MATH-1320 Sample Exam 3 Spring 2017

1. Sketch the graph of the following functions. Label number(s) on the *x*-axis and/or *y*-axis to help identify your sketch. Then state the domain and range of the function in interval notation. (14 pts.)

   a. $f(x) = -3e^{x-4} - 1$ b. $h(x) = \log_{2/3}(x+5) + 8$

2. Use the properties of logarithms to write the following as a sum and/or difference of logarithms. All variables represent positive numbers. (7 pts.) **Put a box around your answer.**

$$\ln \frac{x(x^2-5)^3}{\sqrt[3]{(4x+7)^4}}$$

3. Write $3\log_{1/3} x + \frac{1}{4}\log_{1/3}(3x+5) - 2\log_{1/3}(x+7) - \log_{1/3}(4x^3-9)$ as a single logarithm. (6 pts.) **Put a box around your answer.**

4. Solve the following equations.

   a. $2^{3x-11} = 32$ (6 pts.)   b. $3^{7x+4} = 49$ (7 pts.)

   c. $\log_2 x + \log_2(x-12) = 6$ (9 pts.)

5. Solve the following systems of equations by the indicated method.

   a. $\begin{aligned} 4x + y &= -7 \\ 3x + 5y &= 16 \end{aligned}$ using the substitution method (7 pts.)

   b. $\begin{aligned} 8x + 3y &= -16 \\ 2x - 5y &= 19 \end{aligned}$ using the addition method (6 pts.)

c. $\begin{aligned} x^2 + y^2 &= 28 \\ x^2 + (y+4)^2 &= 4 \end{aligned}$ using any method (9 pts.)

6. How many liters of a 6% salt solution and how many liter of a 25% salt solution are needed to make 38 liters of a 20% salt solution? Set up a system of equations to solve this problem. Don't forget to identify your variables. (6 pts.) **Do NOT solve the system. Put a box around your answer.**

7. Determine the solution for the system represented by each augmented matrix. (10 pts.)

a. $\left[\begin{array}{ccc|c} 1 & -2 & 5 & 9 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 3 & -5 & 7 & -11 \\ 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$

8. Solve the following system of equations using Gaussian elimination. Indicate your row operations. (12 pts.)

$$\begin{aligned} x - 3y - 2z &= -1 \\ 3x + y + 5z &= 32 \\ -4x + 6y - z &= -29 \end{aligned}$$

NOTE: There will be a 3-point problem on the exam that is not on this sample exam.

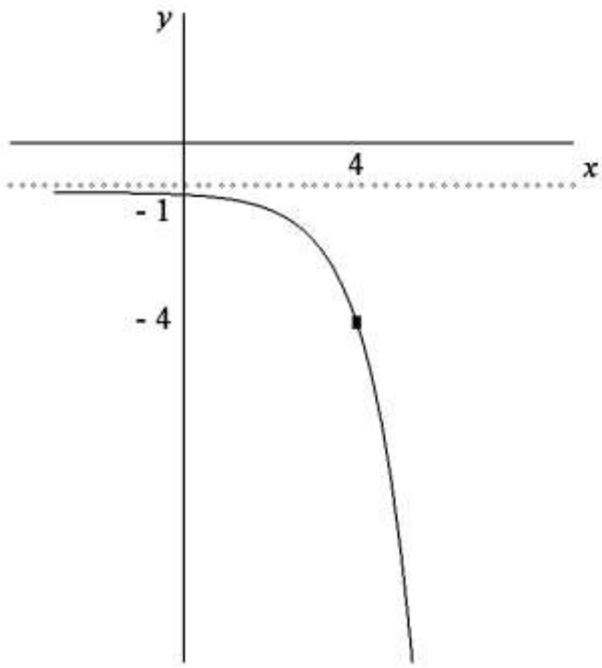
SOLUTIONS:

1a. $f(x) = -3e^{x-4} - 1$ Back to Problem 1.

To graph the function f, we set $f(x) = y$ and graph the equation $y = -3e^{x-4} - 1$.

$$y = -3e^{x-4} - 1 \Rightarrow y + 1 = -3e^{x-4}$$

The graph of $y + 1 = -3e^{x-4}$ is the graph of $y = -3e^{x}$ shifted 4 units to the right and 1 unit downward.



The Drawing of this Sketch

Domain: $(-\infty, \infty)$

Range: $(-\infty, -1)$.

The *y*-coordinate of the *y*-intercept is obtained by setting $x = 0$ in the equation $y = -3e^{x-4} - 1$. Thus, we have that $y = -3e^{-4} - 1$. Thus, the *y*-intercept is the point $(0, -3e^{-4} - 1)$.

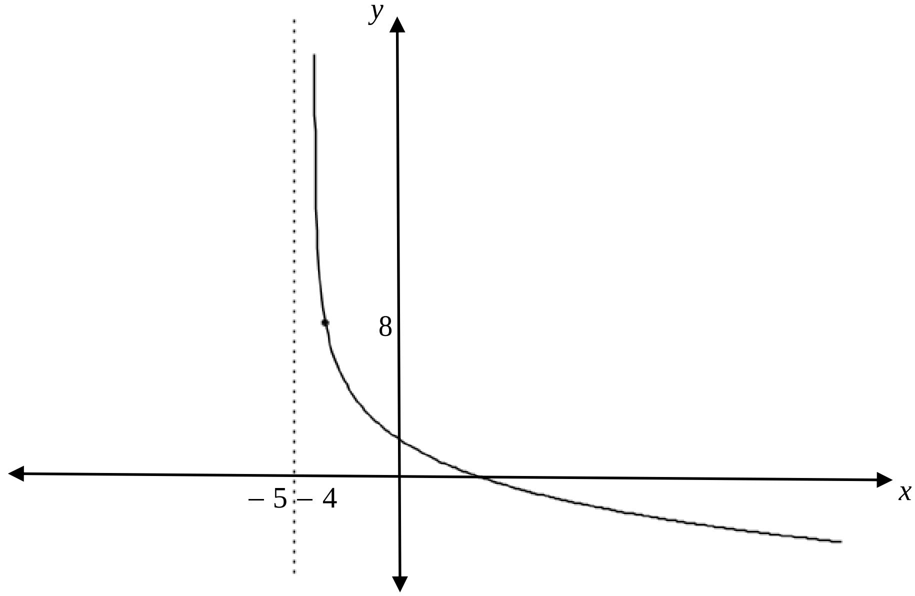
NOTE: The horizontal shift of 4 units to the right is determined from the expression $x - 4$ in the equation $y + 1 = -3e^{x-4}$ and the vertical shift of 1 unit downward is determined from the expression $y + 1$ in the equation.

1b. $h(x) = \log_{2/3}(x + 5) + 8$ Back to Problem 1.

To graph the function h, we set $h(x) = y$ and graph the equation $y = \log_{2/3}(x + 5) + 8$.

$y = \log_{2/3}(x + 5) + 8 \Rightarrow y - 8 = \log_{2/3}(x + 5)$

The graph of $y - 8 = \log_{2/3}(x + 5)$ is the graph of $y = \log_{2/3} x$ shifted 5 units to the left and 8 units upward.



Since we can only take the logarithm of positive numbers, we need that $x + 5$ be positive. That is, we need that $x + 5 > 0 \Rightarrow x > -5$

Domain: $(-5, \infty)$

Range: $(-\infty, \infty)$.

2. $\ln \dfrac{x(x^2 - 5)^3}{\sqrt[3]{(4x + 7)^4}}$ Back to Problem 2.

$$\ln \frac{x(x^2-5)^3}{\sqrt[3]{(4x+7)^4}} = \boxed{\ln x + 3\ln(x^2-5) - \frac{4}{3}\ln(4x+7)}$$

NOTE: $$\ln \frac{x(x^2-5)^3}{\sqrt[3]{(4x+7)^4}} = \ln x(x^2-5)^3 - \ln \sqrt[3]{(4x+7)^4}$$

$$\ln x(x^2-5)^3 = \ln x + \ln(x^2-5)^3 = \ln x + 3\ln(x^2-5)$$

$$\ln \sqrt[3]{(4x+7)^4} = \ln(4x+7)^{4/3} = \frac{4}{3}\ln(4x+7)$$

3. $$3\log_{1/3} x + \frac{1}{4}\log_{1/3}(3x+5) - 2\log_{1/3}(x+7) - \log_{1/3}(4x^3-9) =$$

$$\log_{1/3} x^3 + \log_{1/3} \sqrt[4]{3x+5} - \log_{1/3}(x+7)^2 - \log_{1/3}(4x^3-9) =$$

$$\boxed{\log_{1/3} \frac{x^3 \sqrt[4]{3x+5}}{(x+7)^2(4x^3-9)}}$$

Back to Problem 3.

NOTE: $$\frac{1}{4}\log_{1/3}(3x+5) = \log_{1/3}(3x+5)^{1/4} = \log_{1/3}\sqrt[4]{3x+5}$$

NOTE: Positive logarithms go in the numerator and negative logarithms go in the denominator.

4a. $2^{3x-11} = 32$

Back to Problem 4.

Using the one-to-one property: $2^{3x-11} = 32 \Rightarrow 2^{3x-11} = 2^5 \Rightarrow$

$3x - 11 = 5 \Rightarrow 3x = 16 \Rightarrow x = \dfrac{16}{3}$

Answer: $x = \dfrac{16}{3}$

4b. $3^{7x+4} = 49$

Back to Problem 4.

Using natural logarithms: $3^{7x+4} = 49 \Rightarrow \ln 3^{7x+4} = \ln 49 \Rightarrow$

$(7x + 4)\ln 3 = \ln 49 \Rightarrow 7x\ln 3 + 4\ln 3 = \ln 49 \Rightarrow$

$7x\ln 3 = \ln 49 - 4\ln 3 \Rightarrow x = \dfrac{\ln 49 - 4\ln 3}{7\ln 3}$

Answer: $x = \dfrac{\ln 49 - 4\ln 3}{7\ln 3}$

4c. $\log_2 x + \log_2 (x - 12) = 6$

Back to Problem 4.

$\log_2 x + \log_2 (x - 12) = 6 \Rightarrow \log_2 x(x - 12) = 6$

$\log_2 x(x - 12) = 6 \Rightarrow x(x - 12) = 2^6 \Rightarrow x(x - 12) = 64$

Solving the equation $x(x - 12) = 64$, we have that

$x(x - 12) = 64 \Rightarrow x^2 - 12x = 64 \Rightarrow x^2 - 12x - 64 = 0 \Rightarrow$

$(x+4)(x-16)=0 \Rightarrow x=-4,\ x=16$

When $x=-4$, we have that $x=-4<0$. Thus, when $x=-4$, we have that $\log_2 x=\log_2(-4)$. However, $\log_2(-4)$ is undefined. Thus, -4 is a solution of the equation $x(x-12)=64$, but it is not a solution of the equation $\log_2 x+\log_2(x-12)=6$.

When $x=16$, we have that $x=16>0$ and $x-12=16-12>0$. Thus, 16 is a solution of the equation $\log_2 x+\log_2(x-12)=6$.

Answer: $x=16$

5a. $\begin{aligned} 4x+y&=-7 \\ 3x+5y&=16 \end{aligned}$

Back to Problem 5.

$4x+y=-7 \Rightarrow y=-4x-7$

$3x+5y=16 \Rightarrow 3x+5(-4x-7)=16 \Rightarrow 3x-20x-35=16 \Rightarrow$

$-17x-35=16 \Rightarrow -17x=51 \Rightarrow x=-3$

$y=-4x-7,\ x=-3$:

$y=-4x-7,\ x=-3 \Rightarrow y=-4(-3)-7=12-7=5$

Answer: $(-3,\ 5)$

5b. $\begin{aligned} 8x + 3y &= -16 \\ 2x - 5y &= 19 \end{aligned}$

Back to Problem 5.

$$\begin{aligned} 8x + 3y &= -16 \\ 2x - 5y &= 19 \end{aligned} \Rightarrow \begin{array}{r} 8x + 3y = -16 \\ \underline{-8x + 20y = -76} \\ 23y = -92 \end{array} \Rightarrow y = -4$$

Now, use the second equation to find the value of x when $y = -4$:

$$2x - 5y = 19,\ y = -4 \Rightarrow 2x + 20 = 19 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$$

Answer: $\left(-\frac{1}{2}, -4\right)$

5c. $\begin{aligned} x^2 + y^2 &= 28 \\ x^2 + (y+4)^2 &= 4 \end{aligned}$

Back to Problem 5.

Using the addition method: $\begin{aligned} x^2 + y^2 &= 28 \\ x^2 + (y+4)^2 &= 4 \end{aligned} \Rightarrow \begin{array}{r} -x^2 - y^2 = -28 \\ \underline{x^2 + (y+4)^2 = 4} \\ (y+4)^2 - y^2 = -24 \end{array}$

$$(y+4)^2 - y^2 = -24 \Rightarrow y^2 + 8y + 16 - y^2 = -24 \Rightarrow$$

$$8y + 16 = -24 \Rightarrow 8y = -40 \Rightarrow y = -5$$

$$x^2 + y^2 = 28,\ y = -5 \Rightarrow x^2 + 25 = 28 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

Using substitution: $x^2 + y^2 = 28 \Rightarrow x^2 = 28 - y^2$

$$x^2 + (y+4)^2 = 4,\ x^2 = 28 - y^2 \Rightarrow 28 - y^2 + (y+4)^2 = 4 \Rightarrow$$

$$28 - y^2 + y^2 + 8y + 16 = 4 \Rightarrow 44 + 8y = 4 \Rightarrow 8y = -40 \Rightarrow$$

$$y = -5$$

$$x^2 + y^2 = 28,\ y = -5 \Rightarrow x^2 + 25 = 28 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

Answer: $(-\sqrt{3}, -5)\ (\sqrt{3}, -5)$

6.

Solution	Amount of Solution	Percent of Salt	Amount of Salt
6% Salt	x	6% = 0.06	$0.06x$
25% Salt	y	25% = 0.25	$0.25y$
20% Salt	38	20% = 0.2	0.2(38) = 7.6

$x + y = 38$ and $0.06x + 0.25y = 7.6$.

We can simplify the second equation by multiplying both sides of the equation by 100:

$$0.06x + 0.25y = 7.6 \Rightarrow 6x + 25y = 760$$

$$\boxed{\begin{aligned} x + y &= 38 \\ 6x + 25y &= 760 \end{aligned}}$$

Back to Problem 6.

7a. $\left[\begin{array}{ccc|c} 1 & -2 & 5 & 9 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Back to Problem 7.

Row 3 reads $0x + 0y + 0z = 0$, which is a true equation.

Row 2 reads $y + 3z = 7$

Let $z = t$, where *t* is any real number. Then $y + 3z = 7 \Rightarrow$

$y + 3t = 7 \Rightarrow y = 7 - 3t$

Row 1 reads $x - 2y + 5z = 9$

Since $y = 7 - 3t$ and $z = t$, then $x - 2(7 - 3t) + 5t = 9 \Rightarrow$

$x - 14 + 6t + 5t = 9 \Rightarrow x - 14 + 11t = 9 \Rightarrow x = 23 - 11t$

Answer: $(23 - 11t, 7 - 3t, t)$, where *t* is any real number

7b. $\left[\begin{array}{ccc|c} 3 & -5 & 7 & -11 \\ 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$ Back to Problem 7.

Row 3 reads $0x + 0y + 0z = 4$, which is a false equation.

Answer: No solution

8. $\begin{aligned} x - 3y - 2z &= -1 \\ 3x + y + 5z &= 32 \\ -4x + 6y - z &= -29 \end{aligned}$ Back to Problem 8.

$$\left[\begin{array}{cccc} 1 & -3 & -2 & -1 \\ 3 & 1 & 5 & 32 \\ -4 & 6 & -1 & -29 \end{array}\right] \xrightarrow[4R_1+R_3]{-3R_1+R_2} \left[\begin{array}{cccc} 1 & -3 & -2 & -1 \\ 0 & 10 & 11 & 35 \\ 0 & -6 & -9 & -33 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3}$$

$$\left[\begin{array}{cccc} 1 & -3 & -2 & -1 \\ 0 & 10 & 11 & 35 \\ 0 & 2 & 3 & 11 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc} 1 & -3 & -2 & -1 \\ 0 & 2 & 3 & 11 \\ 0 & 10 & 11 & 35 \end{array}\right] \xrightarrow{-5R_2+R_3}$$

$$\left[\begin{array}{cccc} 1 & -3 & -2 & -1 \\ 0 & 2 & 3 & 11 \\ 0 & 0 & -4 & -20 \end{array}\right] \xrightarrow{-\frac{1}{4}R_3} \left[\begin{array}{cccc} 1 & -3 & -2 & -1 \\ 0 & 2 & 3 & 11 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Row 3 reads $0x + 0y + 1z = 5 \Rightarrow z = 5$

Row 2 reads $0x + 2y + 3z = 11 \Rightarrow 2y + 3z = 11$

$2y + 3z = 11,\ z = 5 \Rightarrow 2y + 15 = 11 \Rightarrow 2y = -4 \Rightarrow y = -2$

Row 1 reads $x - 3y - 2z = -1$

$x - 3y - 2z = -1,\ y = -2,\ z = 5 \Rightarrow$

$x + 6 - 10 = -1 \Rightarrow x - 4 = -1 \Rightarrow x = 3$

Answer: $(3, -2, 5)$