) $\int_0^1 \sqrt{1 + 3t^2}\ dt$

(D) $\int_0^1 \sqrt{t^2 + 4t^3}\ dt$ (E) $\int_0^1 \sqrt{9t^4 + 16t^6}\ dt$ (F) $\int_0^1 \sqrt{4t^4 + 9t^6}\ dt$

(G) $\int_0^1 \sqrt{t^5 + t^7}\ dt$ (H) $\int_0^1 \sqrt{8t^6 + 6t^8}\ dt$ (I) None of the above

7. Convert the polar coordinates $(3, 5\pi)$ to Cartesian coordinates.

(A) $(-\sqrt{3}/2, -\sqrt{3}/2)$ (B) $(3, 0)$ (C) $(0, 3)$

(D) $(\sqrt{3}/2, \sqrt{3}/2)$ (E) $(-3, 0)$ (F) $(\sqrt{3}/2, -\sqrt{3}/2)$

(G) $(0, -3)$ (H) $(\sqrt{3}/2, -\sqrt{3}/2)$ (I) None of the above

8. Convert the Cartesian coordinates $(1, 1)$ to polar coordinates.

(A) $(1, \pi/4)$ (B) $(\sqrt{2}, \pi/4)$ (C) $(\sqrt{2}, \pi/2)$

(D) $(1, 2\pi)$ (E) $(1, \pi)$ (F) $(\sqrt{2}, \pi)$

(G) $(1, \pi/2)$ (H) $(\sqrt{2}, 2\pi)$ (I) None of the above

9. Find the sum of the series $1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \cdots$.

(A) 7/4 (B) 11/6 (C) 3/2

(D) 4/3 (E) 5/3 (F) 7/6

(G) 5/4 (H) divergent (I) None of the above

10. Which of the three series below converge?

1) $\sum_{n=1}^{\infty} \frac{1}{n}$ 2)$\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$ 3)$\sum_{n=1}^{\infty} \frac{1}{n^{0.9}}$

(A) $1, 2$ (B) 1 (C) none (D) 3

(E) 2 (F) $2, 3$ (G) $1, 2, 3$ (H) $1, 3$

11. Which of the following three tests will establish that the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{2n^5+1}}$ converges?

1) Comparison test with $\sum_{n=1}^{\infty} n^{-5/2}$

2) Comparison test with $\sum_{n=1}^{\infty} n^{-3/2}$

3) Comparison test with $\sum_{n=1}^{\infty} n^{-1/2}$

(A) none (B) 1 (C) 2 (D) 3

(E) $1, 2$ (F) $1, 3$ (G) $2, 3$ (H) $1, 2, 3$

12. Find the terms in the Maclaurin series for the function $f(x) = e^{-x}$, as far as the term in x^3.

(A) $1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$ (B) $1 + x + \frac{1}{2}x^2 - \frac{1}{6}x^3$ (C) $1 - x + x^2 - x^3$

(D) $1 + x + x^2 + x^3$ (E) $1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3$ (F) $1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3$

(G) $-x + x^3$ (H) $x - x^3$ (I) None of the above

13. Find the radius of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{n^3 x^n}{3^n}$$

Select the correct answer.

(A) $R = \infty$ (B) $R = 0$ (C) $R = \frac{1}{3}$

(D) $R = 1$ (E) $R = 3$ (F) None of the above

14. Find the sum of the given vectors.

$$\langle -6, 1, 5 \rangle, \qquad \langle 1, 4, 9 \rangle$$

Select the correct answer.

(A) $\langle -5, 5, 14 \rangle$ (B) $\langle 5, -5, -14 \rangle$ (C) $\langle 5, -14, -5 \rangle$

(D) $\langle 5, -5, 14 \rangle$ (E) $\langle -14, -14, -5 \rangle$ (F) None of the above

15. Find the cross product $\mathbf{a} \times \mathbf{b}$, where $\mathbf{a} = \langle 3, 5, 1 \rangle$, $\mathbf{b} = \langle -5, 2, -2 \rangle$. Select the correct answer.

(A) $\mathbf{a} \times \mathbf{b} = \langle -10, -5, 6 \rangle$
(B) $\mathbf{a} \times \mathbf{b} = \langle -12, 1, 31 \rangle$
(C) $\mathbf{a} \times \mathbf{b} = \langle 2, -6, -25 \rangle$
(D) $\mathbf{a} \times \mathbf{b} = \langle -8, -11, -19 \rangle$
(E) $\mathbf{a} \times \mathbf{b} = \langle -16, -23, 25 \rangle$
(F) None of the above

16. Find the dot product $\mathbf{v} \cdot \mathbf{u}$ if $\mathbf{v} = \langle 2, 3, 6 \rangle$ and $\mathbf{u} = \langle -1, 2, -1 \rangle$.

(A) -14
(B) 14
(C) -2
(D) 2
(E) $\langle -2, 6, -6 \rangle$
(F) $\langle 2, -6, 6 \rangle$
(G) $\langle 2, 6, 6 \rangle$
(H) $\langle -2, -6, 6 \rangle$
(I) None of the above

PART II SHOW WORK 136 POINTS

INSTRUCTIONS. Show work. Give details. Show how the answer is obtained. Simplify answer.

1. (10 pts) Let R be the region bounded by $y = x^2$, $x = 2$, and the x-axis. Find the volume of the solid obtained by rotating R about the x-axis.

2. (20 pts) Evaluate the following integrals. Simply your answer.

(a) (10 pts) $\int \sin^3 x \cos^4 x \; dx$

(b) (10 pts) $\int xe^{2x} dx$

3. (10 pts) Compute the definite integral $\displaystyle\int_0^2 \frac{1}{\sqrt{16-x^2}}\,dx$.

4. (10 pts) Evaluate $\int \frac{1}{x\left(x^2+1\right)} dx$ using partial fractions.

5. (10 pts) Find the length of the curve $y = x^{3/2}$ on the interval $[1, 3]$.

6. (10 pts) Determine convergence or divergence of $\sum_{n=2}^{\infty} \frac{2n+1}{n-1}$. Show how you obtain your answer, and state the test that you use.

7. (20 pts) Determine convergence or divergence of $\sum_{n=1}^{\infty} \frac{10^n}{n^n}$ by each of the following tests:

(a) (10 pts) root test.

(b) (10 pts) ratio test.

8. (10 pts) Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}.$$

9. (10 pts) Find the area of the region bounded by the polar curve $r = \sqrt{\cos\theta}$ and that lies in the sector $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

10. (8 pts) Find the parametric equations for the line that passes through the points $(1, -1, 2)$ and $(2, 1, 5)$.

11. (10 pts) Find an equation of the plane which contains the points $P(1,1,1)$, $Q(2,2,1)$ and $R(1,3,3)$.

12. (8 pts) Find the Taylor series centered at $a = 0$ for $f(x) = \ln(1 - x^2)$.