MATH-1320 Sample Exam 2 Spring 2017

1. Find the domain of the following functions. Write your answer using interval notation. (9 pts.)

   a. $f(x) = \dfrac{x^2 - 9}{12x^2 - 80x + 48}$ b. $g(x) = \sqrt{16 - 9x}$

2. Write the equation of the circle in standard form given the following information.

   a. Center: $(-1, 3)$; Radius: $\sqrt{5}$ (5 pts.)

   b. The endpoints of a diameter are $(-6, -2)$ and $(-5, 3)$. (8 pts.)

   c. The center is $(4, 0)$ and the point $(-3, 6)$ is a point on the circle. (6 pts.)

3. Find the center and radius of the circle whose equation is given by $x^2 + y^2 - 10x + 18y + 20 = 0$. (8 pts.)

4. If $f(x) = 3x^2 - 5x$, then find the average rate of change of the function f on the interval $[4, 4 + h]$, where $h > 0$. (8 pts.)

5. If $f(x) = 8 - 3x$ and $g(x) = 4x - 7$, then find $(f \circ g)(x)$. (5 pts.)

6. Sketch the graph of $h(x) = -\sqrt[3]{6x + 11}$. (5 pts.)

7. For the function $f(x) = 2(x - 4)^2 - 3$, sketch the graph and identify the following.

   a. horizontal shift (2 pts.)
   b. vertical shift (2 pts.)
   c. range of the function (3 pts.)
   d. interval(s) on which the function is increasing (2 pts.)
   e. interval(s) on which the function is decreasing (2 pts.)

f. value of relative (local) maximum(s) and location(s) (2 pts.)
g. value of relative (local) minimum(s) and location(s) (2 pts.)
h. x-intercept(s) (5 pts.)

8. Find the zeros (roots) and their multiplicities of $g(x) = (6 - x)(x - 3)^2$. Determine the sign of the infinity that the polynomial values approaches as x approaches positive infinity and negative infinity. Sketch a graph of the polynomial. (10 pts.)

9. If $h(x) = 2x^4 + x^3 - 15x + 12$, then use the Remainder Theorem to find $h(7)$. (4 pts.)

10. Identify the possible rational zeros (roots) of the polynomial $f(x) = 4x^3 - 4x^2 - 9x + 30$. Then find the zeros (roots). Write a factorization for $4x^3 - 4x^2 - 9x + 30$. (12 pts.)

11. Given the rational function $g(x) = \dfrac{2x + 3}{x^2 + 4x - 12}$, then find the following.

a. vertical asymptote(s) (3 pts.)
b. horizontal asymptote(s) (3 pts.)
c. if the function has a horizontal asymptote, determine if the graph crosses the asymptote (3 pts.)
d. sketch the graph of the function (6 pts.)

12. Solve $\dfrac{6 - x}{3x - 14} \le 0$. Write your answer using interval notation. (8 pts.)

SOLUTIONS:

1a. $f(x) = \dfrac{x^2 - 9}{12x^2 - 80x + 48}$

Want (Need): $12x^2 - 80x + 48 \neq 0$

$12x^2 - 80x + 48 \neq 0 \Rightarrow 4(3x^2 - 20x + 12) \neq 0 \Rightarrow 4(x-6)(3x-2) \neq 0 \Rightarrow$

$x - 6 \neq 0$ **and** $3x - 2 \neq 0 \Rightarrow x \neq 6$ **and** $x \neq \dfrac{2}{3}$

Answer: $\left(-\infty, \dfrac{2}{3}\right) \cup \left(\dfrac{2}{3}, 6\right) \cup (6, \infty)$ Back to Problem 1.

1b. $g(x) = \sqrt{16 - 9x}$

Want (Need): $16 - 9x \geq 0$

$16 - 9x \geq 0 \Rightarrow 16 \geq 9x \Rightarrow \dfrac{16}{9} \geq x \Rightarrow x \leq \dfrac{16}{9}$

Answer: $\left(-\infty, \dfrac{16}{9}\right]$ Back to Problem 1.

2a. Center: $(-1, 3)$; Radius: $\sqrt{5}$

The standard form of the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$, where the point (h, k) is the center of the circle and r is the radius.

$h = -1,\ k = 3,\ r = \sqrt{5} \Rightarrow (x + 1)^2 + (y - 3)^2 = 5$

Answer: $(x+1)^2 + (y-3)^2 = 5$ Back to Problem 2.

2b. The endpoints of a diameter are $(-6, -2)$ and $(-5, 3)$.

NOTE: The diameter of a circle is a line segment passing through the center of the circle and whose endpoints lie on the circle. The midpoint of the diameter is the center of the circle and the length of the diameter is twice the radius of the circle.

Use the midpoint formula to find the center of the circle:

$$\left(\frac{-6-5}{2}, \frac{-2+3}{2}\right) = \left(-\frac{11}{2}, \frac{1}{2}\right)$$

Use the distance formula to find the length of the diameter in order to find the radius:

$$d = \sqrt{(-6+5)^2 + (-2-3)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$$

Since the diameter is $\sqrt{26}$, then the radius is $\dfrac{\sqrt{26}}{2}$.

Center: $\left(-\dfrac{11}{2}, \dfrac{1}{2}\right)$; Radius: $\dfrac{\sqrt{26}}{2}$

The standard form of the equation for a circle: $(x-h)^2 + (y-k)^2 = r^2$, where the point (h, k) is the center of the circle and r is the radius.

$$h = -\frac{11}{2},\ k = \frac{1}{2},\ r = \frac{\sqrt{26}}{2} \Rightarrow \left(x + \frac{11}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{13}{2}$$

NOTE: $\left(\frac{\sqrt{26}}{2}\right)^2 = \frac{26}{4} = \frac{13}{2}$

Answer: $\left(x + \frac{11}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{13}{2}$ Back to Problem 2.

2c. The center is $(4, 0)$ and the point $(-3,\ 6)$ is a point on the circle.

The standard form of the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$, where the point (h, k) is the center of the circle and r is the radius.

$$h = 4,\ k = 0,\ r = ? \Rightarrow (x - 4)^2 + (y - 0)^2 = r^2 \Rightarrow (x - 4)^2 + y^2 = r^2$$

In order for the point $(-3,\ 6)$ to be on the circle, it must satisfy the equation for the circle. Thus,

$$(-3 - 4)^2 + 6^2 = r^2 \Rightarrow r^2 = (-7)^2 + 36 = 49 + 36 = 85$$

$$r^2 = 85 \Rightarrow (x - 4)^2 + y^2 = 85$$

Answer: $(x - 4)^2 + y^2 = 85$ Back to Problem 2.

3. $x^2 + y^2 - 10x + 18y + 20 = 0$

We will need to complete the squares for both x and y in order to put the equation in standard form.

$$x^2 + y^2 - 10x + 18y + 20 = 0$$

$$x^2 - 10x + \underline{\ 25\ } + y^2 + 18y + \underline{\ 81\ } = -20 + 25 + 81$$

$\downarrow$ *Half* $\qquad\qquad$ $\downarrow$ *Half*

5 $\qquad\qquad\qquad$ 9

$\downarrow$ *Square* $\qquad\qquad$ $\downarrow$ *Square*

25 $\qquad\qquad\qquad$ 81

$$(x - 5)^2 + (y + 9)^2 = 86$$

Center: $(5, -9)$ **Radius:** $\sqrt{86}$

Back to Problem 3.

4. $f(x) = 3x^2 - 5x$

$$m_{\text{sec}} = \frac{\Delta y}{\Delta x} = \frac{f(4 + h) - f(4)}{4 + h - 4} = \frac{28 + 19h + 3h^2 - 28}{h} = \frac{19h + 3h^2}{h} =$$

$$\frac{h(19 + 3h)}{h} = 19 + 3h$$

$$f(4 + h) = 3(4 + h)^2 - 5(4 + h) = 3(16 + 8h + h^2) - 20 - 5h =$$

$$48 + 24h + 3h^2 - 20 - 5h = 28 + 19h + 3h^2$$

$f(4) = 48 - 20 = 28$

Answer: $19 + 3h$ Back to Problem 4.

5. $f(x) = 8 - 3x$ and $g(x) = 4x - 7$

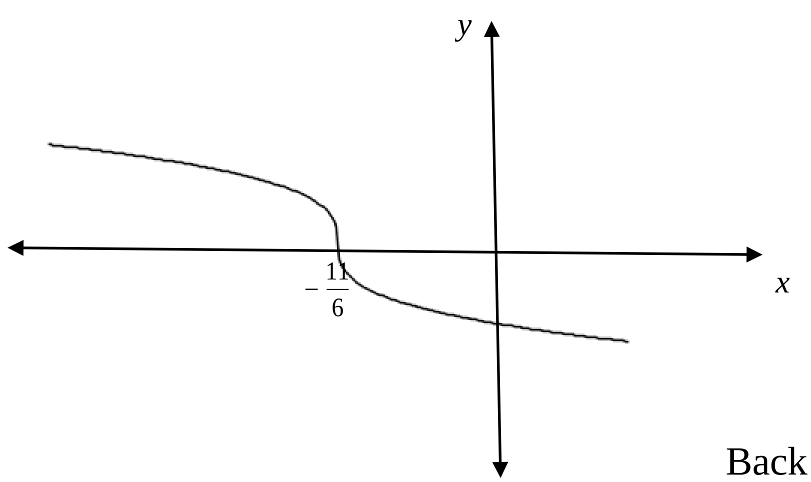
$(f \circ g)(x) = f(g(x)) = f(4x - 7) = 8 - 3(4x - 7) =$

$8 - 12x + 21 = 29 - 12x$

Answer: $29 - 12x$ Back to Problem 5.

6. $h(x) = -\sqrt[3]{6x + 11} = -\sqrt[3]{6\left(x + \frac{11}{6}\right)}$

The graph of $y = -\sqrt[3]{6x + 11}$ is the graph of $y = -\sqrt[3]{6x}$ shifted $\frac{11}{6}$ units to the left.

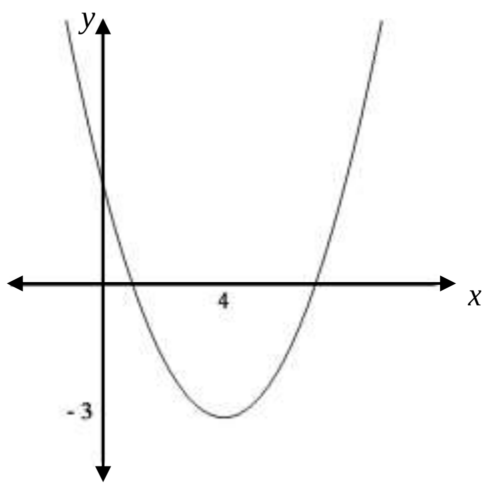


Back to Problem 6.

7. $f(x) = 2(x-4)^2 - 3$

Graph the equation $y = 2(x-4)^2 - 3$

The graph of $y + 3 = 2(x-4)^2$ is the graph of $y = 2x^2$ shifted 4 units to the right and 3 units downward.



a. **horizontal shift:** 4 units to the right

b. **vertical shift:** 3 units downward

c. **range of the function:** $[-3, \infty)$

d. **interval(s) on which the function is increasing:** $(4, \infty)$

e. **interval(s) on which the function is decreasing:** $(-\infty, 4)$

f. **value of relative (local) maximum(s) and location(s):** None

g. **value of relative (local) minimum(s) and location(s):** -3 at $x = 4$

h. ***x*-intercept(s):** $\left(\dfrac{8 \pm \sqrt{6}}{2}, 0 \right)$

Set $y = 0$: $0 = 2(x-4)^2 - 3 \Rightarrow \dfrac{3}{2} = (x-4)^2 \Rightarrow x - 4 = \pm\sqrt{\dfrac{3}{2}}$

$$x - 4 = \pm\frac{\sqrt{6}}{2} \Rightarrow x = 4 \pm \frac{\sqrt{6}}{2} = \frac{8 \pm \sqrt{6}}{2}$$

Back to Problem 7.

8. $g(x) = (6 - x)(x - 3)^2$

Zeros (Roots) of g: $q(x) = 0 \Rightarrow (6 - x)(x - 3)^2 = 0 \Rightarrow$
$(6 - x)(x - 3)^2 = 0 \Rightarrow x = 6, \ x = 3$

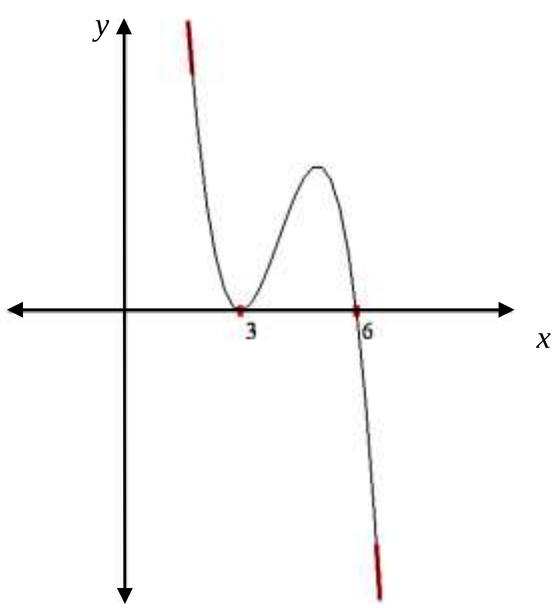
Since the factor $6 - x$ produces the zero (root) of 6, its multiplicity is one. Since the factor $(x - 3)^2$ produces the zero (root) of 3, its multiplicity is two.

Zero (Root)	**Multiplicity**	**Implication for the Graph of the Polynomial**
3	2	Touches the *x*-axis at $(3, 0)$
6	1	Crosses the *x*-axis at $(6, 0)$

For infinitely large values of *x*, $g(x) \approx -x(x^2) = -x^3$

As $x \to \infty$, $x^3 \to \infty$. Thus, since $-1 < 0$, then $q(x) \approx -x^3 \to -\infty$.

As $x \to -\infty$, $x^3 \to -\infty$. Thus, since $-1 < 0$, then $f(x) \approx -x^3 \to \infty$.



Back to Problem 8.

9. $h(x) = 2x^4 + x^3 - 15x + 12$

Find $h(7)$.

$$\begin{array}{rrrrr|l} 2 & 1 & 0 & -15 & 12 & 7 \\ & 14 & 105 & 735 & 5040 & \\ \hline 2 & 15 & 105 & 720 & 5052 & \end{array}$$

Answer: 5052

Back to Problem 9.

10. $f(x) = 4x^3 - 4x^2 - 9x + 30$

Back to Problem 10.

Factors of 30: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

Factors of 4: 1, 2, 4

Using a denominator of 1: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

Using a denominator of 2: $\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

Using a denominator of 4: $\pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{5}{4}, \pm \frac{15}{4}$

Trying 1:

$$\begin{array}{rrrr|l} \multicolumn{4}{c}{\overbrace{\qquad\qquad\qquad}^{Coeff\ of\ 4x^3 - 4x^2 - 9x + 30}} & \\ 4 & -4 & -9 & 30 & 1 \\ & 4 & 0 & -9 & \\ \hline 4 & 0 & -9 & 21 & \end{array}$$

Thus, $f(1) = 21 \neq 0 \Rightarrow x - 1$ is not a factor of f and 1 is not a zero (root) of f.

Trying 2:

$$\begin{array}{rrrr|l} \multicolumn{4}{c}{\overbrace{4 \quad -4 \quad -9 \quad 30}^{Coeff\ \ of\ \ 4x^3 - 4x^2 - 9x + 30}} & \ 2 \\ & 8 & 8 & -2 & \\ \hline 4 & 4 & -1 & 28 & \end{array}$$

Thus, $f(2) = 28 \neq 0 \Rightarrow x - 2$ is not a factor of f and 2 is not a zero (root) of f.

Trying 3:

$$\begin{array}{rrrr|l} \multicolumn{4}{c}{\overbrace{4 \quad -4 \quad -9 \quad 30}^{Coeff\ \ of\ \ 4x^3 - 4x^2 - 9x + 30}} & \ 3 \\ & 12 & 24 & 45 & \\ \hline 4 & 8 & 15 & 75 & \end{array}$$

Thus, $f(3) = 75 \neq 0 \Rightarrow x - 3$ is not a factor of f and 3 is not a zero (root) of f.

NOTE: By the Bound Theorem, 3 is an upper bound for the positive zeros (roots) of f since all the numbers are positive in the third row of the synthetic division. Thus, $\dfrac{15}{4}$, 5, 6, $\dfrac{15}{2}$, 10, 15, and 30 can't be rational zeros (roots) of f.

Trying -1:

$$\begin{array}{rrrr|l} \multicolumn{4}{c}{\overbrace{4 \quad -4 \quad -9 \quad 30}^{Coeff\ \ of\ \ 4x^3 - 4x^2 - 9x + 30}} & \ -1 \\ & -4 & 8 & 1 & \\ \hline 4 & -8 & -1 & 31 & \end{array}$$

Thus, $f(-1) = 31 \neq 0 \Rightarrow x + 1$ is not a factor of f and -1 is not a zero (root) of f.

$$\text{Trying } -2: \qquad \begin{array}{rrrr|l} \multicolumn{4}{c}{\overbrace{\hphantom{4 \quad -4 \quad -9 \quad 30}}^{Coeff\ of\ 4x^3 - 4x^2 - 9x + 30}} & \\ 4 & -4 & -9 & 30 & \underline{-2} \\ & -8 & 24 & -30 & \\ \hline 4 & -12 & 15 & 0 & \end{array}$$

Thus, $f(-2) = 0 \Rightarrow x + 2$ is factor of f and -2 is a zero (root) of f.

The third row in the synthetic division gives us the coefficients of the other factor starting with x^2. Thus, the other factor is $4x^2 - 12x + 15$.

Thus, we have that $4x^3 - 4x^2 - 9x + 30 = (x + 2)(4x^2 - 12x + 15)$.

Now, we can try to find a factorization for the expression $4x^2 - 12x + 15$. However, it does not factor.

Thus, we have that $4x^3 - 4x^2 - 9x + 30 = (x + 2)(4x^2 - 12x + 15)$.

Thus, $4x^3 - 4x^2 - 9x + 30 = 0 \Rightarrow (x + 2)(4x^2 - 12x + 15) = 0 \Rightarrow$

$x = -2,\ 4x^2 - 12x + 15 = 0$

We will need to use the Quadratic Formula to solve $4x^2 - 12x + 15 = 0$.

Thus, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{12 \pm \sqrt{12 \cdot 12 - 4(4)15}}{8} =$

$$\frac{12 \pm \sqrt{4 \cdot 4[3 \cdot 3 - 1(1)15]}}{8} = \frac{12 \pm 4\sqrt{9 - 15}}{8} = \frac{12 \pm 4\sqrt{-6}}{8} =$$

$$\frac{12 \pm 4i\sqrt{6}}{8} = \frac{3 \pm i\sqrt{6}}{2}$$

Answer: Zeros (Roots): $-2, \ \dfrac{3 + i\sqrt{6}}{2}, \ \dfrac{3 - i\sqrt{6}}{2}$

Factorization: $4x^3 - 4x^2 - 9x + 30 = (x + 2)(4x^2 - 12x + 15)$

Back to Problem 10.

11. $g(x) = \dfrac{2x + 3}{x^2 + 4x - 12} = \dfrac{2x + 3}{(x + 6)(x - 2)}$

a. **vertical asymptotes:** $(x + 6)(x - 2) = 0 \Rightarrow x = -6, \ x = 2$

Answer: $x = -6, \ x = 2$

b. **horizontal asymptotes:** $y = 0$

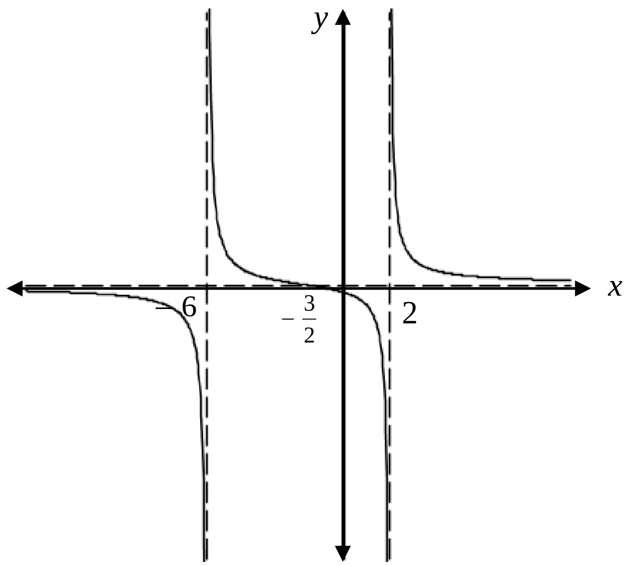
$$g(x) = \frac{2x + 3}{x^2 + 4x - 12} = \frac{x^2\left(\dfrac{2}{x} + \dfrac{3}{x^2}\right)}{x^2\left(1 + \dfrac{4}{x} - \dfrac{12}{x^2}\right)} = \frac{\dfrac{2}{x} + \dfrac{3}{x^2}}{1 + \dfrac{4}{x} - \dfrac{12}{x^2}}$$

As $x \to \pm\infty$, $g(x) \to \dfrac{0 + 0}{1 + 0 - 0} = \dfrac{0}{1} = 0$

c. $g(x) = 0 \Rightarrow 2x + 3 = 0 \Rightarrow x = -\dfrac{3}{2}$

Answer: $\left(-\dfrac{3}{2}, \ 0\right)$

d.



Back to Problem 11.

12. $\dfrac{6 - x}{3x - 14} \leq 0$

Back to Problem 12.

NOTE: This is a **two** part problem. One part of the problem is to solve the nonlinear inequality $\dfrac{6 - x}{3x - 14} < 0$. The other part of the problem is to solve the equation $\dfrac{6 - x}{3x - 14} = 0$.

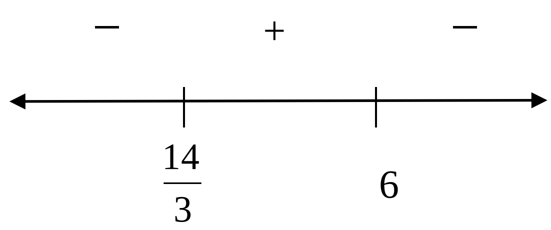
We will use the three step method to solve the nonlinear inequality $\dfrac{6 - x}{3x - 14} < 0$:

Step 1:

$$\frac{6 - x}{3x - 14} = 0 \Rightarrow 6 - x = 0 \Rightarrow x = 6$$

$$\frac{6-x}{3x-14} \text{ undefined } \Rightarrow 3x - 14 = 0 \Rightarrow x = \frac{14}{3}$$

Step 2: Plot all the numbers found in Step 1 on the real number line.



Sign of $\frac{6-x}{3x-14}$

Step 3: Use the real number line to identify the open intervals determined by the plotted numbers. Pick a test value for each open interval.

Interval	Test Value	Sign of $\frac{6-x}{3x-14}$
$\left(-\infty, \frac{14}{3}\right)$	0	$\frac{6-0}{0-14} = \frac{(+)}{(-)} = -$
$\left(\frac{14}{3}, 6\right)$	5	$\frac{6-5}{15-14} = \frac{(+)}{(+)} = +$
$(6, \infty)$	7	$\frac{6-7}{21-14} = \frac{(-)}{(+)} = -$

Thus, the solution for the nonlinear inequality $\frac{6-x}{3x-14} < 0$ is the set of real numbers given by $\left(-\infty, \frac{14}{3}\right) \cup (6, \infty)$. The solution for $\frac{6-x}{3x-14} = 0$ was found in Step 1 above. Thus, the solution for $\frac{6-x}{3x-14} = 0$ is the set

$\{6\}$. Putting these two solutions together, we have that the solution for $\frac{6-x}{3x-14} \leq 0$ is the set of real numbers $\left(-\infty, \frac{14}{3}\right) \cup [6, \infty)$.

Answer: $\left(-\infty, \frac{14}{3}\right) \cup [6, \infty)$