Pre-Class Problems 5 for Wednesday, February 13, and Friday, February 15

Earn one bonus point because you looked at these problems. Send me an email with the following in the Subject line: PC5003 if you are in Section 003 (8:30 class), PC5005 if you are in Section 005 (10:00 class), or PC5002 if you are in Section 002 (11:30 class).

**These are the type of problems that you will be working on in class. These problems are from Lesson 4.**

**Solution to Problems on the Pre-Exam.**

**You can go to the solution for each problem by clicking on the problem letter.**

Objective of the following problems: To find the exact value of the six trigonometric functions for an angle, which is numerically greater than $2\pi$ or $360°$, using the coterminal angle, which is numerically less than $2\pi$ or $360°$.

1. Find the angle between 0 ( $0°$ ) and $2\pi$ ( $360°$ ) that is coterminal with the following angles. Then use this coterminal angle to find the exact value of the cosine, sine, and tangent of the original angle.

a. $960°$ b. $\frac{11\pi}{3}$ c. $\frac{101\pi}{6}$ d. $\frac{89\pi}{4}$

e. $\frac{115\pi}{6}$ f. $\frac{183\pi}{4}$ g. $\frac{57\pi}{2}$ h. $\frac{79\pi}{2}$

i. $\frac{206\pi}{3}$ j. $42\pi$ k. $25\pi$

2. Find the angle between $-2\pi$ ( $-360°$ ) and 0 ( $0°$ ) that is coterminal with the following angles. Then use this coterminal angle to find the exact value of the cosine, sine, and tangent of the original angle.

a. $-1305°$ b. $-\frac{43\pi}{6}$ c. $-\frac{67\pi}{4}$ d. $-\frac{106\pi}{3}$

e. $-\dfrac{85\pi}{6}$ f. $-\dfrac{179\pi}{6}$ g. $-\dfrac{170\pi}{3}$ h. $-\dfrac{125\pi}{6}$

i. $-\dfrac{83\pi}{2}$ j. $-17\pi$ k. $-26\pi$

Additional problems available in the textbook: Page 457 … 57 – 70. Examples 5 and 6 starting on page 451. Page 478 … 39c, 40c, 41b, and 42e. Page 505 … 23cd, 24cd, 25cd, 26cd, 27bd, 28acd, 37, 43 – 54. Example 4 on page 502.

Explanation of the calculations of the coterminal angles for Problems 1 and 2.

**Solutions:**

1a. $960°$ Animation of the making of the $960°$ angle.

$960° - 720° = 240°$

NOTE: The angle $240°$ is the angle between $0°$ and $360°$ which is coterminal with the angle $960°$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos 960° = \cos 240° = -\cos 60° = -\frac{1}{2}$$

$$\sin 960° = \sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$$

$$\tan 960° = \tan 240° = \tan 60° = \sqrt{3}$$

**Answer:** $\cos 960° = -\dfrac{1}{2}$, $\sin 960° = -\dfrac{\sqrt{3}}{2}$, $\tan 960° = \sqrt{3}$

NOTE: $\cos 960^\circ = -\dfrac{1}{2} \Rightarrow \sec 960^\circ = -2$

$$\sin 960^\circ = -\frac{\sqrt{3}}{2} \Rightarrow \csc 960^\circ = -\frac{2}{\sqrt{3}}$$

$$\tan 960^\circ = \sqrt{3} \Rightarrow \cot 960^\circ = \frac{1}{\sqrt{3}}$$

NOTE: The terminal side of the angle $240^\circ$ (and the angle $960^\circ$) is in the third quadrant.

NOTE: In the third quadrant, cosine is negative, sine is negative, and tangent is positive.

NOTE: The reference angle of $240^\circ$ is $60^\circ$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos 60^\circ = \dfrac{1}{2}$, $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$, and $\tan 60^\circ = \sqrt{3}$.

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1b. $\dfrac{11\pi}{3}$ Animation of the making of the $\dfrac{11\pi}{3}$ angle.

$$\frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$$

NOTE: The angle $\frac{5\pi}{3}$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $\frac{11\pi}{3}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\frac{11\pi}{3} = \cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\sin\frac{11\pi}{3} = \sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\tan\frac{11\pi}{3} = \tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$$

**Answer:** $\cos\frac{11\pi}{3} = \frac{1}{2}$, $\sin\frac{11\pi}{3} = -\frac{\sqrt{3}}{2}$, $\tan\frac{11\pi}{3} = -\sqrt{3}$

NOTE: $\cos\frac{11\pi}{3} = \frac{1}{2} \Rightarrow \sec\frac{11\pi}{3} = 2$

$$\sin\frac{11\pi}{3} = -\frac{\sqrt{3}}{2} \Rightarrow \csc\frac{11\pi}{3} = -\frac{2}{\sqrt{3}}$$

$$\tan\frac{11\pi}{3} = -\sqrt{3} \Rightarrow \cot\frac{11\pi}{3} = -\frac{1}{\sqrt{3}}$$

NOTE: The terminal side of the angle $\frac{5\pi}{3}$ (and the angle $\frac{11\pi}{3}$) is in the fourth quadrant.

NOTE: In the fourth quadrant, cosine is positive, sine is negative, and tangent is negative.

NOTE: The reference angle of $\frac{5\pi}{3}$ is $\frac{\pi}{3}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, and $\tan\frac{\pi}{3} = \sqrt{3}$.

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1c. $\frac{101\pi}{6}$ Animation of the making of the $\frac{101\pi}{6}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{101}{6} \Rightarrow 6\overline{)101} = 16 \text{ remainder } 5 \quad \begin{array}{r} 16 \\ 6\overline{)101} \\ \underline{6\phantom{1}} \\ 41 \\ \underline{36} \\ 5 \end{array}$$

$$\frac{101}{6} = 16 + \frac{5}{6} \Rightarrow \frac{101\pi}{6} = 16\pi + \frac{5\pi}{6} = 8(2\pi) + \frac{5\pi}{6}$$

**Using subtraction to find the coterminal angle:**

$$\frac{101\pi}{6} - 8(2\pi) = \frac{101\pi}{6} - 8\left(\frac{12\pi}{6}\right) = \frac{101\pi}{6} - \frac{96\pi}{6} = \frac{5\pi}{6}$$

NOTE: This one calculation could be done in a couple calculations:

$$\frac{101\pi}{6} - 5\left(\frac{12\pi}{6}\right) = \frac{101\pi}{6} - \frac{60\pi}{6} = \frac{41\pi}{6}$$

$$\frac{41\pi}{6} - 3\left(\frac{12\pi}{6}\right) = \frac{41\pi}{6} - \frac{36\pi}{6} = \frac{5\pi}{6}$$

NOTE: The angle $\frac{5\pi}{6}$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $\frac{101\pi}{6}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\frac{101\pi}{6} = \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\frac{101\pi}{6} = \sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\tan\frac{101\pi}{6} = \tan\frac{5\pi}{6} = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$$

**Answer:** $\cos\frac{101\pi}{6} = -\frac{\sqrt{3}}{2}$, $\sin\frac{101\pi}{6} = \frac{1}{2}$, $\tan\frac{101\pi}{6} = -\frac{1}{\sqrt{3}}$

NOTE: $\cos\frac{101\pi}{6} = -\frac{\sqrt{3}}{2} \Rightarrow \sec\frac{101\pi}{6} = -\frac{2}{\sqrt{3}}$

$$\sin\frac{101\pi}{6} = \frac{1}{2} \Rightarrow \csc\frac{101\pi}{6} = 2$$

$$\tan\frac{101\pi}{6} = -\frac{1}{\sqrt{3}} \Rightarrow \cot\frac{101\pi}{6} = -\sqrt{3}$$

NOTE: The terminal side of the angle $\frac{5\pi}{6}$ (and the angle $\frac{101\pi}{6}$) is in the second quadrant.

NOTE: In the second quadrant, cosine is negative, sine is positive, and tangent is negative.

NOTE: The reference angle of $\frac{5\pi}{6}$ is $\frac{\pi}{6}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = \frac{1}{2}$, and $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

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1d. $\frac{89\pi}{4}$ Animation of the making of the $\frac{89\pi}{4}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{89}{4} \Rightarrow 4\overline{)89} = 22 \text{ R } 1$$

$$\begin{array}{r} 22 \\ 4\,\overline{)\,89} \\ \underline{8\phantom{9}} \\ 9 \\ \underline{8} \\ 1 \end{array}$$

$$\frac{89}{4} = 22 + \frac{1}{4} \Rightarrow \frac{89\pi}{4} = 22\pi + \frac{\pi}{4} = 11(2\pi) + \frac{\pi}{4}$$

**Using subtraction to find the coterminal angle:**

$$\frac{89\pi}{4} - 11(2\pi) = \frac{89\pi}{4} - 11\left(\frac{8\pi}{4}\right) = \frac{89\pi}{4} - \frac{88\pi}{4} = \frac{\pi}{4}$$

NOTE: This one calculation could be done in a couple calculations:

$$\frac{89\pi}{4} - 10\left(\frac{8\pi}{4}\right) = \frac{89\pi}{4} - \frac{80\pi}{4} = \frac{9\pi}{4}$$

$$\frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$$

NOTE: The angle $\frac{\pi}{4}$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $\frac{89\pi}{4}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\frac{89\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sin\frac{89\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\tan\frac{89\pi}{4} = \tan\frac{\pi}{4} = 1$$

**Answer:** $\cos\frac{89\pi}{4} = \frac{\sqrt{2}}{2}$, $\sin\frac{89\pi}{4} = \frac{\sqrt{2}}{2}$, $\tan\frac{89\pi}{4} = 1$

NOTE: $\cos\frac{89\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \sec\frac{89\pi}{4} = \sqrt{2}$

$$\sin\frac{89\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \csc\frac{89\pi}{4} = \sqrt{2}$$

$$\tan\frac{89\pi}{4} = 1 \Rightarrow \cot\frac{89\pi}{4} = 1$$

NOTE: The terminal side of the angle $\frac{\pi}{4}$ (and the angle $\frac{89\pi}{4}$) is in the first quadrant.

NOTE: In the first quadrant, cosine is positive, sine is positive, and tangent is positive.

NOTE: The reference angle of $\frac{\pi}{4}$ is $\frac{\pi}{4}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, and $\tan\frac{\pi}{4} = 1$.

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1e. $\frac{115\pi}{6}$ Animation of the making of the $\frac{115\pi}{6}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{115}{6} \Rightarrow 6\overline{)115}$$

$$\begin{array}{r} 19 \\ 6\,)\overline{115} \\ \underline{6\phantom{0}} \\ 55 \\ \underline{54} \\ 1 \end{array}$$

$$\frac{115}{6} = 19 + \frac{1}{6} = 18 + \frac{7}{6} \Rightarrow \frac{115\pi}{6} = 18\pi + \frac{7\pi}{6} = 9(2\pi) + \frac{7\pi}{6}$$

**Using subtraction to find the coterminal angle:**

$$\frac{115\pi}{6} - 9(2\pi) = \frac{115\pi}{6} - 9\left(\frac{12\pi}{6}\right) = \frac{115\pi}{6} - \frac{108\pi}{6} = \frac{7\pi}{6}$$

NOTE: This one calculation could be done in a couple calculations:

$$\frac{115\pi}{6} - 5\left(\frac{12\pi}{6}\right) = \frac{115\pi}{6} - \frac{60\pi}{6} = \frac{55\pi}{6}$$

$$\frac{55\pi}{6} - 4\left(\frac{12\pi}{6}\right) = \frac{55\pi}{6} - \frac{48\pi}{6} = \frac{7\pi}{6}$$

NOTE: The angle $\frac{7\pi}{6}$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $\frac{115\pi}{6}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\frac{115\pi}{6} = \cos\frac{7\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\frac{115\pi}{6} = \sin\frac{7\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\tan\frac{115\pi}{6} = \tan\frac{7\pi}{6} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

**Answer:** $\cos\frac{115\pi}{6} = -\frac{\sqrt{3}}{2}$, $\sin\frac{115\pi}{6} = -\frac{1}{2}$, $\tan\frac{115\pi}{6} = \frac{1}{\sqrt{3}}$

NOTE: $\cos\frac{115\pi}{6} = -\frac{\sqrt{3}}{2} \Rightarrow \sec\frac{115\pi}{6} = -\frac{2}{\sqrt{3}}$

$$\sin \frac{115\pi}{6} = -\frac{1}{2} \Rightarrow \csc \frac{115\pi}{6} = -2$$

$$\tan \frac{115\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow \cot \frac{115\pi}{6} = \sqrt{3}$$

NOTE: The terminal side of the angle $\frac{7\pi}{6}$ (and the angle $\frac{115\pi}{6}$) is in the third quadrant.

NOTE: In the third quadrant, cosine is negative, sine is negative, and tangent is positive.

NOTE: The reference angle of $\frac{7\pi}{6}$ is $\frac{\pi}{6}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin \frac{\pi}{6} = \frac{1}{2}$, and $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

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1f. $\frac{183\pi}{4}$ Animation of the making of the $\frac{183\pi}{4}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{183}{4} \Rightarrow 4\overline{)183}$$

| | |
|---|---|
| | 45 |
| 4 | )183 |
| | 16 |
| | 23 |
| | 20 |
| | 3 |

$$\frac{183}{4} = 45 + \frac{3}{4} = 44 + \frac{7}{4} \Rightarrow \frac{183\pi}{4} = 44\pi + \frac{7\pi}{4} = 22(2\pi) + \frac{7\pi}{4}$$

**Using subtraction to find the coterminal angle:**

$$\frac{183\pi}{4} - 22(2\pi) = \frac{183\pi}{4} - 22\left(\frac{8\pi}{4}\right) = \frac{183\pi}{4} - \frac{176\pi}{4} = \frac{7\pi}{4}$$

NOTE: This one calculation could be done in a couple calculations:

$$\frac{183\pi}{4} - 20\left(\frac{8\pi}{4}\right) = \frac{183\pi}{4} - \frac{160\pi}{4} = \frac{23\pi}{4}$$

$$\frac{23\pi}{4} - 2\left(\frac{8\pi}{4}\right) = \frac{23\pi}{4} - \frac{16\pi}{4} = \frac{7\pi}{4}$$

NOTE: The angle $\frac{7\pi}{4}$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $\frac{183\pi}{4}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\frac{183\pi}{4} = \cos\frac{7\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sin\frac{183\pi}{4} = \sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\tan\frac{183\pi}{4} = \tan\frac{7\pi}{4} = -\tan\frac{\pi}{4} = -1$$

**Answer:** $\cos\dfrac{183\pi}{4} = \dfrac{\sqrt{2}}{2}$, $\sin\dfrac{183\pi}{4} = -\dfrac{\sqrt{2}}{2}$, $\tan\dfrac{183\pi}{4} = -1$

NOTE: $\cos\dfrac{183\pi}{4} = \dfrac{\sqrt{2}}{2} \Rightarrow \sec\dfrac{183\pi}{4} = \sqrt{2}$

$\sin\dfrac{183\pi}{4} = -\dfrac{\sqrt{2}}{2} \Rightarrow \csc\dfrac{183\pi}{4} = -\sqrt{2}$

$\tan\dfrac{183\pi}{4} = -1 \Rightarrow \cot\dfrac{183\pi}{4} = -1$

NOTE: The terminal side of the angle $\dfrac{7\pi}{4}$ (and the angle $\dfrac{183\pi}{4}$) is in the fourth quadrant.

NOTE: In the fourth quadrant, cosine is positive, sine is negative, and tangent is negative.

NOTE: The reference angle of $\dfrac{7\pi}{4}$ is $\dfrac{\pi}{4}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$, $\sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$, and $\tan\dfrac{\pi}{4} = 1$.

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1g. $\dfrac{57\pi}{2}$ Animation of the making of the $\dfrac{57\pi}{2}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{57}{2} \Rightarrow 2\overline{)57}^{\,28}$$

$$\begin{array}{r} 28 \\ 2\,\big)\overline{57} \\ \underline{4\;\;} \\ 17 \\ \underline{16} \\ 1 \end{array}$$

$$\frac{57}{2} = 28 + \frac{1}{2} \Rightarrow \frac{57\pi}{2} = 28\pi + \frac{\pi}{2} = 14(2\pi) + \frac{\pi}{2}$$

**Using subtraction to find the coterminal angle:**

$$\frac{57\pi}{2} - 14(2\pi) = \frac{57\pi}{2} - 14\left(\frac{4\pi}{2}\right) = \frac{57\pi}{2} - \frac{56\pi}{2} = \frac{\pi}{2}$$

NOTE: This one calculation could be done in a couple calculations:

$$\frac{57\pi}{2} - 10\left(\frac{4\pi}{2}\right) = \frac{57\pi}{2} - \frac{40\pi}{2} = \frac{17\pi}{2}$$

$$\frac{17\pi}{2} - 4\left(\frac{4\pi}{2}\right) = \frac{17\pi}{2} - \frac{16\pi}{2} = \frac{\pi}{2}$$

NOTE: The angle $\frac{\pi}{2}$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $\frac{57\pi}{2}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\frac{57\pi}{2} = \cos\frac{\pi}{2} = 0$$

$$\sin \frac{57\pi}{2} = \sin \frac{\pi}{2} = 1$$

$$\tan \frac{57\pi}{2} = \tan \frac{\pi}{2} = \frac{1}{0} = \text{undefined}$$

**Answer:** $\cos \frac{57\pi}{2} = 0$, $\sin \frac{57\pi}{2} = 1$, $\tan \frac{57\pi}{2} = \text{undefined}$

NOTE: $\cos \frac{57\pi}{2} = 0 \Rightarrow \sec \frac{57\pi}{2} = \text{undefined}$

$$\sin \frac{57\pi}{2} = 1 \Rightarrow \csc \frac{57\pi}{2} = 1$$

$$\cot \frac{57\pi}{2} = \frac{0}{1} = 0$$

NOTE: The terminal side of the angle $\frac{\pi}{2}$ (and the angle $\frac{57\pi}{2}$) is on the positive *y*-axis. The angle $\frac{\pi}{2}$ does not have a reference angle. You will have to use

Unit Circle Trigonometry to find that $\cos \frac{\pi}{2} = 0$, $\sin \frac{\pi}{2} = 1$, and $\tan \frac{\pi}{2} = \text{undefined}$.

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1h. $\frac{79\pi}{2}$ Animation of the making of the $\frac{79\pi}{2}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{79}{2} \quad \Rightarrow \quad \begin{array}{r} 39 \\ 2\overline{)79} \\ \underline{6\phantom{9}} \\ 19 \\ \underline{18} \\ 1 \end{array}$$

$$\frac{79}{2} = 39 + \frac{1}{2} = 38 + \frac{3}{2} \Rightarrow \frac{79\pi}{2} = 38\pi + \frac{3\pi}{2} = 19(2\pi) + \frac{3\pi}{2}$$

**Using subtraction to find the coterminal angle:**

$$\frac{79\pi}{2} - 19(2\pi) = \frac{79\pi}{2} - 19\left(\frac{4\pi}{2}\right) = \frac{79\pi}{2} - \frac{76\pi}{2} = \frac{3\pi}{2}$$

NOTE: This one calculation could be done in a couple calculations:

$$\frac{79\pi}{2} - 15\left(\frac{4\pi}{2}\right) = \frac{79\pi}{2} - \frac{60\pi}{2} = \frac{19\pi}{2}$$

$$\frac{19\pi}{2} - 4\left(\frac{4\pi}{2}\right) = \frac{19\pi}{2} - \frac{16\pi}{4} = \frac{3\pi}{2}$$

NOTE: The angle $\frac{3\pi}{2}$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $\frac{79\pi}{2}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\frac{79\pi}{2} = \cos\frac{3\pi}{2} = 0$$

$$\sin\frac{79\pi}{2} = \sin\frac{3\pi}{2} = -1$$

$$\tan\frac{79\pi}{2} = \tan\frac{\pi}{2} = \frac{-1}{0} = \text{undefined}$$

**Answer:** $\cos\frac{79\pi}{2} = 0$ , $\sin\frac{79\pi}{2} = -1$, $\tan\frac{79\pi}{2} = \text{undefined}$

NOTE: $\cos\frac{79\pi}{2} = 0 \Rightarrow \sec\frac{79\pi}{2} = \text{undefined}$

$$\sin\frac{79\pi}{2} = 1 \Rightarrow \csc\frac{79\pi}{2} = 1$$

$$\cot\frac{79\pi}{2} = \frac{0}{-1} = 0$$

NOTE: The terminal side of the angle $\frac{3\pi}{2}$ (and the angle $\frac{79\pi}{2}$) is on the negative $y$-axis. The angle $\frac{3\pi}{2}$ does not have a reference angle. You will have to use Unit Circle Trigonometry to find that $\cos\frac{3\pi}{2} = 0$, $\sin\frac{3\pi}{2} = -1$, and $\tan\frac{3\pi}{2} = \text{undefined}$.

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1i. $\frac{206\pi}{3}$ Animation of the making of the $\frac{206\pi}{3}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{206}{3} \Rightarrow \begin{array}{r} 68 \\ 3\overline{)206} \\ \underline{18\phantom{0}} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

$$\frac{206}{3} = 68 + \frac{2}{3} \Rightarrow \frac{206\pi}{3} = 68\pi + \frac{2\pi}{3} = 34(2\pi) + \frac{2\pi}{3}$$

**Using subtraction to find the coterminal angle:**

$$\frac{206\pi}{3} - 34(2\pi) = \frac{206\pi}{3} - 34\left(\frac{6\pi}{3}\right) = \frac{206\pi}{3} - \frac{204\pi}{3} = \frac{2\pi}{3}$$

NOTE: This one calculation could be done in a couple calculations:

$$\frac{206\pi}{3} - 30\left(\frac{6\pi}{3}\right) = \frac{206\pi}{3} - \frac{180\pi}{3} = \frac{26\pi}{3}$$

$$\frac{26\pi}{3} - 4\left(\frac{6\pi}{3}\right) = \frac{26\pi}{3} - \frac{24\pi}{3} = \frac{2\pi}{3}$$

NOTE: The angle $\frac{2\pi}{3}$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $\frac{206\pi}{3}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\frac{206\pi}{3} = \cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\sin \frac{206\pi}{3} = \sin \frac{2\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\tan \frac{206\pi}{3} = \tan \frac{2\pi}{3} = -\tan \frac{\pi}{3} = -\sqrt{3}$$

**Answer:** $\cos \frac{206\pi}{3} = -\frac{1}{2}$, $\sin \frac{206\pi}{3} = \frac{\sqrt{3}}{2}$, $\tan \frac{206\pi}{3} = -\sqrt{3}$

NOTE: $\cos \frac{206\pi}{3} = -\frac{1}{2} \Rightarrow \sec \frac{206\pi}{3} = -2$

$$\sin \frac{206\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \csc \frac{206\pi}{3} = \frac{2}{\sqrt{3}}$$

$$\tan \frac{206\pi}{3} = -\sqrt{3} \Rightarrow \cot \frac{206\pi}{3} = -\frac{1}{\sqrt{3}}$$

NOTE: The terminal side of the angle $\frac{2\pi}{3}$ (and the angle $\frac{206\pi}{3}$) is in the second quadrant.

NOTE: In the second quadrant, cosine is negative, sine is positive, and tangent is negative.

NOTE: The reference angle of $\frac{2\pi}{3}$ is $\frac{\pi}{3}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos \frac{\pi}{3} = \frac{1}{2}$, $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$, and $\tan \frac{\pi}{3} = \sqrt{3}$.

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1j. $42\pi$ Animation of the making of the $42\pi$ angle.

$42\pi = 21(2\pi)$

NOTE: The angle 0 is the angle between $0$ and $2\pi$ which is coterminal with the angle $42\pi$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$\cos 42\pi = \cos 0 = 1$

$\sin 42\pi = \sin 0 = 0$

$\tan 42\pi = \tan 0 = \dfrac{0}{1} = 0$

**Answer:** $\cos 42\pi = 1$, $\sin 42\pi = 0$, $\tan 42\pi = 0$

NOTE: $\cos 42\pi = 1 \Rightarrow \sec 42\pi = 1$

$\sin 42\pi = 0 \Rightarrow \csc 42\pi =$ undefined

$\tan 42\pi = 0 \Rightarrow \cot 42\pi =$ undefined

NOTE: The terminal side of the angle 0 (and the angle $42\pi$) is on the positive *x*-axis. The angle 0 does not have a reference angle. You will have to use Unit Circle Trigonometry to find that $\cos 0 = 1$, $\sin 0 = 0$, and $\tan 0 = 0$.

Back to Problem 1.

1k. $25\pi$ Animation of the making of the $25\pi$ angle.

$$25\pi = 24\pi + \pi = 12(2\pi) + \pi$$

NOTE: The angle $\pi$ is the angle between $0$ and $2\pi$ which is coterminal with the angle $25\pi$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos 25\pi = \cos \pi = -1$$

$$\sin 25\pi = \sin \pi = 0$$

$$\tan 25\pi = \tan \pi = \frac{0}{-1} = 0$$

**Answer:** $\cos 25\pi = -1$, $\sin 25\pi = 0$, $\tan 25\pi = 0$

NOTE: $\cos 25\pi = -1 \Rightarrow \sec 25\pi = -1$

$\sin 25\pi = 0 \Rightarrow \csc 25\pi =$ undefined

$\tan 25\pi = 0 \Rightarrow \cot 25\pi =$ undefined

NOTE: The terminal side of the angle $\pi$ (and the angle $25\pi$) is on the negative *x*-axis. The angle $\pi$ does not have a reference angle. You will have to use Unit Circle Trigonometry to find that $\cos \pi = -1$, $\sin \pi = 0$, and $\tan \pi = 0$.

Back to Problem 1.

2a. $-1305°$ Animation of the making of the $-1305°$ angle.

$$-1305° = -1080° - 225° = 3(-360°) - 225°$$

**Using addition to find the coterminal angle:** $-1305° + 1080° = -225°$

NOTE: The angle $-225°$ is the angle between $-360°$ and $0°$ which is coterminal with the angle $-1305°$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos(-1305°) = \cos(-225°) = -\cos 45° = -\frac{\sqrt{2}}{2}$$

$$\sin(-1305°) = \sin(-225°) = \sin 45° = \frac{\sqrt{2}}{2}$$

$$\tan(-1305°) = \tan(-225°) = -\tan 45° = -1$$

**Answer:** $\cos(-1305°) = -\frac{\sqrt{2}}{2}$, $\sin(-1305°) = \frac{\sqrt{2}}{2}$, $\tan(-1305°) = -1$

NOTE: $\cos(-1305°) = -\frac{\sqrt{2}}{2} \Rightarrow \sec(-1305°) = -\sqrt{2}$

$\sin(-1305°) = \frac{\sqrt{2}}{2} \Rightarrow \csc(-1305°) = \sqrt{2}$

$\tan(-1305°) = -1 \Rightarrow \cot(-1305°) = -1$

NOTE: The terminal side of the angle $-225°$ (and the angle $-1305°$) is in the second quadrant.

NOTE: In the second quadrant, cosine is negative, sine is positive, and tangent is negative.

NOTE: The reference angle of $-225°$ is $45°$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos 45° = \frac{\sqrt{2}}{2}$, $\sin 45° = \frac{\sqrt{2}}{2}$, and $\tan 45° = 1$.

Back to Problem 2.

2b. $-\frac{43\pi}{6}$ Animation of the making of the $-\frac{43\pi}{6}$ angle.

$$-\frac{43\pi}{6} = -\frac{36\pi}{6} - \frac{7\pi}{6} = 3(-2\pi) - \frac{7\pi}{6}$$

**Using addition to find the coterminal angle:**

$$-\frac{43\pi}{6} + 3(2\pi) = -\frac{43\pi}{6} + 3\left(\frac{12\pi}{6}\right) = -\frac{43\pi}{6} + \frac{36\pi}{6} = -\frac{7\pi}{6}$$

NOTE: The angle $-\frac{7\pi}{6}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{43\pi}{6}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{43\pi}{6}\right) = \cos\left(-\frac{7\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{43\pi}{6}\right) = \sin\left(-\frac{7\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\tan\left(-\frac{43\pi}{6}\right) = \tan\left(-\frac{7\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$$

**Answer:** $\cos\left(-\frac{43\pi}{6}\right) = -\frac{\sqrt{3}}{2}$, $\sin\left(-\frac{43\pi}{6}\right) = \frac{1}{2}$,

$\tan\left(-\frac{43\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

NOTE: $\cos\left(-\frac{43\pi}{6}\right) = -\frac{\sqrt{3}}{2} \Rightarrow \sec\left(-\frac{43\pi}{6}\right) = -\frac{2}{\sqrt{3}}$

$$\sin\left(-\frac{43\pi}{6}\right) = \frac{1}{2} \Rightarrow \csc\left(-\frac{43\pi}{6}\right) = 2$$

$$\tan\left(-\frac{43\pi}{6}\right) = -\frac{1}{\sqrt{3}} \Rightarrow \cot\left(-\frac{43\pi}{6}\right) = -\sqrt{3}$$

NOTE: The terminal side of the angle $-\frac{7\pi}{6}$ (and the angle $-\frac{43\pi}{6}$) is in the second quadrant.

NOTE: In the second quadrant, cosine is negative, sine is positive, and tangent is negative.

NOTE: The reference angle of $-\frac{7\pi}{6}$ is $\frac{\pi}{6}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = \frac{1}{2}$, and $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

Back to Problem 2.

2c. $-\dfrac{67\pi}{4}$ Animation of the making of the $-\dfrac{67\pi}{4}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{67}{4} \Rightarrow 4\overline{)67} \quad \begin{array}{r} 16 \\ 4\,)\overline{67} \\ \underline{4\phantom{7}} \\ 27 \\ \underline{24} \\ 3 \end{array}$$

$$\frac{67}{4} = 16 + \frac{3}{4} \Rightarrow -\frac{67\pi}{4} = -16\pi - \frac{3\pi}{4} = 8(-2\pi) - \frac{3\pi}{4}$$

**Using addition to find the coterminal angle:**

$$-\frac{67\pi}{4} + 8(2\pi) = -\frac{67\pi}{4} + 8\left(\frac{8\pi}{4}\right) = -\frac{67\pi}{4} + \frac{64\pi}{4} = -\frac{3\pi}{4}$$

NOTE: This one calculation could be done in a couple calculations:

$$-\frac{67\pi}{4} + 5\left(\frac{8\pi}{4}\right) = -\frac{67\pi}{4} + \frac{40\pi}{4} = -\frac{27\pi}{4}$$

$$-\frac{27\pi}{4} + 3\left(\frac{8\pi}{4}\right) = -\frac{27\pi}{4} + \frac{24\pi}{3} = -\frac{3\pi}{4}$$

NOTE: The angle $-\frac{3\pi}{4}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{67\pi}{4}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{67\pi}{4}\right) = \cos\left(-\frac{3\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\sin\left(-\frac{67\pi}{4}\right) = \sin\left(-\frac{3\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\tan\left(-\frac{67\pi}{4}\right) = \tan\left(-\frac{3\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

**Answer:** $\cos\left(-\frac{67\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, $\sin\left(-\frac{67\pi}{4}\right) = -\frac{\sqrt{2}}{2}$,
$\tan\left(-\frac{67\pi}{4}\right) = 1$

NOTE: $$\cos\left(-\frac{67\pi}{4}\right) = -\frac{\sqrt{2}}{2} \Rightarrow \sec\left(-\frac{67\pi}{4}\right) = -\sqrt{2}$$

$$\sin\left(-\frac{67\pi}{4}\right) = -\frac{\sqrt{2}}{2} \Rightarrow \csc\left(-\frac{67\pi}{4}\right) = -\sqrt{2}$$

$$\tan\left(-\frac{67\pi}{4}\right) = 1 \Rightarrow \cot\left(-\frac{67\pi}{4}\right) = 1$$

NOTE: The terminal side of the angle $-\frac{3\pi}{4}$ (and the angle $-\frac{67\pi}{4}$) is in the third quadrant.

NOTE: In the third quadrant, cosine is negative, sine is negative, and tangent is positive.

NOTE: The reference angle of $-\dfrac{3\pi}{4}$ is $\dfrac{\pi}{4}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$, $\sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$, and $\tan\dfrac{\pi}{4} = 1$.

Back to Problem 2.

2d. $-\dfrac{106\pi}{3}$ Animation of the making of the $-\dfrac{106\pi}{3}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{106}{3} \Rightarrow 3\overline{)106} = 35 \text{ R } 1$$

$$\begin{array}{r} 35 \\ 3\,)\overline{106} \\ \underline{9\phantom{0}} \\ 16 \\ \underline{15} \\ 1 \end{array}$$

$$\frac{106}{3} = 35 + \frac{1}{3} = 34 + \frac{4}{3} \Rightarrow -\frac{106\pi}{3} = -34\pi - \frac{4\pi}{3} = 17(-2\pi) - \frac{4\pi}{3}$$

**Using addition to find the coterminal angle:**

$$-\frac{106\pi}{3} + 17(2\pi) = -\frac{106\pi}{3} + 17\left(\frac{6\pi}{3}\right) = -\frac{106\pi}{3} + \frac{102\pi}{3} = -\frac{4\pi}{3}$$

NOTE: This one calculation could be done in a couple calculations:

$$-\frac{106\pi}{3} + 15\left(\frac{6\pi}{3}\right) = -\frac{106\pi}{3} + \frac{90\pi}{3} = -\frac{16\pi}{3}$$

$$-\frac{16\pi}{3} + 2\left(\frac{6\pi}{3}\right) = -\frac{16\pi}{3} + \frac{12\pi}{3} = -\frac{4\pi}{3}$$

NOTE: The angle $-\frac{4\pi}{3}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{106\pi}{3}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{106\pi}{3}\right) = \cos\left(-\frac{4\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\sin\left(-\frac{106\pi}{3}\right) = \sin\left(-\frac{4\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\tan\left(-\frac{106\pi}{3}\right) = \tan\left(-\frac{4\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$$

**Answer:** $\cos\left(-\frac{106\pi}{3}\right) = -\frac{1}{2}$, $\sin\left(-\frac{106\pi}{3}\right) = \frac{\sqrt{3}}{2}$,

$\tan\left(-\frac{106\pi}{3}\right) = -\sqrt{3}$

NOTE: $\cos\left(-\frac{106\pi}{3}\right) = -\frac{1}{2} \Rightarrow \sec\left(-\frac{106\pi}{3}\right) = -2$

$\sin\left(-\frac{106\pi}{3}\right) = \frac{\sqrt{3}}{2} \Rightarrow \csc\left(-\frac{106\pi}{3}\right) = \frac{2}{\sqrt{3}}$

$\tan\left(-\frac{106\pi}{3}\right) = -\sqrt{3} \Rightarrow \cot\left(-\frac{106\pi}{3}\right) = -\frac{1}{\sqrt{3}}$

NOTE: The terminal side of the angle $-\frac{4\pi}{3}$ (and the angle $-\frac{106\pi}{3}$) is in the second quadrant.

NOTE: In the second quadrant, cosine is negative, sine is positive, and tangent is negative.

NOTE: The reference angle of $-\frac{4\pi}{3}$ is $\frac{\pi}{3}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, and $\tan\frac{\pi}{3} = \sqrt{3}$.

Back to Problem 2.

2e. $-\frac{85\pi}{6}$ Animation of the making of the $-\frac{85\pi}{6}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{85}{6} \Rightarrow 6\overline{)85}^{\,14}$$

$$\begin{array}{r} 14 \\ 6\,\overline{)\,85} \\ \underline{6\phantom{5}} \\ 25 \\ \underline{24} \\ 1 \end{array}$$

$$\frac{85}{6} = 14 + \frac{1}{6} \Rightarrow -\frac{85\pi}{6} = -14\pi - \frac{\pi}{6} = 7(-2\pi) - \frac{\pi}{6}$$

**Using addition to find the coterminal angle:**

$$-\frac{85\pi}{6} + 7(2\pi) = -\frac{85\pi}{6} + 7\left(\frac{12\pi}{6}\right) = -\frac{85\pi}{6} + \frac{84\pi}{6} = -\frac{\pi}{6}$$

NOTE: This one calculation could be done in a couple calculations:

$$-\frac{85\pi}{6} + 5\left(\frac{12\pi}{6}\right) = -\frac{85\pi}{6} + \frac{60\pi}{6} = -\frac{25\pi}{6}$$

$$-\frac{25\pi}{6} + 2\left(\frac{12\pi}{6}\right) = -\frac{25\pi}{6} + \frac{24\pi}{6} = -\frac{\pi}{6}$$

NOTE: The angle $-\frac{\pi}{6}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{85\pi}{6}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{85\pi}{6}\right) = \cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{85\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\tan\left(-\frac{85\pi}{6}\right) = \tan\left(-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$$

**Answer:** $\cos\left(-\frac{85\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\sin\left(-\frac{85\pi}{6}\right) = -\frac{1}{2}$,
$\tan\left(-\frac{85\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

NOTE: $$\cos\left(-\frac{85\pi}{6}\right) = \frac{\sqrt{3}}{2} \Rightarrow \sec\left(-\frac{85\pi}{6}\right) = \frac{2}{\sqrt{3}}$$

$$\sin\left(-\frac{85\pi}{6}\right) = -\frac{1}{2} \Rightarrow \csc\left(-\frac{85\pi}{6}\right) = -2$$

$$\tan\left(-\frac{85\pi}{6}\right) = -\frac{1}{\sqrt{3}} \Rightarrow \cot\left(-\frac{85\pi}{6}\right) = -\sqrt{3}$$

NOTE: The terminal side of the angle $-\frac{\pi}{6}$ (and the angle $-\frac{85\pi}{6}$) is in the fourth quadrant.

NOTE: In the fourth quadrant, cosine is positive, sine is negative, and tangent is negative.

NOTE: The reference angle of $-\frac{\pi}{6}$ is $\frac{\pi}{6}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = \frac{1}{2}$, and $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

Back to Problem 2.

2f. $-\frac{179\pi}{6}$ Animation of the making of the $-\frac{179\pi}{6}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{179}{6} \Rightarrow 6\overline{)179} = 29$$

$$\begin{array}{r} 29 \\ 6\,\overline{)179} \\ \underline{12\phantom{9}} \\ 59 \\ \underline{54} \\ 5 \end{array}$$

$$\frac{179}{6} = 29 + \frac{5}{6} = 28 + \frac{11}{6} \Rightarrow -\frac{179\pi}{6} = -28\pi - \frac{11\pi}{6} = 14(-2\pi) - \frac{11\pi}{6}$$

**Using addition to find the coterminal angle:**

$$-\frac{179\pi}{6} + 14(2\pi) = -\frac{179\pi}{6} + 14\left(\frac{12\pi}{6}\right) = -\frac{179\pi}{6} + \frac{168\pi}{6} = -\frac{11\pi}{6}$$

NOTE: This one calculation could be done in a couple calculations:

$$-\frac{179\pi}{6} + 10\left(\frac{12\pi}{6}\right) = -\frac{179\pi}{6} + \frac{120\pi}{6} = -\frac{59\pi}{6}$$

$$-\frac{59\pi}{6} + 4\left(\frac{12\pi}{6}\right) = -\frac{59\pi}{6} + \frac{48\pi}{6} = -\frac{11\pi}{6}$$

NOTE: The angle $-\frac{11\pi}{6}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{179\pi}{6}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{179\pi}{6}\right) = \cos\left(-\frac{11\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{179\pi}{6}\right) = \sin\left(-\frac{11\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\tan\left(-\frac{179\pi}{6}\right) = \tan\left(-\frac{11\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

**Answer:** $\cos\left(-\frac{179\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\sin\left(-\frac{179\pi}{6}\right) = \frac{1}{2}$,
$\tan\left(-\frac{179\pi}{6}\right) = \frac{1}{\sqrt{3}}$

NOTE: $\cos\left(-\frac{179\pi}{6}\right) = \frac{\sqrt{3}}{2} \Rightarrow \sec\left(-\frac{179\pi}{6}\right) = \frac{2}{\sqrt{3}}$

$$\sin\left(-\frac{179\pi}{6}\right) = \frac{1}{2} \Rightarrow \csc\left(-\frac{179\pi}{6}\right) = 2$$

$$\tan\left(-\frac{179\pi}{6}\right) = \frac{1}{\sqrt{3}} \Rightarrow \cot\left(-\frac{179\pi}{6}\right) = \sqrt{3}$$

NOTE: The terminal side of the angle $-\dfrac{11\pi}{6}$ (and the angle $-\dfrac{179\pi}{6}$) is in the first quadrant.

NOTE: In the first quadrant, cosine is positive, sine is positive, and tangent is positive.

NOTE: The reference angle of $-\dfrac{11\pi}{6}$ is $\dfrac{\pi}{6}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$, $\sin\dfrac{\pi}{6} = \dfrac{1}{2}$, and $\tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}}$.

Back to Problem 2.

2g. $-\dfrac{170\pi}{3}$ Animation of the making of the $-\dfrac{170\pi}{3}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{170}{3} \Rightarrow 3\,\overline{)170} = 56$$

$$\begin{array}{r} 56 \\ 3\,\overline{)170} \\ \underline{15\phantom{0}} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

$$\frac{170}{3} = 56 + \frac{2}{3} \Rightarrow -\frac{170\pi}{3} = -56\pi - \frac{2\pi}{3} = 28(-2\pi) - \frac{2\pi}{3}$$

**Using addition to find the coterminal angle:**

$$-\frac{170\pi}{3} + 28(2\pi) = -\frac{170\pi}{3} + 28\left(\frac{6\pi}{3}\right) = -\frac{170\pi}{3} + \frac{168\pi}{3} = -\frac{2\pi}{3}$$

NOTE: This one calculation could be done in a couple calculations:

$$-\frac{170\pi}{3} + 20\left(\frac{6\pi}{3}\right) = -\frac{170\pi}{3} + \frac{120\pi}{3} = -\frac{50\pi}{3}$$

$$-\frac{50\pi}{3} + 8\left(\frac{6\pi}{3}\right) = -\frac{50\pi}{3} + \frac{48\pi}{3} = -\frac{2\pi}{3}$$

NOTE: The angle $-\frac{2\pi}{3}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{170\pi}{3}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{170\pi}{3}\right) = \cos\left(-\frac{2\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\sin\left(-\frac{170\pi}{3}\right) = \sin\left(-\frac{2\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\tan\left(-\frac{170\pi}{3}\right) = \tan\left(-\frac{2\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$$

**Answer:** $\cos\left(-\frac{170\pi}{3}\right) = -\frac{1}{2}$, $\sin\left(-\frac{170\pi}{3}\right) = -\frac{\sqrt{3}}{2}$,
$\tan\left(-\frac{170\pi}{3}\right) = \sqrt{3}$

NOTE: $\cos\left(-\dfrac{170\pi}{3}\right) = -\dfrac{1}{2} \Rightarrow \sec\left(-\dfrac{170\pi}{3}\right) = -2$

$$\sin\left(-\frac{170\pi}{3}\right) = -\frac{\sqrt{3}}{2} \Rightarrow \csc\left(-\frac{170\pi}{3}\right) = -\frac{2}{\sqrt{3}}$$

$$\tan\left(-\frac{170\pi}{3}\right) = \sqrt{3} \Rightarrow \cot\left(-\frac{170\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

NOTE: The terminal side of the angle $-\dfrac{2\pi}{3}$ (and the angle $-\dfrac{170\pi}{3}$) is in the third quadrant.

NOTE: In the third quadrant, cosine is negative, sine is negative, and tangent is positive.

NOTE: The reference angle of $-\dfrac{2\pi}{3}$ is $\dfrac{\pi}{3}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\dfrac{\pi}{3} = \dfrac{1}{2}$, $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$, and $\tan\dfrac{\pi}{3} = \sqrt{3}$.

Back to Problem 2.

2h. $-\dfrac{125\pi}{6}$ Animation of the making of the $-\dfrac{125\pi}{6}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{125}{6} \Rightarrow 6\overline{)125}$$

```
    20
6 )125
   12
   --
    5
```

$$\frac{125}{6} = 20 + \frac{5}{6} \Rightarrow -\frac{125\pi}{6} = -20\pi - \frac{5\pi}{6} = 10(-2\pi) - \frac{5\pi}{6}$$

**Using addition to find the coterminal angle:**

$$-\frac{125\pi}{6} + 10(2\pi) = -\frac{125\pi}{6} + 10\left(\frac{12\pi}{6}\right) = -\frac{125\pi}{6} + \frac{120\pi}{6} = -\frac{5\pi}{6}$$

NOTE: The angle $-\frac{5\pi}{6}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{125\pi}{6}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{125\pi}{6}\right) = \cos\left(-\frac{5\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{125\pi}{6}\right) = \sin\left(-\frac{5\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\tan\left(-\frac{125\pi}{6}\right) = \tan\left(-\frac{5\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

**Answer:** $\cos\left(-\frac{125\pi}{6}\right) = -\frac{\sqrt{3}}{2}$, $\sin\left(-\frac{125\pi}{6}\right) = -\frac{1}{2}$, $\tan\left(-\frac{125\pi}{6}\right) = \frac{1}{\sqrt{3}}$

NOTE: $\cos\left(-\frac{125\pi}{6}\right) = -\frac{\sqrt{3}}{2} \Rightarrow \sec\left(-\frac{125\pi}{6}\right) = -\frac{2}{\sqrt{3}}$

$$\sin\left(-\frac{125\pi}{6}\right) = -\frac{1}{2} \Rightarrow \csc\left(-\frac{125\pi}{6}\right) = -2$$

$$\tan\left(-\frac{125\pi}{6}\right) = \frac{1}{\sqrt{3}} \Rightarrow \cot\left(-\frac{125\pi}{6}\right) = \sqrt{3}$$

NOTE: The terminal side of the angle $-\frac{5\pi}{6}$ (and the angle $-\frac{125\pi}{6}$) is in the third quadrant.

NOTE: In the third quadrant, cosine is negative, sine is negative, and tangent is positive.

NOTE: The reference angle of $-\frac{5\pi}{6}$ is $\frac{\pi}{6}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = \frac{1}{2}$, and $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

Back to Problem 2.

2i. $-\frac{83\pi}{2}$ Animation of the making of the $-\frac{83\pi}{2}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{83}{2} \Rightarrow \begin{array}{r} 41 \\ 2\,\overline{)\,83} \\ \underline{8\phantom{0}} \\ 3 \\ \underline{2} \\ 1 \end{array}$$

$$\frac{83}{2} = 41 + \frac{1}{2} = 40 + \frac{3}{2} \Rightarrow -\frac{83\pi}{2} = -40\pi - \frac{3\pi}{2} = 20(-2\pi) - \frac{3\pi}{2}$$

**Using addition to find the coterminal angle:**

$$-\frac{83\pi}{2} + 20(2\pi) = -\frac{83\pi}{2} + 20\left(\frac{4\pi}{2}\right) = -\frac{83\pi}{2} + \frac{80\pi}{2} = -\frac{3\pi}{2}$$

NOTE: The angle $-\frac{3\pi}{2}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{83\pi}{2}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{83\pi}{2}\right) = \cos\left(-\frac{3\pi}{2}\right) = 0$$

$$\sin\left(-\frac{83\pi}{2}\right) = \sin\left(-\frac{3\pi}{2}\right) = 1$$

$$\tan\left(-\frac{83\pi}{2}\right) = \tan\left(-\frac{3\pi}{2}\right) = \frac{1}{0} = \text{undefined}$$

**Answer:** $\cos\left(-\frac{83\pi}{2}\right) = 0$, $\sin\left(-\frac{83\pi}{2}\right) = 1$,

$\tan\left(-\frac{83\pi}{2}\right) = \text{undefined}$

NOTE: $\cos\left(-\frac{83\pi}{2}\right) = 0 \Rightarrow \sec\left(-\frac{83\pi}{2}\right) = \text{undefined}$

$\sin\left(-\frac{83\pi}{2}\right) = 1 \Rightarrow \csc\left(-\frac{83\pi}{2}\right) = 1$

$$\cot\left(-\frac{83\pi}{2}\right) = \frac{0}{1} = 0$$

NOTE: The terminal side of the angle $-\frac{3\pi}{2}$ (and the angle $-\frac{83\pi}{2}$) is on the positive $y$-axis. The angle $-\frac{3\pi}{2}$ does not have a reference angle. You will have to use Unit Circle Trigonometry to find that $\cos\left(-\frac{3\pi}{2}\right) = 0$, $\sin\left(-\frac{3\pi}{2}\right) = 1$, and $\tan\left(-\frac{3\pi}{2}\right) =$ undefined.

Back to Problem 2.

2j. $-17\pi$ Animation of the making of the $-17\pi$ angle.

$$-17\pi = -16\pi - \pi = 8(-2\pi) - \pi$$

NOTE: The angle $-\pi$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-17\pi$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos(-17\pi) = \cos(-\pi) = -1$$

$$\sin(-17\pi) = \sin(-\pi) = 0$$

$$\tan(-17\pi) = \tan(-\pi) = \frac{0}{-1} = 0$$

**Answer:** $\cos(-17\pi) = -1$, $\sin(-17\pi) = 0$, $\tan(-17\pi) = 0$

NOTE: $\cos(-17\pi) = -1 \Rightarrow \sec(-17\pi) = -1$

$\sin(-17\pi) = 0 \Rightarrow \csc(-17\pi) =$ undefined

$\tan(-17\pi) = 0 \Rightarrow \cot(-17\pi) =$ undefined

NOTE: The terminal side of the angle $-\pi$ (and the angle $-17\pi$) is on the negative $x$-axis. The angle $-\pi$ does not have a reference angle. You will have to use Unit Circle Trigonometry to find that $\cos(-\pi) = -1$, $\sin(-\pi) = 0$, and $\tan(-\pi) = 0$.

Back to Problem 2.

2k. $-26\pi$ Animation of the making of the $-26\pi$ angle.

$-26\pi = 13(-2\pi)$

NOTE: The angle 0 is the angle between $-2\pi$ and 0 which is coterminal with the angle $-26\pi$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$\cos(-26\pi) = \cos 0 = 1$

$\sin(-26\pi) = \sin 0 = 0$

$\tan(-26\pi) = \tan 0 = \dfrac{0}{1} = 0$

**Answer:** $\cos(-26\pi) = 1$, $\sin(-26\pi) = 0$, $\tan(-26\pi) = 0$

NOTE: $\cos(-26\pi) = 1 \Rightarrow \sec(-26\pi) = 1$

$$\sin(-26\pi) = 0 \Rightarrow \csc(-26\pi) = \text{undefined}$$

$$\tan(-26\pi) = 0 \Rightarrow \cot(-26\pi) = \text{undefined}$$

NOTE: The terminal side of the angle 0 (and the angle $-26\pi$) is on the positive *x*-axis. The angle 0 does not have a reference angle. You will have to use Unit Circle Trigonometry to find that $\cos 0 = 1$, $\sin 0 = 0$, and $\tan 0 = 0$.

Back to Problem 2.

**Solution to Problems on the Pre-Exam:** Back to Page 1.

2. Find the angle between 0 and $2\pi$ that is coterminal with the angle $\frac{167\pi}{6}$ and then find the exact value of $\cot \frac{167\pi}{6}$.

Animation of the making of the $\frac{167\pi}{6}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{167}{6} \Rightarrow 6\overline{)167} = 27$$

```
    27
6 )167
   12
   --
    47
    42
    --
     5
```

$$\frac{167}{6} = 27 + \frac{5}{6} = 26 + \frac{11}{6} \Rightarrow \frac{167\pi}{6} = 26\pi + \frac{11\pi}{6} = 13(2\pi) + \frac{11\pi}{6}$$

**Using subtraction to find the coterminal angle:**

$$\frac{167\pi}{6} - 13(2\pi) = \frac{167\pi}{6} - 13\left(\frac{12\pi}{6}\right) = \frac{167\pi}{6} - \frac{156\pi}{6} = \frac{11\pi}{6}$$

NOTE: This one calculation could be done in a couple calculations:

$$\frac{167\pi}{6} - 10\left(\frac{12\pi}{6}\right) = \frac{167\pi}{6} - \frac{120\pi}{6} = \frac{47\pi}{6}$$

$$\frac{47\pi}{6} - 3\left(\frac{12\pi}{6}\right) = \frac{47\pi}{6} - \frac{36\pi}{6} = \frac{11\pi}{6}$$

NOTE: The angle $\frac{11\pi}{6}$ is the angle between 0 and $2\pi$ which is coterminal with the angle $\frac{167\pi}{6}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\tan\frac{167\pi}{6} = \tan\frac{11\pi}{6} = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$$

$$\tan\frac{167\pi}{6} = -\frac{1}{\sqrt{3}} \Rightarrow \cot\frac{167\pi}{6} = -\sqrt{3}$$

Coterminal Angle: $\frac{11\pi}{6}$ $\qquad$ $\cot\frac{167\pi}{6} = -\sqrt{3}$

NOTE: The terminal side of the angle $\frac{11\pi}{6}$ (and the angle $\frac{167\pi}{6}$) is in the fourth quadrant.

NOTE: In the fourth quadrant, tangent is negative.

NOTE: The reference angle of $\frac{11\pi}{6}$ is $\frac{\pi}{6}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

3. Find the angle between $-2\pi$ and 0 that is coterminal with the angle $-\frac{55\pi}{3}$ and then find the exact value of $\cos\left(-\frac{55\pi}{3}\right)$.

Animation of the making of the $-\frac{55\pi}{3}$ angle.

**Using long division to find the coterminal angle:**

$$\frac{55}{3} \Rightarrow 3\overline{)55} = 18 \text{ R } 1$$

$$\begin{array}{r} 18 \\ 3\,)\overline{55} \\ \underline{3\phantom{5}} \\ 25 \\ \underline{24} \\ 1 \end{array}$$

$$\frac{55}{3} = 18 + \frac{1}{3} \Rightarrow -\frac{55\pi}{3} = -18\pi - \frac{\pi}{3} = 9(-2\pi) - \frac{\pi}{3}$$

**Using addition to find the coterminal angle:**

$$-\frac{55\pi}{3} + 9(2\pi) = -\frac{55\pi}{3} + 9\left(\frac{6\pi}{3}\right) = -\frac{55\pi}{3} + \frac{54\pi}{3} = -\frac{\pi}{3}$$

NOTE: This one calculation could be done in a couple calculations:

$$-\frac{55\pi}{3} + 5\left(\frac{6\pi}{3}\right) = -\frac{55\pi}{3} + \frac{30\pi}{3} = -\frac{25\pi}{3}$$

$$-\frac{25\pi}{3} + 4\left(\frac{6\pi}{3}\right) = -\frac{25\pi}{3} + \frac{24\pi}{3} = -\frac{\pi}{3}$$

NOTE: The angle $-\frac{\pi}{3}$ is the angle between $-2\pi$ and 0 which is coterminal with the angle $-\frac{55\pi}{3}$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$$\cos\left(-\frac{55\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$$

Coterminal Angle: $-\frac{\pi}{3}$ $\cos\left(-\frac{55\pi}{3}\right) = \frac{1}{2}$

NOTE: The terminal side of the angle $-\frac{\pi}{3}$ (and the angle $-\frac{55\pi}{3}$) is in the fourth quadrant.

NOTE: In the fourth quadrant, cosine is positive.

NOTE: The reference angle of $-\frac{\pi}{3}$ is $\frac{\pi}{3}$.

NOTE: You can use either Unit Circle Trigonometry or Right Triangle Trigonometry to find that $\cos\frac{\pi}{3} = \frac{1}{2}$.

4. Find the angle between $0°$ and $360°$ that is coterminal with the angle $810°$ and then find the exact value of $\sec 810°$.

Animation of the making of the $810°$ angle.

$810° - 720° = 90°$

NOTE: The angle $90°$ is the angle between $0°$ and $360°$ which is coterminal with the angle $810°$. Even though the coterminal angles are not equal, the trigonometric functions of the coterminal angles are equal.

$\cos 810° = \cos 90° = 0$

$\cos 810° = 0 \Rightarrow \sec 810° =$ undefined

Coterminal Angle: $90°$ $\sec 810° =$ undefined

NOTE: The terminal side of the angle $90°$ (and the angle $810°$) is on the positive *y*-axis. The angle $90°$ does not have a reference angle. You will have to use Unit Circle Trigonometry to find that $\cos 90° = 0$.