Pre-Class Problems 4 for Monday, February 11

These are the type of problems that you will be working on in class. These problems are from Lesson 3.

Solution to Problems on the Pre-Exam.

1. Verify the following statements:

The reference angle of the Special Angles of $\pm\frac{\pi}{6}$, $\pm\frac{5\pi}{6}$, $\pm\frac{7\pi}{6}$, and $\pm\frac{11\pi}{6}$ is $\frac{\pi}{6}$.
The reference angle of the Special Angles of $\pm\frac{\pi}{4}$, $\pm\frac{3\pi}{4}$, $\pm\frac{5\pi}{4}$, and $\pm\frac{7\pi}{4}$ is $\frac{\pi}{4}$.
The reference angle of the Special Angles of $\pm\frac{\pi}{3}$, $\pm\frac{2\pi}{3}$, $\pm\frac{4\pi}{3}$, and $\pm\frac{5\pi}{3}$ is $\frac{\pi}{3}$.

The reference angle of the Special Angles of $\pm 30^\circ$, $\pm 150^\circ$, $\pm 210^\circ$, and $\pm 330^\circ$ is 30°.
The reference angle of the Special Angles of $\pm 45^\circ$, $\pm 135^\circ$, $\pm 225^\circ$, and $\pm 315^\circ$ is 45°.
The reference angle of the Special Angles of $\pm 60^\circ$, $\pm 120^\circ$, $\pm 240^\circ$, and $\pm 300^\circ$ is 60°.

You can go to the solution for each problem by clicking on the problem letter.

2. Find the exact value of the cosine, sine, and tangent of the given angle using the reference angle of the angle if it has one.

   Objective of these problems: Since the sign (positive or negative) of any of the six trigonometric functions of a given angle whose terminal side lies in the first, second, third, or fourth quadrant is determined by that quadrant, then we only need to be able to supply the numerical number to go with that sign. The reference angle, which is an acute angle, of the given angle is used to find this numerical number using either Unit Circle Trigonometry or Right Triangle Trigonometry. If the terminal side of the angle is located on one of the coordinate axes, then the angle does not have a reference angle and you would use Unit Circle Trigonometry to find the value of the trigonometric function of the angle.

a. $\dfrac{5\pi}{6}$

b. $-135°$

c. $\dfrac{5\pi}{3}$

d. $210°$

e. $-\dfrac{5\pi}{4}$

f. $330°$

g. $-\dfrac{11\pi}{6}$

h. $120°$

i. $-\dfrac{\pi}{4}$

j. $\dfrac{4\pi}{3}$

k. $-\dfrac{7\pi}{4}$

l. $90°$

m. π

Additional problems available in the textbook: Page 505 … 31 – 36, 38 – 42, 55 – 62 (use a reference angle to help you find the angles). Example 3 on page 501.

Solutions:

2a. $\dfrac{5\pi}{6}$

Animation of the making of the $\dfrac{5\pi}{6}$ angle.

NOTE: The angle of $\dfrac{5\pi}{6}$ radians is the same as the angle of $150°$. The terminal side of this angle is in the II quadrant.

NOTE: In the second quadrant, cosine is negative, sine is positive, and tangent is negative.

NOTE: The reference angle of $\dfrac{5\pi}{6}$ is $\dfrac{\pi}{6}$.

$$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$$

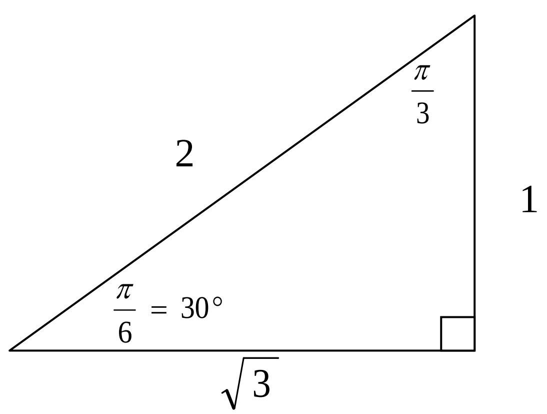
$$\tan\frac{5\pi}{6} = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos\frac{\pi}{6}$, $\sin\frac{\pi}{6}$, and $\tan\frac{\pi}{6}$:

$$P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\cos\frac{\pi}{6} = x = \frac{\sqrt{3}}{2}, \quad \sin\frac{\pi}{6} = y = \frac{1}{2}, \quad \tan\frac{\pi}{6} = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Using Right Triangle Trigonometry to find $\cos\frac{\pi}{6}$, $\sin\frac{\pi}{6}$, and $\tan\frac{\pi}{6}$:



$$\cos\frac{\pi}{6} = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}, \quad \sin\frac{\pi}{6} = \frac{opp}{hyp} = \frac{1}{2}, \quad \tan\frac{\pi}{6} = \frac{opp}{adj} = \frac{1}{\sqrt{3}}$$

Answer: $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}, \quad \sin\frac{5\pi}{6} = \frac{1}{2}, \quad \tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$

NOTE: $\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} \Rightarrow \sec \frac{5\pi}{6} = -\frac{2}{\sqrt{3}}$

$$\sin \frac{5\pi}{6} = \frac{1}{2} \Rightarrow \csc \frac{5\pi}{6} = 2$$

$$\tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}} \Rightarrow \cot \frac{5\pi}{6} = -\sqrt{3}$$

Back to Problem 2.

2b. $-135°$ Animation of the making of the $-135°$ angle.

NOTE: The angle of $-135°$ is the same as the angle of $-\frac{3\pi}{4}$ radians. The terminal side of this angle is in the III quadrant.

NOTE: In the third quadrant, cosine is negative, sine is negative, and tangent is positive.

NOTE: The reference angle of $-135°$ is $45°$.

$$\cos(-135°) = -\cos 45° = -\frac{\sqrt{2}}{2}$$

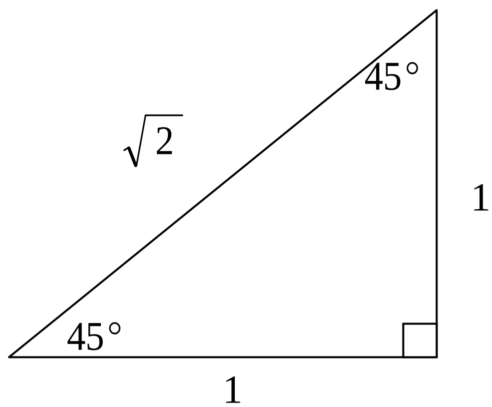
$$\sin(-135°) = -\sin 45° = -\frac{\sqrt{2}}{2}$$

$$\tan(-135°) = \tan 45° = 1$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos 45°$, $\sin 45°$, and $\tan 45°$: $P(45°) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$\cos 45° = x = \frac{\sqrt{2}}{2}, \quad \sin 45° = y = \frac{\sqrt{2}}{2}, \quad \tan 45° = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

Using Right Triangle Trigonometry to find $\cos 45°$, $\sin 45°$, and $\tan 45°$:



$$\cos 45° = \frac{adj}{hyp} = \frac{1}{\sqrt{2}}, \quad \sin 45° = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}, \quad \tan 45° = \frac{opp}{adj} = \frac{1}{1} = 1$$

Answer: $\cos(-135°) = -\frac{\sqrt{2}}{2}$, $\sin(-135°) = -\frac{\sqrt{2}}{2}$,

$\tan(-135°) = 1$

NOTE: $\cos(-135°) = -\frac{\sqrt{2}}{2} \Rightarrow \sec(-135°) = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

$\sin(-135°) = -\frac{\sqrt{2}}{2} \Rightarrow \csc(-135°) = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

$\tan(-135°) = 1 \Rightarrow \cot(-135°) = 1$

Back to Problem 2.

2c. $\dfrac{5\pi}{3}$

Animation of the making of the $\dfrac{5\pi}{3}$ angle.

NOTE: The angle of $\dfrac{5\pi}{3}$ radians is the same as the angle of $300°$. The terminal side of this angle is in the IV quadrant.

NOTE: In the fourth quadrant, cosine is positive, sine is negative, and tangent is negative.

NOTE: The reference angle of $\dfrac{5\pi}{3}$ is $\dfrac{\pi}{3}$.

$$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

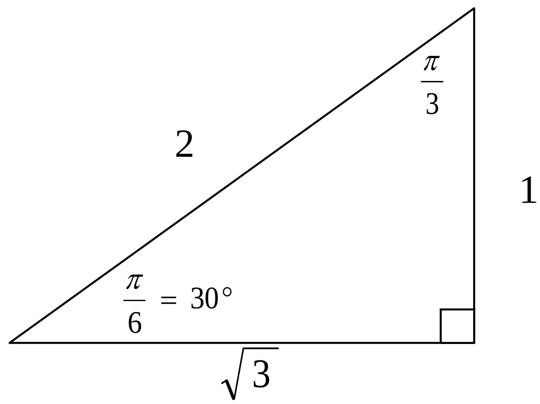
$$\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos\dfrac{\pi}{3}$, $\sin\dfrac{\pi}{3}$, and $\tan\dfrac{\pi}{3}$:

$$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\cos\frac{\pi}{3} = x = \frac{1}{2}, \quad \sin\frac{\pi}{3} = y = \frac{\sqrt{3}}{2}, \quad \tan\frac{\pi}{3} = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Using Right Triangle Trigonometry to find $\cos\frac{\pi}{3}$, $\sin\frac{\pi}{3}$, and $\tan\frac{\pi}{3}$:



$$\cos\frac{\pi}{3} = \frac{adj}{hyp} = \frac{1}{2}, \quad \sin\frac{\pi}{3} = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}, \quad \tan\frac{\pi}{3} = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Answer: $\cos\frac{5\pi}{3} = \frac{1}{2}$, $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$, $\tan\frac{5\pi}{3} = -\sqrt{3}$

NOTE: $\cos\frac{5\pi}{3} = \frac{1}{2} \Rightarrow \sec\frac{5\pi}{3} = 2$

$$\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2} \Rightarrow \csc\frac{5\pi}{3} = -\frac{2}{\sqrt{3}}$$

$$\tan\frac{5\pi}{3} = -\sqrt{3} \Rightarrow \cot\frac{5\pi}{3} = -\frac{1}{\sqrt{3}}$$

Back to Problem 2.

2d. $210°$ Animation of the making of the $210°$ angle.

NOTE: The angle of $210°$ is the same as the angle of $\frac{7\pi}{6}$ radians. The terminal side of this angle is in the III quadrant.

NOTE: In the third quadrant, cosine is negative, sine is negative, and tangent is positive.

NOTE: The reference angle of $210°$ is $30°$.

$$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$$

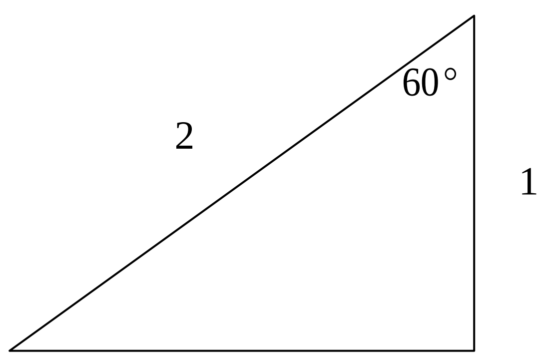
$$\sin 210° = -\sin 30° = -\frac{1}{2}$$

$$\tan 210° = \tan 30° = \frac{1}{\sqrt{3}}$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos 30°$, $\sin 30°$, and $\tan 30°$: $P(30°) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$$\cos 30° = x = \frac{\sqrt{3}}{2}, \quad \sin 30° = y = \frac{1}{2}, \quad \tan 30° = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Using Right Triangle Trigonometry to find $\cos 30°$, $\sin 30°$, and $\tan 30°$:



30°

□

$\sqrt{3}$

$\cos 30^\circ = \dfrac{adj}{hyp} = \dfrac{\sqrt{3}}{2}$, $\sin 30^\circ = \dfrac{opp}{hyp} = \dfrac{1}{2}$, $\tan 30^\circ = \dfrac{opp}{adj} = \dfrac{1}{\sqrt{3}}$

Answer: $\cos 210^\circ = -\dfrac{\sqrt{3}}{2}$, $\sin 210^\circ = -\dfrac{1}{2}$, $\tan 210^\circ = \dfrac{1}{\sqrt{3}}$

NOTE: $\cos 210^\circ = -\dfrac{\sqrt{3}}{2} \Rightarrow \sec 210^\circ = -\dfrac{2}{\sqrt{3}}$

$\sin 210^\circ = -\dfrac{1}{2} \Rightarrow \csc 210^\circ = -2$

$\tan 210^\circ = \dfrac{1}{\sqrt{3}} \Rightarrow \cot 210^\circ = \sqrt{3}$

Back to Problem 2.

2e. $-\dfrac{5\pi}{4}$ Animation of the making of the $-\dfrac{5\pi}{4}$ angle.

NOTE: The angle of $-\dfrac{5\pi}{4}$ radians is the same as the angle of -225°. The terminal side of this angle is in the II quadrant.

NOTE: In the second quadrant, cosine is negative, sine is positive, and tangent is negative.

NOTE: The reference angle of $-\dfrac{5\pi}{4}$ is $\dfrac{\pi}{4}$.

$$\cos\left(-\frac{5\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

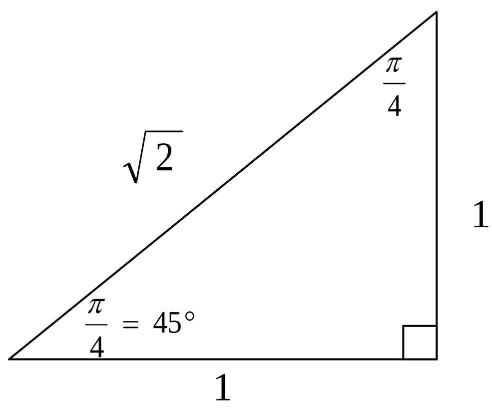
$$\sin\left(-\frac{5\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\tan\left(-\frac{5\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos\frac{\pi}{4}$, $\sin\frac{\pi}{4}$, and $\tan\frac{\pi}{4}$: $P\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$\cos\frac{\pi}{4} = x = \frac{\sqrt{2}}{2}, \quad \sin\frac{\pi}{4} = y = \frac{\sqrt{2}}{2}, \quad \tan\frac{\pi}{4} = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

Using Right Triangle Trigonometry to find $\cos\frac{\pi}{4}$, $\sin\frac{\pi}{4}$, and $\tan\frac{\pi}{4}$:



$$\cos\frac{\pi}{4} = \frac{adj}{hyp} = \frac{1}{\sqrt{2}}, \quad \sin\frac{\pi}{4} = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}, \quad \tan\frac{\pi}{4} = \frac{opp}{adj} = \frac{1}{1} = 1$$

Answer: $\cos\left(-\dfrac{5\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$, $\sin\left(-\dfrac{5\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$,

$\tan\left(-\dfrac{5\pi}{4}\right) = -1$

NOTE: $\cos\left(-\dfrac{5\pi}{4}\right) = -\dfrac{\sqrt{2}}{2} \Rightarrow \sec\left(-\dfrac{5\pi}{4}\right) = -\dfrac{2}{\sqrt{2}} = -\sqrt{2}$

$\sin\left(-\dfrac{5\pi}{4}\right) = \dfrac{\sqrt{2}}{2} \Rightarrow \csc\left(-\dfrac{5\pi}{4}\right) = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

$\tan\left(-\dfrac{5\pi}{4}\right) = -1 \Rightarrow \cot\left(-\dfrac{5\pi}{4}\right) = -1$

Back to Problem 2.

2f. $330°$ Animation of the making of the $330°$ angle.

NOTE: The angle of $330°$ is the same as the angle of $\dfrac{11\pi}{6}$ radians. The terminal side of this angle is in the IV quadrant.

NOTE: In the fourth quadrant, cosine is positive, sine is negative, and tangent is negative.

NOTE: The reference angle of $330°$ is $30°$.

$$\cos 330° = \cos 30° = \frac{\sqrt{3}}{2}$$

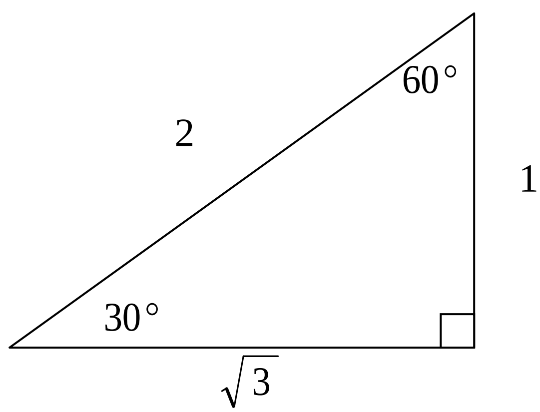
$$\sin 330^\circ = -\sin 30^\circ = -\frac{1}{2}$$

$$\tan 330^\circ = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos 30^\circ$, $\sin 30^\circ$, and $\tan 30^\circ$: $P(30^\circ) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$$\cos 30^\circ = x = \frac{\sqrt{3}}{2}, \quad \sin 30^\circ = y = \frac{1}{2}, \quad \tan 30^\circ = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Using Right Triangle Trigonometry to find $\cos 30^\circ$, $\sin 30^\circ$, and $\tan 30^\circ$:



$$\cos 30^\circ = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}, \quad \sin 30^\circ = \frac{opp}{hyp} = \frac{1}{2}, \quad \tan 30^\circ = \frac{opp}{adj} = \frac{1}{\sqrt{3}}$$

Answer: $\cos 330^\circ = \frac{\sqrt{3}}{2}$, $\sin 330^\circ = -\frac{1}{2}$, $\tan 330^\circ = -\frac{1}{\sqrt{3}}$

NOTE: $\cos 330° = \dfrac{\sqrt{3}}{2} \Rightarrow \sec 330° = \dfrac{2}{\sqrt{3}}$

$\sin 330° = -\dfrac{1}{2} \Rightarrow \csc 330° = -2$

$\tan 330° = -\dfrac{1}{\sqrt{3}} \Rightarrow \cot 210° = -\sqrt{3}$

Back to Problem 2.

2g. $-\dfrac{11\pi}{6}$ Animation of the making of the $-\dfrac{11\pi}{6}$ angle.

NOTE: The angle of $-\dfrac{11\pi}{6}$ radians is the same as the angle of $-330°$. The terminal side of this angle is in the I quadrant.

NOTE: In the first quadrant, cosine is positive, sine is positive, and tangent is positive.

NOTE: The reference angle of $-\dfrac{11\pi}{6}$ is $\dfrac{\pi}{6}$.

$$\cos\left(-\frac{11\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{11\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

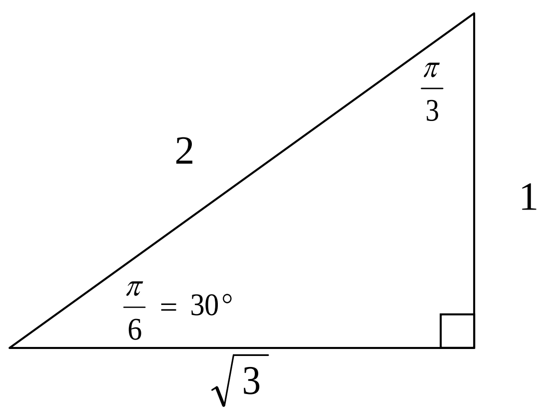
$$\tan\left(-\frac{11\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos\frac{\pi}{6}$, $\sin\frac{\pi}{6}$, and $\tan\frac{\pi}{6}$:

$$P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\cos\frac{\pi}{6} = x = \frac{\sqrt{3}}{2}, \quad \sin\frac{\pi}{6} = y = \frac{1}{2}, \quad \tan\frac{\pi}{6} = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Using Right Triangle Trigonometry to find $\cos\frac{\pi}{6}$, $\sin\frac{\pi}{6}$, and $\tan\frac{\pi}{6}$:



$$\cos\frac{\pi}{6} = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}, \quad \sin\frac{\pi}{6} = \frac{opp}{hyp} = \frac{1}{2}, \quad \tan\frac{\pi}{6} = \frac{opp}{adj} = \frac{1}{\sqrt{3}}$$

Answer: $\cos\left(-\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\sin\left(-\frac{11\pi}{6}\right) = \frac{1}{2}$, $\tan\left(-\frac{11\pi}{6}\right) = \frac{1}{\sqrt{3}}$

NOTE: $\cos\left(-\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2} \Rightarrow \sec\left(-\frac{11\pi}{6}\right) = \frac{2}{\sqrt{3}}$

$$\sin\left(-\frac{11\pi}{6}\right) = \frac{1}{2} \Rightarrow \csc\left(-\frac{11\pi}{6}\right) = 2$$

$$\tan\left(-\frac{11\pi}{6}\right) = \frac{1}{\sqrt{3}} \Rightarrow \cot\left(-\frac{11\pi}{6}\right) = \sqrt{3}$$

Back to Problem 2.

2h. $120°$ Animation of the making of the $120°$ angle.

NOTE: The angle of $120°$ is the same as the angle of $\frac{2\pi}{3}$ radians. The terminal side of this angle is in the II quadrant.

NOTE: In the second quadrant, cosine is negative, sine is positive, and tangent is negative.

NOTE: The reference angle of $120°$ is $60°$.

$$\cos 120° = -\cos 60° = -\frac{1}{2}$$

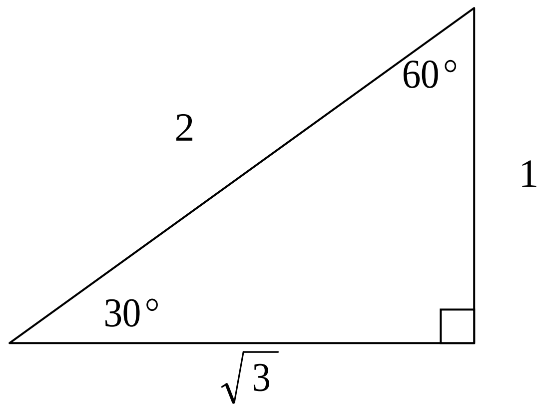
$$\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$$

$$\tan 120° = -\tan 60° = -\sqrt{3}$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos 60°$, $\sin 60°$, and $\tan 60°$: $P(60°) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$\cos 60^\circ = x = \frac{1}{2}, \quad \sin 60^\circ = y = \frac{\sqrt{3}}{2}, \quad \tan 60^\circ = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Using Right Triangle Trigonometry to find $\cos 60^\circ$, $\sin 60^\circ$, and $\tan 60^\circ$:



$$\cos 60^\circ = \frac{adj}{hyp} = \frac{1}{2}, \ \sin 60^\circ = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}, \ \tan 60^\circ = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Answer: $\cos 120^\circ = -\frac{1}{2}, \quad \sin 120^\circ = \frac{\sqrt{3}}{2}, \quad \tan 120^\circ = -\sqrt{3}$

NOTE: $\cos 120^\circ = -\frac{1}{2} \Rightarrow \sec 120^\circ = -2$

$$\sin 120^\circ = \frac{\sqrt{3}}{2} \Rightarrow \csc 120^\circ = \frac{2}{\sqrt{3}}$$

$$\tan 120^\circ = -\sqrt{3} \Rightarrow \cot 120^\circ = -\frac{1}{\sqrt{3}}$$

Back to Problem 2.

2i. $-\frac{\pi}{4}$ Animation of the making of the $-\frac{\pi}{4}$ angle.

NOTE: The angle of $-\frac{\pi}{4}$ radians is the same as the angle of $-45°$. The terminal side of this angle is in the IV quadrant.

NOTE: In the fourth quadrant, cosine is positive, sine is negative, and tangent is negative.

NOTE: The reference angle of $-\frac{\pi}{4}$ is $\frac{\pi}{4}$.

$$\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

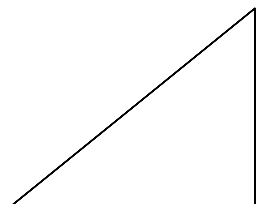
$$\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$$

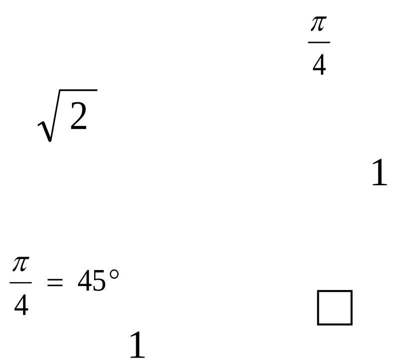
Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos\frac{\pi}{4}$, $\sin\frac{\pi}{4}$, and $\tan\frac{\pi}{4}$:

$$P\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\cos\frac{\pi}{4} = x = \frac{\sqrt{2}}{2}, \quad \sin\frac{\pi}{4} = y = \frac{\sqrt{2}}{2}, \quad \tan\frac{\pi}{4} = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

Using Right Triangle Trigonometry to find $\cos\frac{\pi}{4}$, $\sin\frac{\pi}{4}$, and $\tan\frac{\pi}{4}$:





$$\cos\frac{\pi}{4} = \frac{adj}{hyp} = \frac{1}{\sqrt{2}}\ , \quad \sin\frac{\pi}{4} = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}\ , \quad \tan\frac{\pi}{4} = \frac{opp}{adj} = \frac{1}{1} = 1$$

Answer: $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\ , \quad \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \quad \tan\left(-\frac{\pi}{4}\right) = -1$

NOTE: $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow \sec\left(-\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \Rightarrow \csc\left(-\frac{\pi}{4}\right) = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

$$\tan\left(-\frac{\pi}{4}\right) = -1 \Rightarrow \cot\left(-\frac{\pi}{4}\right) = -1$$

Back to Problem 2.

2j. $\frac{4\pi}{3}$ Animation of the making of the $\frac{4\pi}{3}$ angle.

NOTE: The angle of $\frac{4\pi}{3}$ radians is the same as the angle of $240°$. The terminal side of this angle is in the III quadrant.

NOTE: In the third quadrant, cosine is negative, sine is negative, and tangent is positive.

NOTE: The reference angle of $\dfrac{4\pi}{3}$ is $\dfrac{\pi}{3}$.

$$\cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

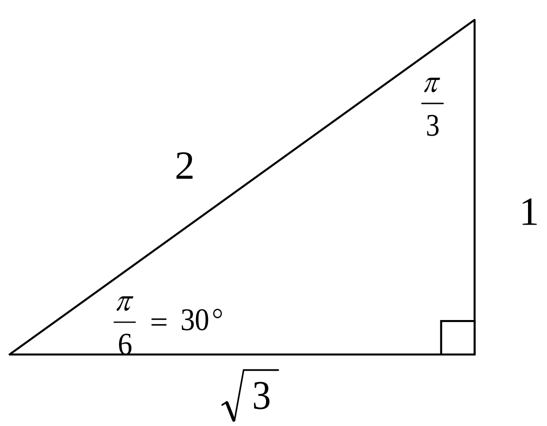
$$\tan\frac{4\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos\dfrac{\pi}{3}$, $\sin\dfrac{\pi}{3}$, and $\tan\dfrac{\pi}{3}$:

$$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\cos\frac{\pi}{3} = x = \frac{1}{2}, \quad \sin\frac{\pi}{3} = y = \frac{\sqrt{3}}{2}, \quad \tan\frac{\pi}{3} = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Using Right Triangle Trigonometry to find $\cos\dfrac{\pi}{3}$, $\sin\dfrac{\pi}{3}$, and $\tan\dfrac{\pi}{3}$:



$$\cos\frac{\pi}{3} = \frac{adj}{hyp} = \frac{1}{2}, \quad \sin\frac{\pi}{3} = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}, \quad \tan\frac{\pi}{3} = \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Answer: $\cos\frac{4\pi}{3} = -\frac{1}{2}, \quad \sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}, \quad \tan\frac{4\pi}{3} = \sqrt{3}$

NOTE: $\cos\frac{4\pi}{3} = -\frac{1}{2} \Rightarrow \sec\frac{4\pi}{3} = -2$

$$\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2} \Rightarrow \csc\frac{4\pi}{3} = -\frac{2}{\sqrt{3}}$$

$$\tan\frac{4\pi}{3} = \sqrt{3} \Rightarrow \cot\frac{4\pi}{3} = \frac{1}{\sqrt{3}}$$

Back to Problem 2.

2k. $-\frac{7\pi}{4}$ Animation of the making of the $-\frac{7\pi}{4}$ angle.

NOTE: The angle of $-\frac{7\pi}{4}$ radians is the same as the angle of $-315°$. The terminal side of this angle is in the I quadrant.

NOTE: In the first quadrant, cosine is positive, sine is positive, and tangent is positive.

NOTE: The reference angle of $-\frac{7\pi}{4}$ is $\frac{\pi}{4}$.

$$\cos\left(-\frac{7\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

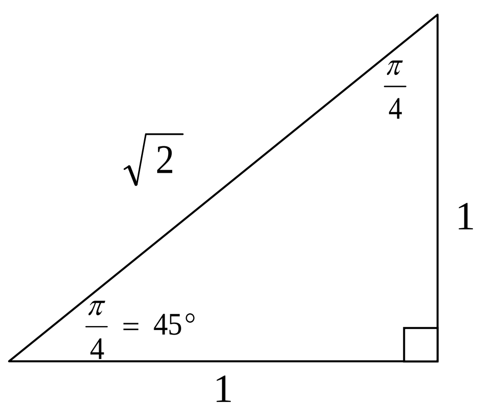
$$\sin\left(-\frac{7\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\tan\left(-\frac{7\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos\frac{\pi}{4}$, $\sin\frac{\pi}{4}$, and $\tan\frac{\pi}{4}$: $P\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$\cos\frac{\pi}{4} = x = \frac{\sqrt{2}}{2}, \quad \sin\frac{\pi}{4} = y = \frac{\sqrt{2}}{2}, \quad \tan\frac{\pi}{4} = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

Using Right Triangle Trigonometry to find $\cos\frac{\pi}{4}$, $\sin\frac{\pi}{4}$, and $\tan\frac{\pi}{4}$:



$$\cos\frac{\pi}{4} = \frac{adj}{hyp} = \frac{1}{\sqrt{2}}, \quad \sin\frac{\pi}{4} = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}, \quad \tan\frac{\pi}{4} = \frac{opp}{adj} = \frac{1}{1} = 1$$

Answer: $\cos\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}, \quad \sin\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}, \quad \tan\left(-\frac{7\pi}{4}\right) = 1$

NOTE: $\cos\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow \sec\left(-\frac{7\pi}{4}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\sin\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow \csc\left(-\frac{7\pi}{4}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\tan\left(-\frac{7\pi}{4}\right) = 1 \Rightarrow \cot\left(-\frac{7\pi}{4}\right) = 1$

Back to Problem 2.

2l. $90°$ Animation of the making of the $90°$ angle.

NOTE: The angle of $90°$ is the same as the angle of $\frac{\pi}{2}$ radians. The terminal side of this angle is on the positive *y*-axis.

NOTE: The angle of $90°$ does **NOT** have a reference angle. You will need to find the values of $\cos 90°$, $\sin 90°$, and $\tan 90°$ using Unit Circle Trigonometry.

Using Unit Circle Trigonometry, $P(\theta) = (\cos\theta, \sin\theta)$, to find $\cos 90°$, $\sin 90°$, and $\tan 90°$: $P(90°) = (0, 1)$

$\cos 90° = x = 0$, $\sin 90° = y = 1$, $\tan 90° = \frac{y}{x} = \frac{1}{0} =$ undefined

Answer: $\cos 90° = 0$, $\sin 90° = 1$, $\tan 90°$ is undefined

NOTE: $\cos 90° = 0 \Rightarrow \sec 90°$ is undefined

$\sin 90° = 1 \Rightarrow \csc 90° = 1$

$$\cot 90° = \frac{x}{y} = \frac{0}{1} = 0$$

Back to Problem 2.

2m. π Animation of the making of the π angle.

NOTE: The angle of π radians is the same as the angle of $180°$. The terminal side of this angle is on the negative *x*-axis.

NOTE: The angle of π does **NOT** have a reference angle. You will need to find the values of $\cos \pi$, $\sin \pi$, and $\tan \pi$ using Unit Circle Trigonometry.

Using Unit Circle Trigonometry, $P(\theta) = (\cos \theta, \sin \theta)$, to find $\cos \pi$, $\sin \pi$, and $\tan \pi$: $P(\pi) = (-1, 0)$

$$\cos \pi = x = -1, \quad \sin \pi = y = 0, \quad \tan \pi = \frac{y}{x} = \frac{0}{-1} = 0$$

Answer: $\cos \pi = -1$, $\sin \pi = 0$, $\tan \pi = 0$

NOTE: $\cos \pi = -1 \Rightarrow \sec \pi = -1$

$\sin \pi = 0 \Rightarrow \csc \pi$ is undefined

$\tan \pi = 0 \Rightarrow \cot \pi$ is undefined

Back to Problem 2.

Solution to Problems on the Pre-Exam: Back to Page 1.

1a. Find the exact value of $\sin \frac{4\pi}{3}$.

Animation of the making of the $\frac{4\pi}{3}$ angle.

NOTE: The terminal side of the angle $\frac{4\pi}{3}$ is in the third quadrant. The reference angle of $\frac{4\pi}{3}$ is $\frac{\pi}{3}$. Sine is negative in the third quadrant.

$$\sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

Answer: $-\frac{\sqrt{3}}{2}$

1c. Find the exact value of $\csc(-315°)$.

Animation of the making of the $-315°$ angle.

NOTE: The terminal side of the angle $-315°$ is in the first quadrant. The reference angle of $-315°$ is $45°$. Sine (and cosecant) are positive in the first quadrant.

$$\sin(-315°) = \sin 45° = \frac{\sqrt{2}}{2}$$

$$\sin(-315°) = \frac{\sqrt{2}}{2} \Rightarrow \csc(-315°) = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Answer: $\sqrt{2}$