

INSTRUCTIONS: You must show enough work to justify your answer on **ALL** problems except for Problem 16. Correct answers with no work (or inconsistent work) shown **will not** receive full credit. **All answers are to be exact; no decimal approximations.** You are **NOT** allowed to use any electronic device for this pre-exam.

1. For each of the following angles, indicate where the terminal side of the angle is located. Then find the reference angle and the **exact** value of the indicated trigonometric function for the angle. (5 pts. each)

a. $\alpha = \frac{4\pi}{3}$ Location of $\frac{4\pi}{3}$ _____ $\alpha' =$ _____ $\sin\left(\frac{4\pi}{3}\right) =$ _____

The solution to the first part of this problem is at the bottom of **Pre-Class Problems 1**.

The solution to the second part of this problem is at the bottom of **Pre-Class Problems 2**.

The solution to the third part of this problem is at the bottom of **Pre-Class Problems 3**.

b. $\beta = 180^\circ$ Location of 180° _____ $\beta' =$ _____ $\tan 180^\circ =$ _____

The solution to the first part of this problem is at the bottom of **Pre-Class Problems 1**.

The solution to the second part of this problem is at the bottom of **Pre-Class Problems 2**.

The solution to the third part of this problem is at the bottom of **Pre-Class Problems 2**.

c. $\theta = -315^\circ$ Location of -315° _____ $\theta' =$ _____ $\csc(-315^\circ) =$ _____

The solution to the first part of this problem is at the bottom of **Pre-Class Problems 1**.

The solution to the second part of this problem is at the bottom of **Pre-Class Problems 2**.

The solution to the third part of this problem is at the bottom of **Pre-Class Problems 3**.

2. Find the angle between 0 and 2π that is coterminal with the angle $\frac{167\pi}{6}$ and then find the exact value of $\cot \frac{167\pi}{6}$. Show how you get your coterminal angle. (6 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 4](#).

Coterminal angle _____ $\cot \frac{167\pi}{6} =$ _____

3. Find the angle between -2π and 0 that is coterminal with the angle $-\frac{55\pi}{3}$ and then find the exact value of $\cos\left(-\frac{55\pi}{3}\right)$. Show how you get your coterminal angle. (6 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 4](#).

Coterminal angle _____ $\cos\left(-\frac{55\pi}{3}\right) =$ _____

4. Find the angle between 0° and 360° that is coterminal with the angle 810° and then find the exact value of $\sec 810^\circ$. Show how you get your coterminal angle. (4 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 4](#).

Coterminal angle _____ $\sec 810^\circ =$ _____

5. Find what quadrant the angle $\alpha = 290^\circ$ is in. Then find the reference angle of the angle. You need to show your work for finding the reference angle. (5 pts.)

The solution to the first part of this problem is at the bottom of [Pre-Class Problems 1](#).

The solution to the second part of this problem is at the bottom of [Pre-Class Problems 2](#).

The angle $\alpha = 290^\circ$ is in the _____ quadrant and the reference angle is _____.

6. Find what quadrant the angle $\theta = -\frac{11\pi}{14}$ is in. Then find the reference angle of the angle. You need to show your work for finding the reference angle. (5 pts.)

The solution to the first part of this problem is at the bottom of [Pre-Class Problems 1](#).

The solution to the second part of this problem is at the bottom of [Pre-Class Problems 2](#).

The angle $\theta = -\frac{11\pi}{14}$ is in the _____ quadrant and the reference angle is _____.

7. Convert the following angles to degrees if it is given in radians or to radians if it is given in degrees. (5 pts. each)

a. $\theta = \frac{8\pi}{15}$

The solution to this problem is at the bottom of [Pre-Class Problems 1](#).

Answer _____

b. $\beta = -165^\circ$

The solution to this problem is at the bottom of [Pre-Class Problems 1](#).

Answer _____

c. $\alpha = 4$

The solution to this problem is at the bottom of [Pre-Class Problems 1](#).

Answer _____

8. If $\tan \theta < 0$ and $\sec \theta < 0$, then θ lies in which quadrant? (5 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 6](#).

Answer _____

9. If the terminal side of α passes through the point $(4, -7)$, then find the exact value of $\sec \alpha$ and $\tan \alpha$. (8 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 5](#).

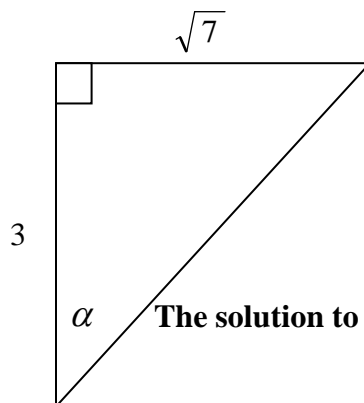
$\sec \alpha =$ _____ $\tan \alpha =$ _____

10. If the terminal side of β is in the III quadrant and lies on the line $15x - 6y = 0$, find the exact value of $\sin \beta$ and $\cot \beta$. (10 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 5](#).

$\sin \beta =$ _____ $\cot \beta =$ _____

11. Given:



Find the exact value of

a. $\cos \alpha =$ _____ (3 pts.)

b. $\csc \alpha =$ _____ (3 pts.)

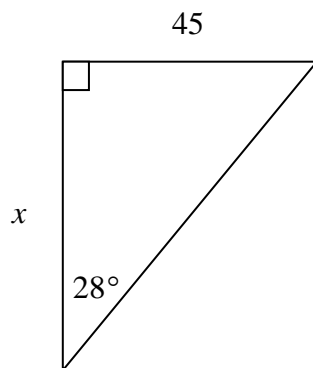
The solution to this problem is at the bottom of [Pre-Class Problems 6](#).

12. Use a **right triangle** to find the exact value of $\sec \beta$ and $\tan \beta$ if $\sin \beta = -\frac{\sqrt{5}}{4}$ and $\cos \beta > 0$. (12 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 7](#).

β lies in the _____ quadrant $\sec \beta =$ _____ $\tan \beta =$ _____

13. Given the triangle below, find x . Set up an equation and solve. (4 pts.)



The solution to this problem is at the bottom of [Pre-Class Problems 8](#).

$x =$ _____

14. Find the exact angle θ , where $0^\circ \leq \theta < 360^\circ$, if the terminal side of θ passes through the point $(-6, -14)$. (5 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 14](#).

$\theta =$ _____

15. From the top of a building, which is 80 meters tall, the angle of depression to an object on level ground below is 15.7° . How far is the object from the top of the building? Draw a picture and label known information. Indicate any variable you use. Set up an equation and solve. (6 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 8](#).

Answer _____

16. Find the exact value of each of the following. (3 pts. each)

a. $\sin^{-1} 1 =$ _____

The solution to this problem is at the bottom of [Pre-Class Problems 13](#).

b. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ _____

The solution to this problem is at the bottom of [Pre-Class Problems 14](#).

c. $\text{Arcsin}\left(-\frac{\sqrt{3}}{2}\right) =$ _____

The solution to this problem is at the bottom of [Pre-Class Problems 13](#).

d. $\tan^{-1}(-1) =$ _____

The solution to this problem is at the bottom of [Pre-Class Problems 13](#).

e. $\text{Arc cos } \frac{1}{2} =$ _____

The solution to this problem is at the bottom of [Pre-Class Problems 14](#).

17. Sketch the graph of two cycles of the following function on the same side of the y-axis. Label the numbers on the x- and y-axes as needed. Label where the cycles begin and end. Then label the numbers between these numbers. (10 pts.)

$$y = \sqrt{2} \cot\left(\frac{8x}{3} + \frac{5\pi}{6}\right)$$

Amplitude _____

Period _____

Phase Shift _____

The solution to this problem is at the bottom of [Pre-Class Problems 12](#).



18. Sketch the graph of two cycles of the following function on the same side of the y-axis. Label the numbers on the x- and y-axes as needed. Label where the cycles begin and end. (6 pts.)

$$y = -\frac{4}{9} \sec \frac{\pi x}{6}$$

Amplitude _____

Period _____

Phase Shift _____

The solution to this problem is at the bottom of [Pre-Class Problems 11](#).



19. Find the exact value of $\sec \left[\sin^{-1} \left(-\frac{3}{8} \right) \right]$ (8 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 15](#).

$$\sec \left[\sin^{-1} \left(-\frac{3}{8} \right) \right] = \underline{\hspace{2cm}}$$

$$\sin^{-1} \left(-\frac{3}{8} \right) \text{ is in } \underline{\hspace{1cm}} \text{ quadrant}$$

20. Use **one Pythagorean Identity and Basic Identities** to find the exact value of $\cos \beta$ and $\sin \beta$ if $\tan \beta = \frac{\sqrt{8}}{5}$ and β is in the III quadrant. (8 pts.)

The solution to this problem is at the bottom of [Pre-Class Problems 17](#).

$$\cos \beta = \underline{\hspace{2cm}} \qquad \sin \beta = \underline{\hspace{2cm}}$$

21. Find all the exact solutions (in degrees) of the equation $(\cos \theta + 1)(\csc \theta + 2) = 0$. (8 pts.) **Put a box around your answer(s).**

The solution to this problem is at the bottom of [Pre-Class Problems 17](#).

22. Find the exact value of $\sin 195^\circ$. (5 pts.) **Put a box around your answer.**

The solution to this problem is at the bottom of [Pre-Class Problems 18](#).

23. Find the exact value of $\sin 2\alpha$ and $\cos \frac{\alpha}{2}$ if $\cot \alpha = -\frac{\sqrt{7}}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$. **Put a box around your answers.** (8 pts.)

The solution to the first part of this problem is at the bottom of [Pre-Class Problems 19](#).

The solution to the second part of this problem is at the bottom of [Pre-Class Problems 20](#).

24. Find the exact value of $\tan \frac{5\pi}{8}$. (5 pts.) **Put a box around your answer.**

The solution to this problem is at the bottom of [Pre-Class Problems 20](#).

25. Find all the solutions (in degrees) to the equation $\cos 3x = -\frac{\sqrt{3}}{2}$ in the interval $[-180^\circ, 180^\circ]$. (9 pts.)
Put a box around your answer(s).

The solution to this problem is at the bottom of [Pre-Class Problems 16](#).

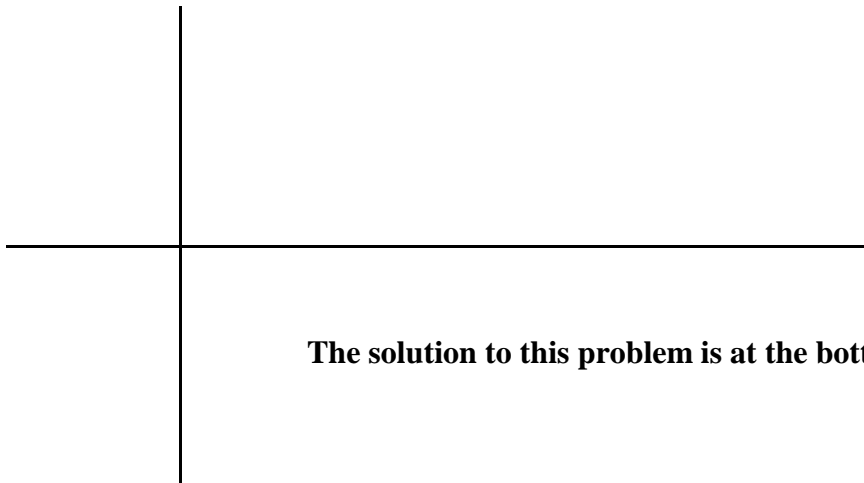
26. Sketch the graph of two cycles of the following function on the same side of the y -axis. Label the numbers on the x - and y -axes as needed. Label where the cycles begin and end. Then label the numbers between these numbers. (10 pts.)

$$y = 3 \sin 7x$$

Amplitude _____

Period _____

Phase Shift _____



The solution to this problem is at the bottom of [Pre-Class Problems 9](#).

FORMULAS

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$