Pre-Class Problems 9 for Wednesday, February 21

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem number or letter.

1. Determine if the graph of the following equations is symmetric with respect to the *x*-axis, *y*-axis, origin, or none of these.

a. $y = 3x^2 - 5$ b. $y^2 = 7x^2 + 2$ c. $y = x^2 - |x| + 4$

d. $y^2 = x^2 - |x| + 4$ e. $x^2 + y^2 = 9$ f. $y = 2x^3$

g. $y = x^2 + 9x$ h. $y = 6x - 15$

<u>Definition</u> A function f is called an even function if and only if $f(-x) = f(x)$ for all *x* in the domain of the function.

NOTE: The graph of an even function is symmetric with respect to the *y*-axis.

<u>Definition</u> A function f is called an odd function if and only if $f(-x) = -f(x)$ for all *x* in the domain of the function.

NOTE: The graph of an odd function is symmetric with respect to the origin.

2. Determine if the following functions are even, odd, or neither.

a. $f(x) = 3x^4 - 2x^2$ b. $g(x) = x^2 + 4|x| - 7$

c. $h(x) = x^5 - 6x^3$ d. $h(x) = x^5 - 6x^3 + 8$

e. $f(x) = \sqrt{16 - 9x^2}$ f. $g(x) = 2x^6 + 5x^3$

g. $h(x) = \dfrac{4x}{x^2 + 9}$ h. $f(x) = \dfrac{4x^3}{x^5 - 6x}$

3. If $h(x) = \begin{cases} -2x^2, & x \le 1 \\ 5 - x, & x > 1 \end{cases}$, then find

a. $h(5)$ b. $h(-3)$ c. $h(1)$ d. sketch the graph of h

4. If $g(x) = \begin{cases} x^2 + 6x + 9, & x \le -5 \\ \sqrt[3]{4x + 9}, & -5 < x \le -2 \\ 3x^2 + \dfrac{11}{2}x, & x > -2 \end{cases}$, then find

a. $g(4)$ b. $g(-2)$ c. $g\left(-\dfrac{17}{4}\right)$ d. $g(-4)$

e. $g(-8)$

5. If $f(x) = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$, then find

a. $f(-4)$ b. $f(0)$ c. $f(6)$ d. sketch the graph of f

<u>Definition</u> Let a function f be defined on an interval (a, b) and let x_1 and x_2 be any numbers in the interval. Then

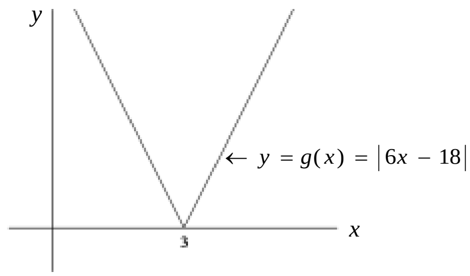
a. f is <u>increasing</u> on the interval if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$,

b. f is <u>decreasing</u> on the interval if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$,

c. f is constant on the interval if $f(x_1) = f(x_2)$ for every x_1 and x_2 in the interval.

6. Determine the interval(s) where the following functions are increasing and decreasing.

a.



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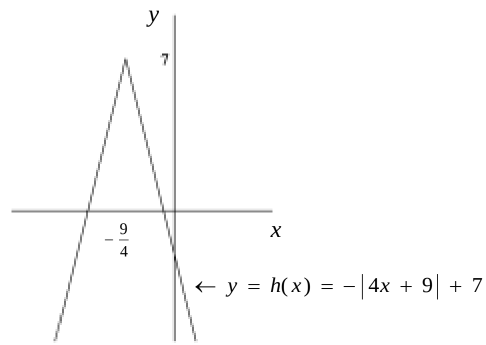

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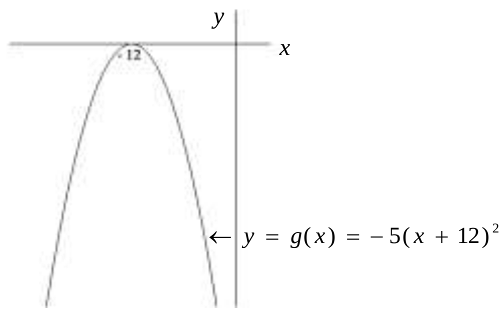

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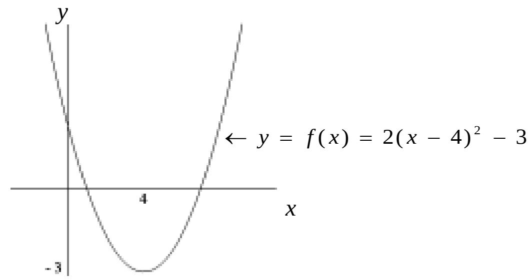

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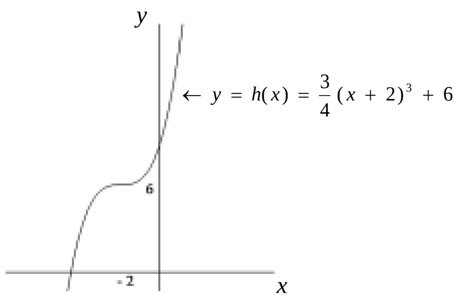

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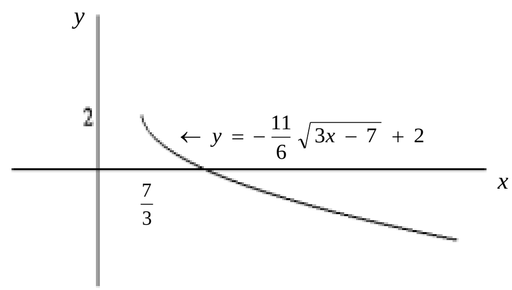

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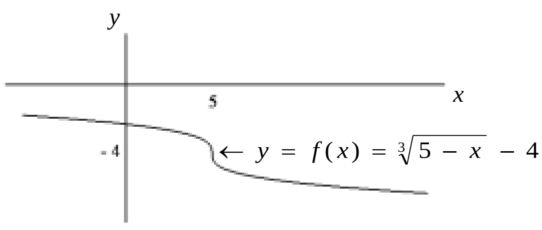

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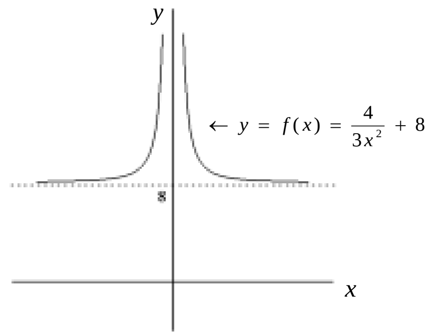
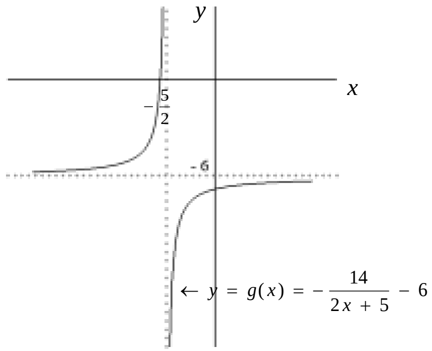


i.



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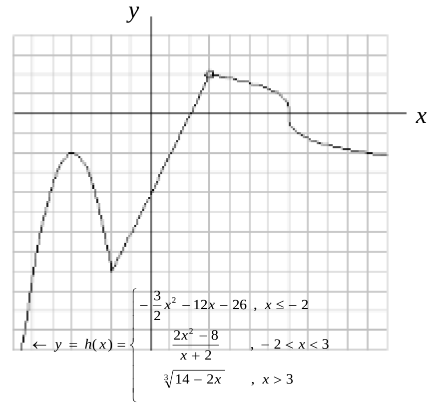
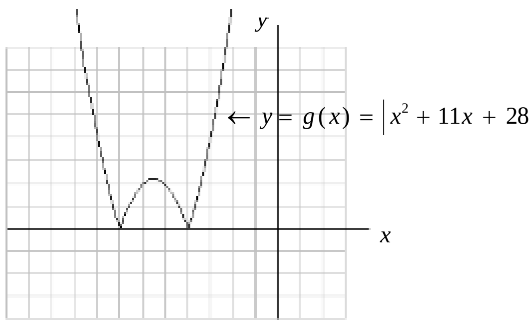


k.



Definition Let c be a number in the domain of the function *f*. Then

a. $f(c)$ is a relative (local) maximum of *f* if there exists an open interval (a, b) containing c such that $f(x) \le f(c)$ for all x in the interval (a, b).

b. $f(c)$ is a relative (local) minimum of *f* if there exists an open interval (a, b) containing c such that $f(x) \ge f(c)$ for all x in the interval (a, b).

On the graph of $y = f(x)$, $f(c)$ is the *y*-coordinate of the point with an *x*-coordinate of c. Part (a) of the definition is saying that $f(c)$ is a **relative (local)** maximum of the function f if it is the largest *y*-coordinate of all the *y*-coordinates of points on the graph of $y = f(x)$ in an **open interval** of $x = c$.

On the graph of $y = f(x)$, $f(c)$ is the *y*-coordinate of the point with an *x*-coordinate of c. Part (b) of the definition is saying that $f(c)$ is a **relative (local)** minimum of the function f if it is the smallest y-coordinate of all the y-coordinates of points on the graph of $y = f(x)$ in an **open interval** of $x = c$.

7. Determine the location and the value of any relative maximum and minimum of the functions in Problem 6.

   a. b. c. d. e. f. g. h. i. j. k.

New functions can be formed from the sum, difference, product, and quotient of two functions.

Definition Given the functions *f* and *g*, the functions $f + g$, $f - g$, $f \cdot g$, and $\dfrac{f}{g}$ are defined by the following.

$$(f + g)(x) = f(x) + g(x)$$
$$(f - g)(x) = f(x) - g(x)$$
$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ provided that } g(x) \neq 0$$

The domain of these new functions is the set of real numbers in the intersection of the domains of the functions *f* and *g*. For the function $\dfrac{f}{g}$, we also exclude any real number *x* for which $g(x) = 0$ since division by zero is undefined.

8. If $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$, then find the following.

   a. $(f + g)(-3)$ b. $(f - g)(-3)$ c. $(f \cdot g)(-3)$ d. $\left(\dfrac{f}{g}\right)(-3)$

9. If $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$, then find the following.

a. $(f+g)(x)$ b. $(f-g)(x)$ c. $(f \cdot g)(x)$ d. $\left(\frac{f}{g}\right)(x)$

10. If $h(x) = 8 - 5x$ and $k(x) = x^2 + 9x - 36$, then find the following.

a. $(h+k)(x)$ b. $(h-k)(x)$ c. $(h \cdot k)(x)$ d. $\left(\frac{h}{k}\right)(x)$

Problems available in the textbook: Page 255 … 7 – 112 and Examples 1 – 11 starting on page 244. Problems available in the textbook: Page 271 … 5 – 44, 99adcd – 102abcd, 103, 104. Examples 1 – 5 and 12abc starting on page 262.

SOLUTIONS:

1a. $y = 3x^2 - 5$ Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$-y = 3x^2 - 5$.

Solving for *y*: $-y = 3x^2 - 5 \Rightarrow y = -3x^2 + 5$

The equations $y = 3x^2 - 5$ and $-y = 3x^2 - 5$ are NOT the same. Thus, there is no symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$y = 3(-x)^2 - 5 = 3x^2 - 5$.

The equations $y = 3x^2 - 5$ and $y = 3(-x)^2 - 5$ ARE the same. Thus, there is symmetry with respect to the *y*-axis.

Answer: *y*-axis

1b. $y^2 = 7x^2 + 2$ Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$(-y)^2 = 7x^2 + 2 \Rightarrow y^2 = 7x^2 + 2$.

The equations $y^2 = 7x^2 + 2$ and $(-y)^2 = 7x^2 + 2$ ARE the same. Thus, there is symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$y^2 = 7(-x)^2 + 2 = 7x^2 + 2$.

The equations $y^2 = 7x^2 + 2$ and $y^2 = 7(-x)^2 + 2$ ARE the same. Thus, there is symmetry with respect to the *y*-axis.

Answer: *x*-axis, *y*-axis, and origin

1c. $y = x^2 - |x| + 4$ Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$-y = x^2 - |x| + 4$.

Solving for *y*: $-y = x^2 - |x| + 4 \Rightarrow y = -x^2 + |x| - 4$

The equations $y = x^2 - |x| + 4$ and $-y = x^2 - |x| + 4$ are NOT the same. Thus, there is no symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$y = (-x)^2 - |-x| + 4 = x^2 - |x| + 4$.

The equations $y = x^2 - |x| + 4$ and $y = (-x)^2 - |-x| + 4$ ARE the same. Thus, there is symmetry with respect to the *y*-axis.

Answer: *y*-axis

1d. $y^2 = x^2 - |x| + 4$ Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$(-y)^2 = x^2 - |x| + 4 \Rightarrow y^2 = x^2 - |x| + 4$.

The equations $y^2 = x^2 - |x| + 4$ and $(-y)^2 = x^2 - |x| + 4$ ARE the same. Thus, there is symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$y^2 = (-x)^2 - |-x| + 4 = x^2 - |x| + 4$.

The equations $y^2 = x^2 - |x| + 4$ and $y^2 = (-x)^2 - |-x| + 4$ ARE the same. Thus, there is symmetry with respect to the *y*-axis.

Answer: *x*-axis, *y*-axis, and origin

1e. $x^2 + y^2 = 9$ Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$x^2 + (-y)^2 = 9 \Rightarrow x^2 + y^2 = 9$.

The equations $x^2 + y^2 = 9$ and $x^2 + (-y)^2 = 9$ ARE the same. Thus, there is symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$(-x)^2 + y^2 = 9 \Rightarrow x^2 + y^2 = 9$.

The equations $x^2 + y^2 = 9$ and $(-x)^2 + y^2 = 9$ ARE the same. Thus, there is symmetry with respect to the *y*-axis.

Answer: *x*-axis, *y*-axis, and origin

1f. $y = 2x^3$

Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$-y = 2x^3$.

Solving for *y*: $-y = 2x^3 \Rightarrow y = -2x^3$

The equations $y = 2x^3$ and $-y = 2x^3$ are NOT the same. Thus, there is no symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$y = 2(-x)^3 = 2(-1)x^3 = -2x^3$.

NOTE: $(-x)^3 = (-1 \cdot x)^3 = (-1)^3 x^3 = -1 \cdot x^3 = -x^3$

The equations $y = 2x^3$ and $y = 2(-x)^3$ are NOT the same. Thus, there is no symmetry with respect to the *y*-axis.

To check for symmetry with respect to the origin, replace x by $-x$ and y by $-y$: $-y = 2(-x)^3$.

Solving for y: $-y = 2(-x)^3 \Rightarrow -y = -2x^3 \Rightarrow y = 2x^3$

The equations $y = 2x^3$ and $-y = 2(-x)^3$ ARE the same. Thus, there is symmetry with respect to the origin.

Answer: origin

1g. $y = x^2 + 9x$ Back to Problem 1.

To check for symmetry with respect to the x-axis, replace y by $-y$: $-y = x^2 + 9x$.

Solving for y: $-y = x^2 + 9x \Rightarrow y = -x^2 - 9x$

The equations $y = x^2 + 9x$ and $-y = x^2 + 9x$ are NOT the same. Thus, there is no symmetry with respect to the x-axis.

To check for symmetry with respect to the y-axis, replace x by $-x$: $y = (-x)^2 + 9(-x) = x^2 - 9x$.

The equations $y = x^2 + 9x$ and $y = (-x)^2 + 9(-x)$ are NOT the same. Thus, there is no symmetry with respect to the y-axis.

To check for symmetry with respect to the origin, replace x by $-x$ and y by $-y$: $-y = (-x)^2 + 9(-x)$.

Solving for y: $-y = (-x)^2 + 9(-x) \Rightarrow -y = x^2 - 9x \Rightarrow$
$y = -x^2 + 9x$

The equations $y = x^2 + 9x$ and $-y = (-x)^2 + 9(-x)$ are NOT the same. Thus, there is no symmetry with respect to the origin.

Answer: none of these

1h. $y = 6x - 15$ Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$-y = 6x - 15$.

Solving for *y*: $-y = 6x - 15 \Rightarrow y = -6x + 15$

The equations $y = 6x - 15$ and $-y = 6x - 15$ are NOT the same. Thus, there is no symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$y = 6(-x) - 15 = -6x - 15$.

The equations $y = 6x - 15$ and $y = 6(-x) - 15$ are NOT the same. Thus, there is no symmetry with respect to the *y*-axis.

To check for symmetry with respect to the origin, replace *x* by $-x$ and *y* by $-y$:
$-y = 6(-x) - 15$.

Solving for *y*: $-y = 6(-x) - 15 \Rightarrow -y = -6x - 15 \Rightarrow y = 6x + 15$

The equations $y = 6x - 15$ and $-y = 6(-x) - 15$ are NOT the same. Thus, there is no symmetry with respect to the origin.

Answer: none of these

2a. $f(x) = 3x^4 - 2x^2$ Back to Problem 2.

$$f(-x) = 3(-x)^4 - 2(-x)^2 = 3x^4 - 2x^2 = f(x)$$

Answer: Even

NOTE:
$$(-x)^4 = (-1 \cdot x)^4 = (-1)^4 x^4 = 1 \cdot x^4 = x^4$$
$$(-x)^2 = (-1 \cdot x)^2 = (-1)^2 x^2 = 1 \cdot x^2 = x^2$$

2b. $g(x) = x^2 + 4|x| - 7$ Back to Problem 2.

$$g(-x) = (-x)^2 + 4|-x| - 7 = x^2 + 4|x| - 7 = g(x)$$

Answer: Even

2c. $h(x) = x^5 - 6x^3$ Back to Problem 2.

$$h(-x) = (-x)^5 - 6(-x)^3 = -x^5 + 6x^3 = -h(x)$$

Answer: Odd

2d. $h(x) = x^5 - 6x^3 + 8$ Back to Problem 2.

$h(-x) = (-x)^5 - 6(-x)^3 + 8 = -x^5 + 6x^3 + 8$

NOTE: $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$

Answer: Neither

2e. $f(x) = \sqrt{16 - 9x^2}$ Back to Problem 2.

$f(-x) = \sqrt{16 - 9(-x)^2} = \sqrt{16 - 9x^2} = f(x)$

Answer: Even

2f. $g(x) = 2x^6 + 5x^3$ Back to Problem 2.

$g(-x) = 2(-x)^6 + 5(-x)^3 = 2x^6 - 5x^3$

NOTE: $g(-x) \neq g(x)$ and $g(-x) \neq -g(x)$

Answer: Neither

2g. $h(x) = \dfrac{4x}{x^2 + 9}$ Back to Problem 2.

$$h(-x) = \frac{4(-x)}{(-x)^2 + 9} = \frac{-4x}{x + 9} = -\frac{4x}{x + 9} = -h(x)$$

Answer: Odd

2h. $f(x) = \dfrac{4x^3}{x^5 - 6x}$ Back to Problem 2.

$$f(-x) = \frac{4x^3}{x^5 - 6x} = \frac{4(-x)^3}{(-x)^5 - 6(-x)} = \frac{-4x^3}{-x^5 + 6x} = \frac{4x^3}{x^5 - 6x} = f(x)$$

Answer: Even

3. $h(x) = \begin{cases} -2x^2, & x \le 1 \\ 5 - x, & x > 1 \end{cases}$ Back to Problem 3.

The function h is called a piecewise function. The number 1 is sometimes called a breakup point of the function h.

3a. To find $h(5)$: Since $5 > 1$ and $h(x) = 5 - x$ when $x > 1$, then $h(5) = 0$.

Answer: 0 Back to Problem 3.

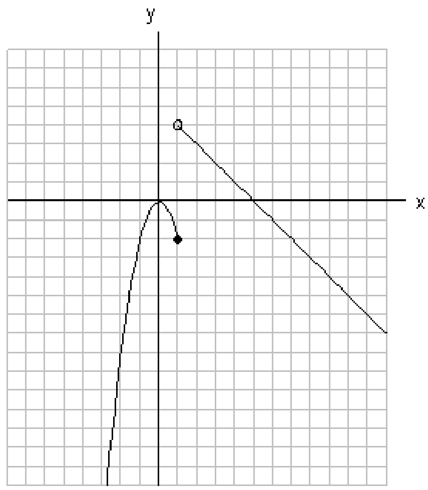
3b. To find $h(-3)$: Since $-3 < 1$ and $h(x) = -2x^2$ when $x \le 1$, then $h(-3) = -18$.

Answer: -18 Back to Problem 3.

3c. To find $h(1)$: Since $1 = 1$ and $h(x) = -2x^2$ when $x \le 1$, then $h(1) = -2$.

Answer: -2 Back to Problem 3.

3d.



Back to Problem 3.

4. $$g(x) = \begin{cases} x^2 + 6x + 9\,, & x \le -5 \\ \sqrt[3]{4x+9}\,, & -5 < x \le -2 \\ 3x^2 + \dfrac{11}{2}x\,, & x > -2 \end{cases}$$

Back to Problem 4.

The function g is a piecewise function. The numbers -5 and -2 are the breakup points of the function g.

4a. To find $g(4)$: Since $4 > -2$ and $g(x) = 3x^2 + \dfrac{11}{2}x$ when $x > -2$, then $g(4) = 48 + 22 = 70$. NOTE: Since $3x^2 + \dfrac{11}{2}x = \dfrac{1}{2}x(6x + 11)$, then we could use this information in order to find $g(4)$. Thus,

$g(4) = \dfrac{1}{2}(4)(35) = 70$.

Answer: 70 Back to Problem 4.

4b. To find $g(-2)$: Since $-2 = -2$ and $g(x) = \sqrt[3]{4x+9}$ when $-5 < x \leq -2$, then $g(-2) = \sqrt[3]{-8+9} = \sqrt[3]{1} = 1$.

Answer: 1

Back to Problem 4.

4c. To find $g\left(-\dfrac{17}{4}\right)$: Since $-5 < -\dfrac{17}{4} \leq -2$ and $g(x) = \sqrt[3]{4x+9}$ when $-5 < x \leq -2$, then $g\left(-\dfrac{17}{4}\right) = \sqrt[3]{-17+9} = \sqrt[3]{-8} = -2$.

Answer: -2

Back to Problem 4.

4d. To find $g(-4)$: Since $-5 < -4 \leq -2$ and $g(x) = \sqrt[3]{4x+9}$ when $-5 < x \leq -2$, then $g(-4) = \sqrt[3]{-16+9} = \sqrt[3]{-7} = -\sqrt[3]{7}$.

Answer: $-\sqrt[3]{7}$

Back to Problem 4.

4e. To find $g(-8)$: Since $-8 \leq -5$ and $g(x) = x^2 + 6x + 9$ when $x \leq -5$, then $g(-8) = 64 - 48 + 9 = 25$. NOTE: Since $x^2 + 6x + 9 = (x+3)^2$, then we could use this information in order to find $g(-8)$. Thus, $g(-8) = (-5)^2 = 25$.

Answer: 25

Back to Problem 4.

5. $f(x) = \begin{cases} x \ , & x \geq 0 \\ -x \ , & x < 0 \end{cases}$

Back to Problem 5.

The number 0 is the breakup point of this piecewise function *f*.

5a. To find $f(-4)$: Since $-4 < 0$ and $f(x) = -x$ when $x < 0$, then $f(-4) = -(-4) = 4$.

Answer: 4 Back to Problem 5.

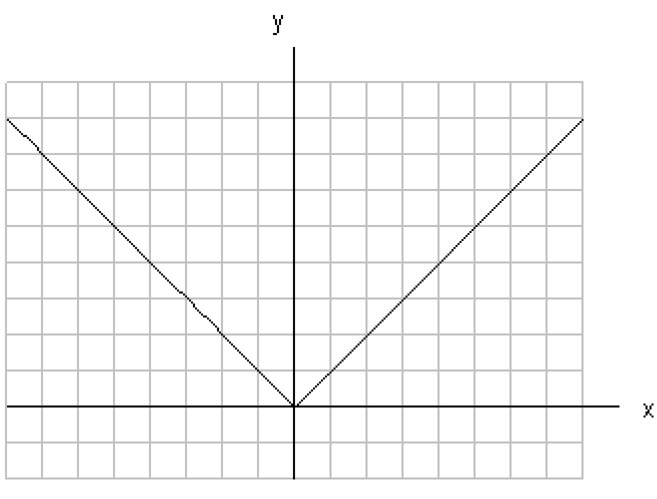
5b. To find $f(0)$: Since $0 \geq 0$ and $f(x) = x$ when $x \geq 0$, then $f(0) = 0$.

Answer: 0 Back to Problem 5.

5c. To find $f(6)$: Since $6 \geq 0$ and $f(x) = x$ when $x \geq 0$, then $f(6) = 6$.

Answer: 6 Back to Problem 5.

5d.



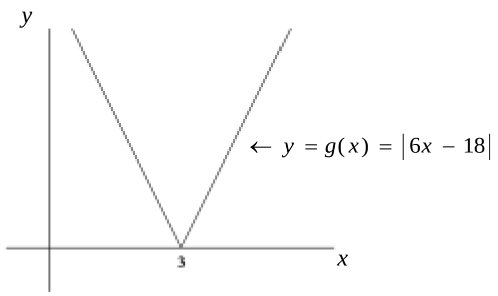
Back to Problem 5.

NOTE: This piecewise function is the absolute value function.

We will have the need to find the absolute value of algebraic expressions. We will do this using the following definition.

Definition Let *a* be an algebraic expression. Then $|a| = \begin{cases} a, & a \ge 0 \\ -a, & a < 0 \end{cases}$.

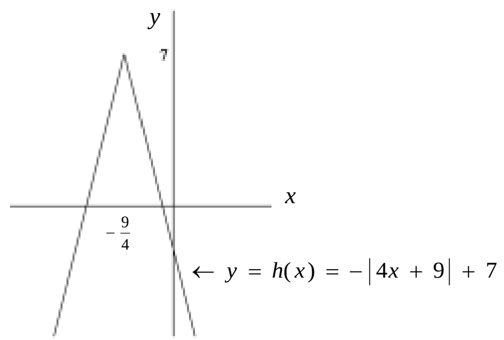
6a.



Back to Problem 6.

Answer: Increasing: $(3, \infty)$
Decreasing: $(-\infty, 3)$

6b.

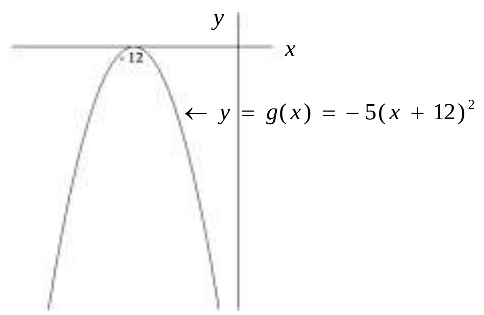


Back to Problem 6.

Answer: Increasing: $\left(-\infty, -\frac{9}{4}\right)$

Decreasing: $\left(-\frac{9}{4}, \infty\right)$

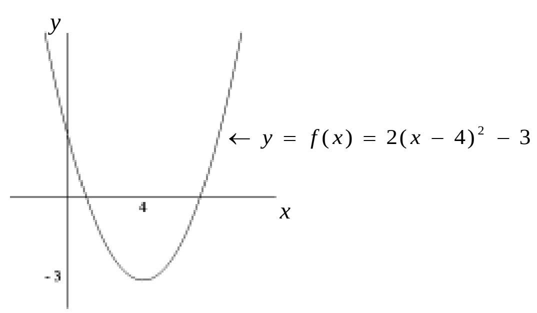
6c.



Back to Problem 6.

Answer: Increasing: $(-\infty, -12)$
Decreasing: $(-12, \infty)$

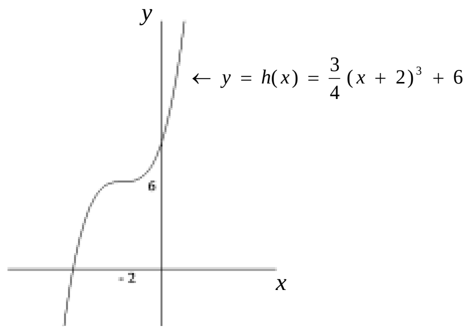
6d.



Back to Problem 6.

Answer: Increasing: $(4, \infty)$
Decreasing: $(-\infty, 4)$

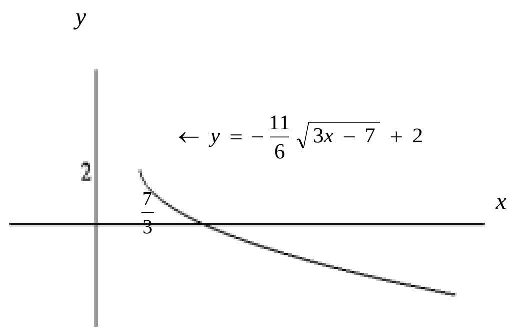
6e.



Back to Problem 6.

Answer: Increasing: $(-\infty, \infty)$
Decreasing: Nowhere

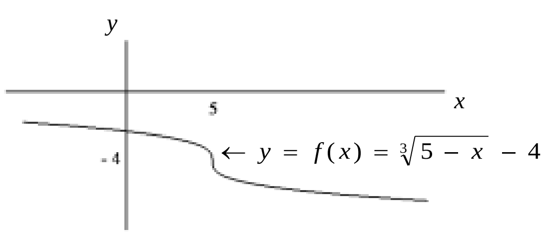
6f.



Back to Problem 6.

Answer: Increasing: Nowhere
Decreasing: $\left(\frac{7}{3}, \infty\right)$

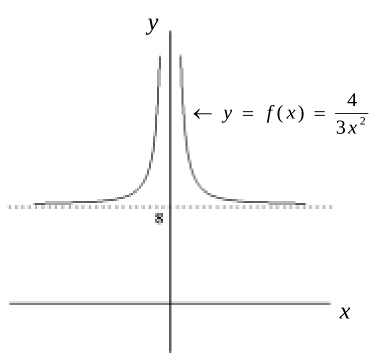
6g.



Back to Problem 6.

Answer: Increasing: Nowhere
Decreasing: $(-\infty, \infty)$

6h.

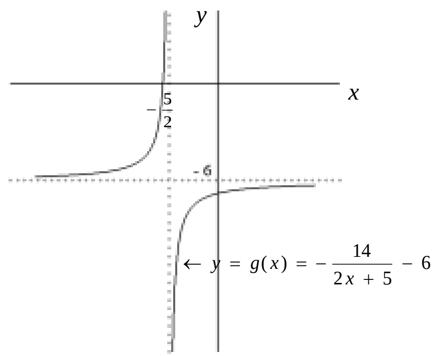


Back to Problem 6.

Answer: Increasing: $(-\infty, 0)$
Decreasing: $(0, \infty)$

6i.

Back to Problem 6.

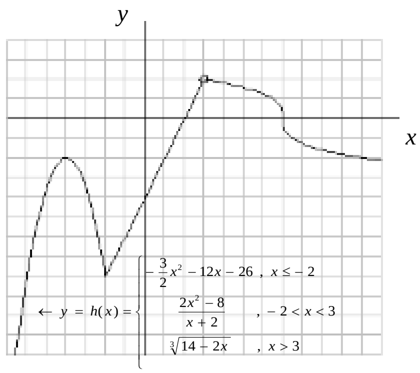


Answer: Increasing: $\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, \infty\right)$

Decreasing: Nowhere

6j.

Back to Problem 6.

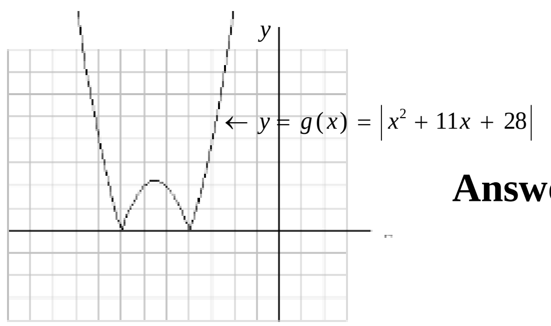


Answer: Increasing: $(-\infty, -4) \cup (-2, 3)$

Decreasing: $(-4, -2) \cup (3, \infty)$

6k.

Back to Problem 6.

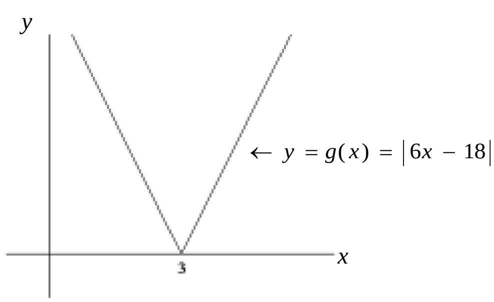


Answer: Increasing: $\left(-7, -\frac{11}{2}\right) \cup (-4, \infty)$

Decreasing: $(-\infty, -7) \cup \left(-\frac{11}{2}, -4\right)$

7a.

Back to Problem 7.

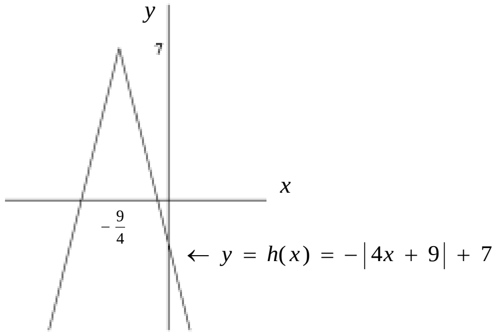


Answer: Rel. Max: None

Rel. Min: 0 at $x = 3$

7b.

Back to Problem 7.

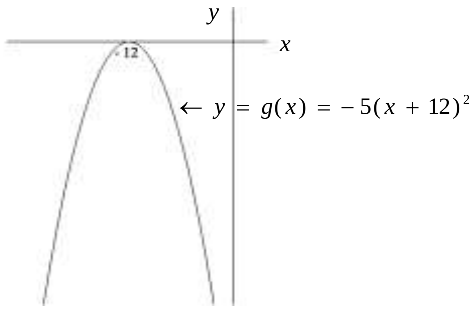


Answer: Rel. Max: 7 at $x = -\frac{9}{4}$

Rel. Min: None

7c.

Back to Problem 7.

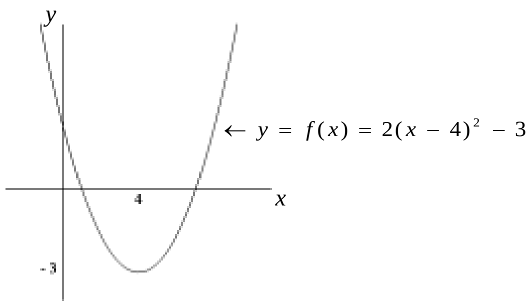


Answer: Rel. Max: 0 at $x = -12$

Rel. Min: None

7d.

Back to Problem 7.

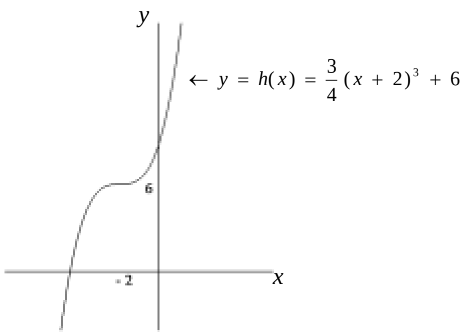


Answer: Rel. Max: None

Rel. Min: -3 at $x = 4$

7e.

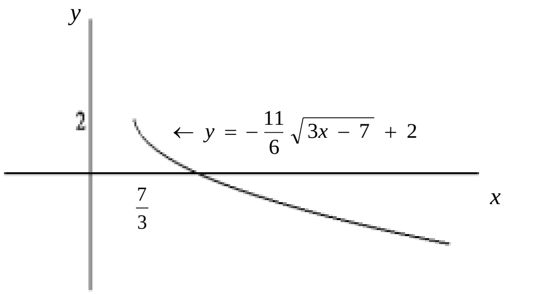
Back to Problem 7.



Answer: Rel. Max: None

Rel. Min: None

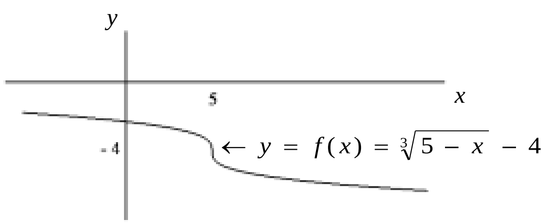
7f.



Back to Problem 7.

Answer: Rel. Max: 2 at $x = \frac{7}{3}$
Rel. Min: None

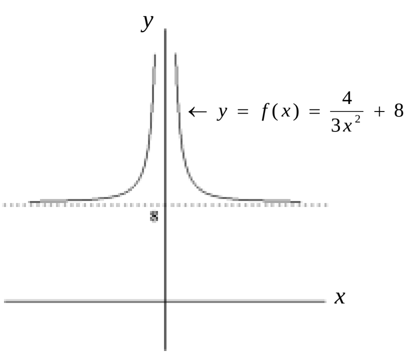
7g.



Back to Problem 7.

Answer: Rel. Max: None
Rel. Min: None

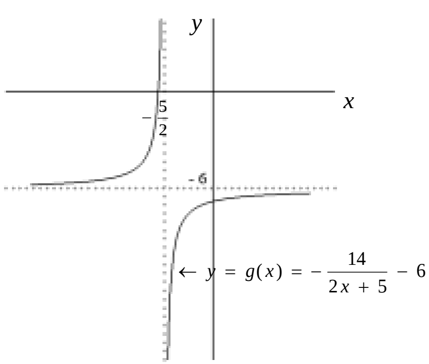
7h.



Back to Problem 7.

Answer: Rel. Max: None
Rel. Min: None

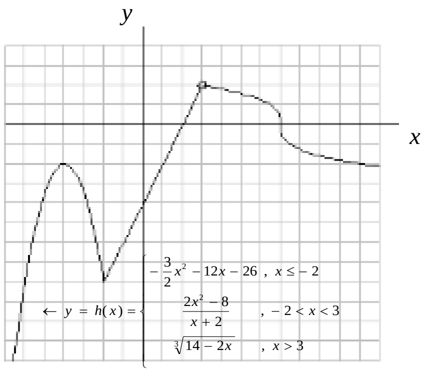
7i.



Back to Problem 7.

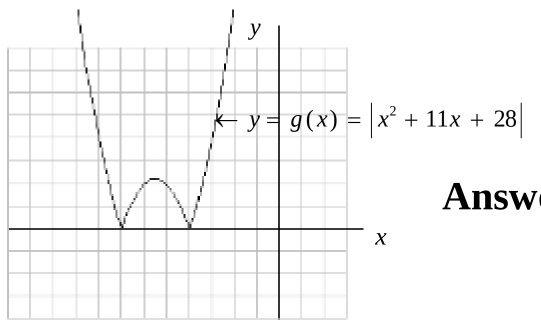
Answer: Rel. Max: None
Rel. Min: None

7j. Back to Problem 7.



Answer: Rel. Max: -2 at $x = -4$
Rel. Min: -8 at $x = -2$

7k. Back to Problem 7.



Answer: Rel. Max: $\frac{9}{4}$ at $x = -\frac{11}{2}$
Rel. Min: 0 at $x = -7$ and $x = -4$

8. $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$

$f(-3) = 36 + 21 - 75 = -18$

$g(-3) = \sqrt{-9 + 25} = \sqrt{16} = 4$

8a. $(f + g)(-3) = f(-3) + g(-3) = -18 + 4 = -14$

Answer: $(f + g)(-3) = -14$

8b. $(f - g)(-3) = f(-3) - g(-3) = -18 - 4 = -22$

Answer: $(f - g)(-3) = -22$ Back to Problem 8.

8c. $(f \cdot g)(-3) = f(-3) \cdot g(-3) = -18(4) = -72$

Answer: $(f \cdot g)(-3) = -72$

8d. $\left(\dfrac{f}{g}\right)(-3) = \dfrac{f(-3)}{g(-3)} = \dfrac{-18}{4} = -\dfrac{9}{2}$

Answer: $\left(\dfrac{f}{g}\right)(-3) = -\dfrac{9}{2}$

9. $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$

The domain of the function f is the set of all real numbers and the domain of the function g is the interval of real numbers given by $\left[-\dfrac{25}{3}, \infty\right)$. Thus, the domain of the functions $f + g$, $f - g$, and $f \cdot g$ is the interval $\left[-\dfrac{25}{3}, \infty\right)$ and the domain of the function $\dfrac{f}{g}$ is the interval $\left(-\dfrac{25}{3}, \infty\right)$.

NOTE: $3x + 25 \geq 0 \Rightarrow 3x \geq -25 \Rightarrow x \geq -\dfrac{25}{3}$

9a. $(f + g)(x) = f(x) + g(x) = 4x^2 - 7x - 75 + \sqrt{3x + 25}$

Answer: $(f + g)(x) = 4x^2 - 7x - 75 + \sqrt{3x + 25}$

9b. $(f - g)(x) = f(x) - g(x) = 4x^2 - 7x - 75 - \sqrt{3x + 25}$

Answer: $(f - g)(x) = 4x^2 - 7x - 75 - \sqrt{3x + 25}$

Back to Problem 9.

9c. $(f \cdot g)(x) = f(x) \cdot g(x) = (4x^2 - 7x - 75)\sqrt{3x + 25}$

Answer: $(f \cdot g)(x) = (4x^2 - 7x - 75)\sqrt{3x + 25}$

9d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{4x^2 - 7x - 75}{\sqrt{3x + 25}}$

Answer: $\left(\dfrac{f}{g}\right)(x) = \dfrac{4x^2 - 7x - 75}{\sqrt{3x + 25}}$

10. $h(x) = 8 - 5x$ and $k(x) = x^2 + 9x - 36$

The domain of the functions h and k is the set of all real numbers. Thus, the domain of the functions $h + k$, $h - k$, and $h \cdot k$ is the set of all real numbers and the domain of the function $\dfrac{h}{k}$ is the interval of real numbers given by $(-\infty, -12) \cup (3, \infty)$.

NOTE: $x^2 + 9x - 36 \neq 0 \Rightarrow (x + 12)(x - 3) \neq 0 \Rightarrow x \neq -12$ and $x \neq 3$

10a. $(h + k)(x) = h(x) + k(x) = 8 - 5x + x^2 + 9x - 36 = x^2 + 4x - 28$

Answer: $(h + k)(x) = x^2 + 4x - 28$

10b. $(h - k)(x) = h(x) - k(x) = 8 - 5x - (x^2 + 9x - 36) =$

$8 - 5x - x^2 - 9x + 36 = 44 - 14x - x^2$

Answer: $(h - k)(x) = 44 - 14x - x^2$

Back to Problem 10.

10c. $(h \cdot k)(x) = h(x) \cdot k(x) = (8 - 5x)(x^2 + 9x - 36) =$

$8x^2 + 72x - 288 - 5x^3 - 45x^2 + 180x = -5x^3 - 37x^2 + 252x - 288$

Answer: $(h \cdot k)(x) = -5x^3 - 37x^2 + 252x - 288$

10d. $\left(\frac{h}{k}\right)(x) = \frac{h(x)}{k(x)} = \frac{8 - 5x}{x^2 + 9x - 36}$

Answer: $\left(\frac{h}{k}\right)(x) = \frac{8 - 5x}{x^2 + 9x - 36}$