MATH-1320 Sample Exam 1 Spring 2017

1. Simplify the following. Write your answer in $a + b\,i$ form.

a. $\dfrac{-24 + \sqrt{-48}}{12}$ b. $(7 - 4i)^2$ c. $\dfrac{3 + 8i}{5 - 2i}$

2. Solve the following equations by the indicated method.

a. $4(x - 6)^2 - 25 = 87$ (using square roots)

b. $3y^2 + 48 = 0$ (using square roots)

c. $3t^2 = 7 - 4t$ (using the Quadratic Formula)

3. Solve the following equations.

a. $45w^3 - 27w^2 - 20w + 12 = 0$

b. $y^4 = 64y$

c. $\dfrac{6x}{x - 3} - \dfrac{4}{x + 6} = \dfrac{5x^2 + 39x}{x^2 + 3x - 18}$

d. $4|3t - 5| - 7 = 17$

e. $\sqrt{3x + 16} - \sqrt{x + 13} = -1$

4. Solve the following inequalities. Write the solution set in interval notation.

a. $3 \le \dfrac{4x - 9}{5} < 11$ b. $-11 > -2|y + 9| - 5$

5. A motorboat travels 75 miles with a current of 5 mph. The return trip against the current takes 2 hours longer. Set up an equation that will find the average

rate of the motorboat in still water. Do **NOT** solve the equation. Don't forget to identify your variable.

SOLUTIONS:

1a. $\dfrac{-24+\sqrt{-48}}{12}=\dfrac{-24+i\sqrt{16\cdot 3}}{12}=\dfrac{-24+4i\sqrt{3}}{12}=\dfrac{-24}{12}+\dfrac{4i\sqrt{3}}{12}=$

$-2+\dfrac{\sqrt{3}}{3}i$

Answer: $-2+\dfrac{\sqrt{3}}{3}i$ Back to Problem 1.

1b. $(7-4i)^2$

NOTE: We will use the following special product formula.

$$(a-b)^2=a^2-2ab+b^2$$

$(7-4i)^2=49-56i+16i^2=33-56i$

NOTE: $i^2=-1$ and $16i^2=-16$

Answer: $33-56i$ Back to Problem 1.

1c. $\dfrac{3+8i}{5-2i}$

The conjugate of $5 - 2i$ is $5 + 2i$.

$$\frac{3 + 8i}{5 - 2i} = \frac{3 + 8i}{5 - 2i} \cdot \frac{5 + 2i}{5 + 2i} = \frac{15 + 6i + 40i + 16i^2}{25 + 4} = \frac{-1 + 46i}{29} =$$

$$-\frac{1}{29} + \frac{46}{29}i$$

Answer: $-\frac{1}{29} + \frac{46}{29}i$ Back to Problem 1.

2a. $4(x - 6)^2 - 25 = 87$ Back to Problem 2.

$$4(x - 6)^2 - 25 = 87 \Rightarrow 4(x - 6)^2 = 112 \Rightarrow (x - 6)^2 = 28 \Rightarrow$$

$$x - 6 = \pm 2\sqrt{7} \Rightarrow x = 6 \pm 2\sqrt{7}$$

Answer: $x = 6 \pm 2\sqrt{7}$ **or** $\{6 \pm 2\sqrt{7}\}$

2b. $3y^2 + 48 = 0$ Back to Problem 2.

$$3y^2 + 48 = 0 \Rightarrow 3y^2 = -48 \Rightarrow y^2 = -16 \Rightarrow y = \pm 4i$$

Answer: $y = \pm 4i$ **or** $\{\pm 4i\}$

2c. $3t^2 = 7 - 4t$ Back to Problem 2.

$3t^2 = 7 - 4t \Rightarrow 3t^2 + 4t - 7 = 0$

$a = 3,\ b = 4,\ c = -7$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(3)(-7)}}{6} = \frac{-4 \pm \sqrt{16 + 84}}{6} =$$

$$\frac{-4 \pm \sqrt{100}}{6} = \frac{-4 \pm 10}{6}$$

$$t = \frac{-4 - 10}{6} = \frac{-14}{6} = -\frac{7}{3} \qquad t = \frac{-4 + 10}{6} = \frac{6}{6} = 1$$

Answer: $t = -\frac{7}{3},\ t = 1$ **or** $\left\{-\frac{7}{3},\ 1\right\}$

3a. $45w^3 - 27w^2 - 20w + 12 = 0$ Back to Problem 3.

We can factor the expression $45w^3 - 27w^2 - 20w + 12$ by grouping:

$$45w^3 - 27w^2 - 20w + 12 = 9w^2(5w - 3) - 4(5w - 3) = (5w - 3)(9w^2 - 4)$$

$$45w^3 - 27w^2 - 20w + 12 = 0 \Rightarrow 9w^2(5w - 3) - 4(5w - 3) = 0 \Rightarrow$$

$$(5w - 3)(9w^2 - 4) = 0$$

$$5w - 3 = 0 \Rightarrow w = \frac{3}{5}$$

$$9w^2 - 4 = 0 \Rightarrow w^2 = \frac{4}{9} \Rightarrow w = \pm\frac{2}{3}$$

Answer: $w = \pm\frac{2}{3}, \frac{3}{5}$ **or** $\left\{\pm\frac{2}{3}, \frac{3}{5}\right\}$

3b. $y^4 = 64y$ Back to Problem 3.

$$y^4 = 64y \Rightarrow y^4 - 64y = 0 \Rightarrow y(y^3 - 64) = 0$$

NOTE: In order to factor the expression $y^3 - 64$, which is a difference of cubes, you will need the following factoring formula:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Thus, $y^3 - 64 = y^3 - 4^3 = (y - 4)(y^2 + 4y + 16)$

$$y^4 = 64y \Rightarrow y^4 - 64y = 0 \Rightarrow y(y^3 - 64) = 0 \Rightarrow$$

$$y(y - 4)(y^2 + 4y + 16) = 0$$

$$y = 0$$

$$y - 4 = 0 \Rightarrow y = 4$$

$$y^2 + 4y + 16 = 0$$

$$a = 1,\ b = 4,\ c = 16$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(1)(16)}}{2} = \frac{-4 \pm \sqrt{16(1-4)}}{2} =$$

$$\frac{-4 \pm 4\sqrt{-3}}{2} = \frac{-4 \pm 4i\sqrt{3}}{2} = -2 \pm 2i\sqrt{3}$$

Answer: $y = 0,\ 4,\ -2 \pm 2i\sqrt{3}$ **or** $\left\{0,\ 4,\ -2 \pm 2i\sqrt{3}\right\}$

3c. $\dfrac{6x}{x-3} - \dfrac{4}{x+6} = \dfrac{5x^2 + 39x}{x^2 + 3x - 18}$ Back to Problem 3.

$x^2 + 3x - 18 = (x+6)(x-3)$

$$\frac{6x}{x-3} - \frac{4}{x+6} = \frac{5x^2 + 39x}{x^2 + 3x - 18} \Rightarrow \frac{6x}{x-3} - \frac{4}{x+6} = \frac{5x^2 + 39x}{(x+6)(x-3)}$$

NOTE: $x \neq -6,\ x \neq 3$

LCD $= (x+6)(x-3)$

$$\frac{6x}{x-3} - \frac{4}{x+6} = \frac{5x^2 + 39x}{(x+6)(x-3)} \Rightarrow$$

$$(x+6)(x-3)\left(\frac{6x}{x-3} - \frac{4}{x+6}\right) = \left[\frac{5x^2 + 39x}{(x+6)(x-3)}\right](x+6)(x-3)$$

$\Rightarrow 6x(x+6) - 4(x-3) = 5x^2 + 39x \Rightarrow$

$6x^2 + 36x - 4x + 12 = 5x^2 + 39x \Rightarrow 6x^2 + 32x + 12 = 5x^2 + 39x \Rightarrow$

$x^2 - 7x + 12 = 0 \Rightarrow (x - 3)(x - 4) = 0 \Rightarrow x = 3,\ 4$

If $x = 3$, then two of the fractions in the equation are undefined because you would have division by zero. Thus, $x = 4$ is the only solution.

Answer: $x = 4$ **or** $\{4\}$

3d. $4|3t - 5| - 7 = 17$ Back to Problem 3.

$4|3t - 5| - 7 = 17 \Rightarrow 4|3t - 5| = 24 \Rightarrow |3t - 5| = 6 \Rightarrow$

$3t - 5 = \pm 6 \Rightarrow 3t = 5 \pm 6 \Rightarrow t = \dfrac{5 \pm 6}{3}$

$t = \dfrac{5 - 6}{3} = -\dfrac{1}{3}, \quad t = \dfrac{5 + 6}{3} = \dfrac{11}{3}$

Answer: $t = -\dfrac{1}{3}, \dfrac{11}{3}$ **or** $\left\{-\dfrac{1}{3}, \dfrac{11}{3}\right\}$

3e. $\sqrt{3x + 16} - \sqrt{x + 13} = -1$ Back to Problem 3.

$\sqrt{3x + 16} - \sqrt{x + 13} = -1 \Rightarrow \sqrt{3x + 16} = \sqrt{x + 13} - 1 \Rightarrow$

$(\sqrt{3x + 16})^2 = (\sqrt{x + 13} - 1)^2 \Rightarrow 3x + 16 = x + 13 - 2\sqrt{x + 13} + 1$

$\Rightarrow\ 3x + 16 = x + 14 - 2\sqrt{x + 13}\ \Rightarrow\ 2x + 2 = -2\sqrt{x + 13}\ \Rightarrow$

$2(x + 1) = -2\sqrt{x + 13}\ \Rightarrow\ x + 1 = -\sqrt{x + 13}\ \Rightarrow$

$(x + 1)^2 = (-\sqrt{x + 13})^2\ \Rightarrow\ x^2 + 2x + 1 = x + 13\ \Rightarrow$

$x^2 + x - 12 = 0\ \Rightarrow\ (x + 4)(x - 3) = 0\ \Rightarrow\ x = -4,\ x = 3$

Check for $x = -4$: $\sqrt{-12 + 16} - \sqrt{-4 + 13} \stackrel{?}{=} -1\ \Rightarrow$

$\sqrt{4} - \sqrt{9} \stackrel{?}{=} -1\ \Rightarrow\ 2 - 3 \stackrel{?}{=} -1\ \Rightarrow\ -1 \stackrel{?}{=} -1$ True

Check for $x = 3$: $\sqrt{9 + 16} - \sqrt{3 + 13} \stackrel{?}{=} -1\ \Rightarrow$

$\sqrt{25} - \sqrt{16} \stackrel{?}{=} -1\ \Rightarrow\ 5 - 4 \stackrel{?}{=} -1\ \Rightarrow\ 1 \stackrel{?}{=} -1$ False

Answer: $x = -4$ **or** $\{-4\}$

4a. $3 \le \dfrac{4x - 9}{5} < 11$ Back to Problem 4.

$3 \le \dfrac{4x - 9}{5} < 11\ \Rightarrow\ 15 \le 4x - 9 < 55\ \Rightarrow\ 24 \le 4x < 64\ \Rightarrow$

$6 \le x < 16$

Answer: $[6, 16)$

4b. $-11 > -2|y + 9| - 5$ Back to Problem 4.

$$-11 > -2|y + 9| - 5 \Rightarrow -6 > -2|y + 9| \Rightarrow 3 < |y + 9| \Rightarrow$$

$$|y + 9| > 3$$

$$|y + 9| > 3 \Rightarrow \quad y + 9 > 3 \quad \text{or} \quad y + 9 < -3$$
$$y > -6 \qquad\qquad y < -12$$

Answer: $(-\infty, -12) \cup (-6, \infty)$

5. A motorboat travels 75 miles with a current of 5 mph. The return trip against the current takes 2 hours longer. Set up an equation that will find the average rate of the motorboat in still water. Do **NOT** solve the equation. Don't forget to identify your variable.

Let m = the rate of the motorboat in still water

	R	$T = \frac{D}{R}$	D
With the current	$m + 5$	$\frac{75}{m + 5}$	75
Against the current	$m - 5$	$\frac{75}{m - 5}$	75

NOTE: The time for the return trip against the current, which is $\frac{75}{m-5}$, is 2 hours longer than the time for the trip with the current, which is $\frac{75}{m+5}$. Thus, $\frac{75}{m+5} + 2 = \frac{75}{m-5}$.

NOTE: In order to set up your equation for this problem, it might have been easier to realize that the difference in the two times is 2 hours. The difference of the time to travel against the current and the time to travel with the current is 2 hours. That is, $\frac{75}{m-5} - \frac{75}{m+5} = 2$.

Answer: $\frac{75}{m+5} + 2 = \frac{75}{m-5}$ **or** $\frac{75}{m-5} - \frac{75}{m+5} = 2$

Back to Problem 5.