Pre-Class Problems 6 for Monday, February 12

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem number or letter.

Definition The domain of the function f is the set of all real numbers x such that f(x) is a real number. That is, domain of $f = \{x : f(x) \text{ is a real number}\}$.

1. Find the domain of the following functions. Write your answer in interval notation.

a.
$$g(x) = \sqrt{9 - 4x}$$
 b. $p(x) = 5x^2 + 3x - 6$

c.
$$r(x) = \frac{x^2 - 9}{12x^2 - 80x + 48}$$
 d. $h(t) = \sqrt[3]{2t^2 - 5t + 3}$

e.
$$f(x) = \sqrt[5]{\frac{5x+3}{x+2}}$$
 f. $g(x) = \frac{\sqrt[4]{45} - 9x}{x^2 - 4x - 12}$

g.
$$h(x) = \frac{x^2 + 3x + 15}{\sqrt[4]{16x + 64}}$$
 h. $f(y) = |7 - 2y|$

i.
$$g(x) = \frac{8x - 3}{4 - |11 - 6x|}$$

2. If
$$g(x) = 6x^2 - 8x + 15$$
, then find $\frac{g(x+h) - g(x)}{h}$.

3. If
$$f(x) = \frac{8}{3x + 16}$$
, then find $\frac{f(-2 + h) - f(-2)}{h}$

- 4. If $f = \{(-4, -7), (-1, 2), (0, 5), (2, -11), (5, -7)\}$, then find the following.
 - a. the domain of f b. the range of f c. f(-1)
 - d. f(2) e. f(5) f. the value(s) of x for which f(x) = -7
 - g. the value(s) of x for which f(x) = 5
- 5. Use the graph of y = g(x) to find the following.



- 6. Find a function of one variable for the following descriptions.
 - a. Betty wishes to fence a rectangular region of area 650 square yards. Express the amount F of fencing that is required as function of x, which is the length of the rectangle.
 - b. A closed rectangular box is to be constructed having a volume of 85 cubic feet. The width of the bottom of the box is w ft. The length of the bottom of the box is five times the width of the bottom. Express the

amount M of material that is needed to make this box as a function of w.

- c. An open rectangular box is to be constructed having a volume of 288 cubic inches. The length of the bottom of the box is three times the width of the bottom. The material for the bottom of the box costs 8 cents per square inch and the material for the four sides costs 5 cents per square inch. Express the cost C to make this box as a function of one variable.
- d. Bill can only afford to buy 100 yards of fencing. He uses the fencing to enclose his rectangular garden. Express the area *A* of the rectangular enclosure as a function of one variable.

Additional problems available in the textbook: Page 192 ... 63 - 84, 87 - 114 and Examples 7 - 10 starting on page 188.

SOLUTIONS:

1a.
$$g(x) = \sqrt{9 - 4x}$$
 Back to Problem 1.

When we replace x by a real number, the functional value of this real number is obtained by multiplying that number by 4. Then subtracting the resulting product from 9. Then taking the square root of this difference. We will obtain a real number as long as we do not take the square root of a negative number. The square root of a negative number is a complex number.

Thus, we want to find all the values of x that will make the expression 9 - 4x be greater than or equal not zero. Thus, we want to solve the inequality $9 - 4x \ge 0$. Solving this inequality, we have that

$$9 - 4x \ge 0 \implies 9 \ge 4x \implies x \le \frac{9}{4}$$

Thus, if we replace x in the expression 9 - 4x by any number that is less than or equal to nine fourths, then the value of 9 - 4x will either be a positive number or will be zero. Thus, the square root of the positive number or zero will be a real number and will not be a complex number. Thus, the domain of the function g is the set of numbers given by the interval $\left(-\infty, \frac{9}{4}\right)$. Note that the number 8 is not in the domain of the function g as we saw in the example above.

Answer:
$$\left(-\infty, \frac{9}{4}\right]$$

1b. $p(x) = 5x^2 + 3x - 6$ Back to Problem 1.

If we replace x in the expression $5x^2 + 3x - 6$ by a real number, the functional value of this real number will real number. Since this is true for all real numbers that we would substitute for x, then the domain of p is the set of all real numbers.

NOTE: The function p is a polynomial function. The domain of any polynomial is the set of all real numbers.

Answer: $(-\infty, \infty)$

1c.
$$r(x) = \frac{x^2 - 9}{12x^2 - 80x + 48}$$

Back to Problem 1.

If we replace x in the expressions $x^2 - 9$ and $12x^2 - 80x + 48$ by a real number, we will obtain a real number for each one. The functional value of the original real number is obtained by dividing the real number, that was obtained from the expression $x^2 - 9$, by the real number that was obtained from the expression $12x^2 - 80x + 48$. This functional value will be a real number if we don't divide by zero. Thus, we want (or need) that the denominator of $12x^2 - 80x + 48$ not equal zero. That is,

Want (Need):
$$12x^2 - 80x + 48 \neq 0$$

 $12x^2 - 80x + 48 \neq 0 \Rightarrow 4(3x^2 - 20x + 12) \neq 0 \Rightarrow 4(x - 6)(3x - 2) \neq 0 \Rightarrow$
 $x - 6 \neq 0$ and $3x - 2 \neq 0 \Rightarrow x \neq 6$ and $x \neq \frac{2}{3}$

Thus, the domain of the function r is the set of all real numbers such that $x \neq 6$ and $x \neq \frac{2}{3}$.

Answer:
$$\left(-\infty,\frac{2}{3}\right)\cup\left(\frac{2}{3},6\right)\cup(6,\infty)$$

NOTE: The function *r* is a rational function. It is the quotient of two polynomials. If *p* and *q* are polynomials, then the function $r(x) = \frac{p(x)}{q(x)}$ is a rational function and its domain is the set $\{x: q(x) \neq 0\}$.

1d.
$$h(t) = \sqrt[3]{2t^2 - 5t + 3}$$
 Back to Problem 1.

The index of the radical is three. This is an odd number (by the mathematical definition of odd). The cube root of a negative real number is also a negative real number. It is not a complex number. Of course, the cube root of a

positive real number is also a positive real number and the cube root of zero is zero. Thus, we can take the cube root of any real number. Thus, the domain of the function h is the set of all real numbers.

In general, if the index of the radical is **odd**, the root of a negative real number is also a negative real number, the root of a positive real number is also a positive real number, and the root of zero is zero.

Answer: $(-\infty, \infty)$

1e.
$$f(x) = \sqrt[5]{\frac{5x+3}{x+2}}$$
 Back to Problem 1.

The index of this radical is five, an odd number. The fifth root of a negative real number is a negative real number, the fifth root of a positive real number is a positive real number, and the fifth root of zero is zero. Thus, we only need to worry about division by zero.

Want (Need): $x + 2 \neq 0 \implies x \neq -2$

Thus, the domain of the function f is the set of all real numbers such that $x \neq -2$.

Answer: $(-\infty, -2) \cup (-2, \infty)$

1f.
$$g(x) = \frac{\sqrt[4]{45} - 9x}{x^2 - 4x - 12}$$

Back to **Problem 1**.

Want (Need): $45 - 9x \ge 0$ and $x^2 - 4x - 12 \ne 0$

 $45 - 9x \ge 0 \implies 45 \ge 9x \implies 5 \ge x \implies x \le 5$

$$x^2 - 4x - 12 \neq 0 \implies (x+2)(x-6) \neq 0 \implies x \neq -2$$
 and $x \neq 6$

Thus, we need that $x \le 5$ and $x \ne -2$ and $x \ne 6$. Since $x \le 5$ and $x \ne 6$ is equilavent to $x \le 5$, then $x \le 5$ and $x \ne -2$ and $x \ne 6$ implies that $x \le 5$ and $x \ne -2$.

Answer: $(-\infty, -2) \cup (-2, 5]$

1g.
$$h(x) = \frac{x^2 + 3x + 15}{\sqrt[4]{16x + 64}}$$
 Back to Problem 1.

Want (Need): 16x + 64 > 0

NOTE: The reason we want or need that 16x + 64 > 0 and not $16x + 64 \ge 0$ is because if 16x + 64 = 0, then $\sqrt[4]{16x + 64x} = \sqrt[4]{0} = 0$ and we would have division by zero.

 $16x + 64 > 0 \implies 16x > -64 \implies x > -4$

Answer: $(-4, \infty)$

1h. f(y) = |7 - 2y| Back to Problem 1.

When we replace *y* by a real number, the functional value of this real number is obtained by multiplying that number by 2. Then subtracting the resulting

product from 7. Then taking the absolute value of this difference. These operations are defined for all real numbers *y*.

Answer: $(-\infty, \infty)$

1i.
$$g(x) = \frac{8x - 3}{4 - |11 - 6x|}$$
 Back to Problem 1.

Want (Need):
$$4 - |11 - 6x| \neq 0$$

 $4 - |11 - 6x| \neq 0 \Rightarrow |11 - 6x| \neq 4 \Rightarrow 11 - 6x \neq \pm 4 \Rightarrow 11 \mp 4 \neq 6x \Rightarrow$
 $\frac{11 \pm 4}{6} \neq x$

$$x \neq \frac{11 \pm 4}{6} \Rightarrow x \neq \frac{11 - 4}{6} \Rightarrow x \neq \frac{7}{6}$$

$$x \neq \frac{11 \pm 4}{6} \Rightarrow x \neq \frac{11 + 4}{6} \Rightarrow x \neq \frac{15}{6}$$

Thus, the domain of the function g is the set of all real numbers such that $x \neq \frac{7}{6}$ and $x \neq \frac{15}{6}$.

Answer:
$$\left(-\infty, \frac{7}{6}\right) \cup \left(\frac{7}{6}, \frac{15}{6}\right) \cup \left(\frac{15}{6}, \infty\right)$$

2.
$$g(x) = 6x^2 - 8x + 15$$

Back to Problem 2.
 $g(x + h) = 6(x + h)^2 - 8(x + h) + 15 =$
 $6(x^2 + 2xh + h^2) - 8x - 8h + 15 = 6x^2 + 12xh + 6h^2 - 8x - 8h + 15$
 $g(x) = 6x^2 - 8x + 15$

NOTE: In the subtraction of g(x) from g(x+h), the $6x^2$ terms will cancel, the -8x terms will cancel, and the 15's will cancel. Thus,

$$g(x + h) - g(x) = 12xh + 6h^{2} - 8h = h(12x + 6h - 8)$$

NOTE: In the division of g(x + h) - g(x) by h, the h's will cancel. Thus,

$$\frac{g(x+h) - g(x)}{h} = 12x + 6h - 8 \text{ provided that } h \neq 0.$$

Answer: 12x + 6h - 8

NOTE: In calculus, you will take the limit as h approaches zero of this expression in order to obtain the derivative of the function g. That is,

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} (12x + 6h - 8) = 12x - 8$$

3.
$$f(x) = \frac{8}{3x + 16}$$
 Back to Problem 3.

$$f(-2+h) = f(h-2) = \frac{8}{3(h-2)+16} = \frac{8}{3h-6+16} = \frac{8}{3h+10}$$

$$f(-2) = \frac{8}{10} = \frac{4}{5}$$
$$f(-2 + h) - f(-2) = \frac{8}{3h + 10} - \frac{4}{5}$$

The least common denominator of 5 and 3h + 10 is 5(3h + 10). Thus,

$$f(-2+h) - f(-2) = \frac{40}{5(3h+10)} - \frac{4(3h+10)}{5(3h+10)} = \frac{40 - 4(3h+10)}{5(3h+10)} = \frac{40}{5(3h+10)} = \frac$$

$$\frac{40 - 12h - 40}{5(3h + 10)} = \frac{-12h}{5(3h + 10)}$$

NOTE: Division by *h* is the same as multiplying by $\frac{1}{h}$, provided that $h \neq 0$. When you multiply f(-2 + h) - f(-2), which is a fraction by $\frac{1}{h}$, the *h*'s will cancel. Thus,

$$\frac{f(-2+h) - f(-2)}{h} = \frac{-12}{5(3h+10)}$$
, provided that $h \neq 0$.

Answer: $\frac{-12}{5(3h+10)}$ or $-\frac{12}{5(3h+10)}$

NOTE: In calculus, you will take the limit as *h* approaches zero of this expression in order to obtain the slope of the tangent line to the graph of the function *f* at the point $(-2, f(-2)) = \left(-2, \frac{4}{5}\right)$. That is,

$$m_{\text{tan}} = f'(-2) = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h \to 0} \left[-\frac{12}{5(3h+10)} \right] = -\frac{12}{50}$$
$$= -\frac{6}{25}$$

4a. $f = \{(-4, -7), (-1, 2), (0, 5), (2, -11), (5, -7)\}$ Back to <u>Problem 4</u>.

NOTE: The points of the function f are of the form (x, f(x)).

The domain of the function is set $\{-4, -1, 0, 2, 5\}$.

Answer: $\{-4, -1, 0, 2, 5\}$

4b. $f = \{(-4, -7), (-1, 2), (0, 5), (2, -11), (5, -7)\}$ Back to <u>Problem 4</u>.

NOTE: The points of the function f are of the form (x, f(x)).

The range of the function is set $\{-7, 2, 5, -11\}$. NOTE: We don't list the value of -7 for a second time. We only list it once.

Answer: $\{-7, 2, 5, -11\}$ or $\{-11, -7, 2, 5\}$

4c. $f = \{(-4, -7), (-1, 2), (0, 5), (2, -11), (5, -7)\}$ Back to <u>Problem 4</u>.

NOTE: The points of the function f are of the form (x, f(x)).

$$(-1, 2) \Rightarrow f(-1) = 2$$

Answer: 2

4d. $f = \{(-4, -7), (-1, 2), (0, 5), (2, -11), (5, -7)\}$ Back to <u>Problem 4</u>. NOTE: The points of the function *f* are of the form (x, f(x)).

 $(2,-11) \implies f(2) = -11$

Answer: -11

4e. $f = \{(-4, -7), (-1, 2), (0, 5), (2, -11), (5, -7)\}$ Back to <u>Problem 4</u>.

NOTE: The points of the function f are of the form (x, f(x)).

 $(5,-7) \Rightarrow f(5) = -7$

Answer: - 7

4f.
$$f = \{(-4, -7), (-1, 2), (0, 5), (2, -11), (5, -7)\}$$
 Back to Problem 4.

NOTE: The points of the function f are of the form (x, f(x)).

$$(-4, -7) \Rightarrow f(-4) = -7$$
 Thus, $f(x) = -7$ when $x = -4$
 $(5, -7) \Rightarrow f(5) = -7$ Thus, $f(x) = -7$ when $x = 5$

Answer: -4, 5

4g. $f = \{(-4, -7), (-1, 2), (0, 5), (2, -11), (5, -7)\}$ Back to <u>Problem 4</u>.

NOTE: The points of the function f are of the form (x, f(x)).

 $(0,5) \Rightarrow f(0) = 5$ Thus, f(x) = 5 when x = 0

Answer: 0



NOTE: The points of the function g are of the form (x, g(x)).

 $(0,0) \Rightarrow g(0) = 0$

Answer: 0

Back to **Problem 5**.



NOTE: The points of the function g are of the form (x, g(x)).

$$(-2, -8) \Rightarrow g(-2) = -8$$

Answer: -8

Back to **Problem 5**.



NOTE: The points of the function g are of the form (x, g(x)).

$$(1,-2) \implies g(1) = -2$$

Answer: – 2 Back to Problem 5.



NOTE: The points of the function g are of the form (x, g(x)).

 $(5,0) \Rightarrow g(5) = 0$

Answer: 0

Back to **Problem 5**.



Answer: (0, 0); (5, 0) Back to <u>Problem 5</u>.

5f. x NO' (x, x)

NOTE: The points of the function g are of the form (x, g(x)).

Answer: (0, 0)

Back to **Problem 5**.





NOTE: The points of the function g are of the form (x, g(x)).

$$(3,2) \Rightarrow g(3) = 2$$
 Thus, $g(x) = 2$ when $x = 3$

Answer: 3

Back to **Problem 5**.



NOTE: The points of the function f are of the form (x, g(x)).

 $(-1, -2) \Rightarrow g(-1) = -2$ Thus, g(x) = -2 when x = -1 $(1, -2) \Rightarrow g(1) = -2$ Thus, g(x) = -2 when x = 1 $(7, -2) \Rightarrow g(7) = -2$ Thus, g(x) = -2 when x = 7

Answer: -1, 1, 7 Back to Problem 5.

6a. Betty wishes to fence a rectangular region of area 650 square yards. Express the amount F of fencing that is required as function of x, which is the length of the rectangle.

We will need to identify the width of the rectangle: Let *y* be the width of the rectangle.



NOTE: The amount of fencing, which is required to fence this rectangular region, is the perimeter of the rectangle. Thus, F = 2x + 2y.

NOTE: *F* is a function of two variables *x* and *y*. In order to get *F* as a function of one variable *x*, we will need to get a relationship between *x* and *y*. A relationship between *x* and *y* is an equation containing only the variables of *x* and *y*. We haven't used the information that the area of the rectangular enclosure is to 650 square yards. Since the area of a rectangle is given by the formula A = lw, then the area of our rectangular enclosure is A = xy. Thus, in order for the area of our enclosure to be 650 square yards, we need that xy = 650. This is our relationship between *x* and *y*.

Now, we can solve for y in terms of x. Thus, $xy = 650 \implies y = \frac{650}{x}$. Since F = 2x + 2y and $y = \frac{650}{x}$, then $F = 2x + 2y = 2x + 2\left(\frac{650}{x}\right)$ $= 2x + \frac{1300}{x}$.

Answer: $F = 2x + \frac{1300}{x}$ (in yards) Back to Problem 6.

NOTE: In calculus, you will find the dimensions of the rectangle which requires the least of amount of fencing in order to enclose 650 square yards.

6b. A closed rectangular box is to be constructed having a volume of 85 cubic feet. The width of the bottom of the box is w ft. The length of the bottom of the box is five times the width of the bottom. Express the amount M of material that is needed to make this box as a function of w.

NOTE: Closed box means that there will be a top to the box.

We will need to identify the height of the box: Let h be the height of the box.



The amount of material, which is needed to construct the base **or** the top of the box, is given by $5w^2$ square feet. (You only need to recognize this statement. You do not need to write it.)

Thus, the amount of material, which is needed to construct the base **and** the top of the box, is given by $2(5w^2)$ square feet. (You only need to recognize this statement. You do not need to write it.)

The amount of material, which is needed to construct the front **or** the back of the box, is given by 5wh square feet. (You only need to recognize this statement. You do not need to write it.)

Thus, the amount of material, which is needed to construct the front **and** the back of the box, is given by 2(5wh) square feet. (You only need to recognize this statement. You do not need to write it.)

The amount of material, which is needed to construct the left-side or the right-side of the box, is given by wh square feet. (You only need to recognize this statement. You do not need to write it.)

Thus, the amount of material, which is needed to construct the front **and** the back of the box, is given by 2wh square feet. (You only need to recognize this statement. You do not need to write it.)

Thus,
$$M = 2(5w^2) + 2(5wh) + 2wh \implies M = 10w^2 + 10wh + 2wh$$

 $\Rightarrow M = 10w^2 + 12wh$.

NOTE: *M* is a function of two variables *w* and *h*. In order to get *M* as a function of one variable *y*, we will need to get a relationship between *w* and *h*. A relationship between *w* and *h* is an equation containing only the variables of *w* and *h*. We haven't used the information that the volume of the box is to 85 cubic feet. Since the volume of a box is given by the formula V = lwh, then the volume of our box is $V = (5w)wh = 5w^2h$. Thus, in order for the volume of our box to be 85 cubic feet, we need that $5w^2h = 85$. This is our relationship between *w* and *h*.

Now, we can solve for *h* in terms of *w*. Thus, $5w^2h = 85 \Rightarrow h = \frac{85}{5w^2} \Rightarrow h = \frac{17}{w^2}$. Since $M = 10w^2 + 12wh$ and $h = \frac{17}{w^2}$, then $M = 10w^2 + 12wh = 10w^2 + 12\left(\frac{17}{w^2}\right)w = 10w^2 + \frac{204}{w}$. NOTE: 17(12) = 17(10+2) = 170 + 34 = 204

Answer: $M = 10w^2 + \frac{204}{w}$ (in ft²) Back to Problem 6.

NOTE: In calculus, you will find the dimensions of the box which require the least of amount of material in order to have a volume of 85 cubic feet.

6c. An open rectangular box is to be constructed having a volume of 288 cubic inches. The length of the bottom of the box is three times the width of the bottom. The material for the bottom of the box costs 8 cents per square inch and the material for the four sides costs 5 cents per square inch. Express the cost C to make this box as a function of one variable.

NOTE: An open box means no top.



NOTE: The amount of material, which is needed to construct the bottom of the box, is given by $3x^2$ square inches. Thus, the cost to construct the bottom of the box is given by $8(3x^2)$ cents.

NOTE: The amount of material, which is needed to construct the front and the back of the box, is given by 2(3xy) square inches. Thus, the cost to construct the front and the back of the box is given by 5[2(3xy)] cents.

NOTE: The amount of material, which is needed to construct the left-side and the right-side of the box, is given by 2xy square inches. Thus, the cost to construct the front and the back of the box is given by 5(2xy) cents.

Thus, the cost, C, to construct this open box, is given by $C = 8(3x^2) + 5[2(3xy) + 2xy]$ in cents. Simplifying, we have that $C = 24x^2 + 5(6xy + 2xy) = 24x^2 + 5(8xy) = 24x^2 + 40xy$ in cents.

NOTE: *C* is a function of two variables *x* and *y*. In order to get *C* as a function of one variable in *x* or *y*, we will need to get a relationship between *x* and *y*. A relationship between *x* and *y* is an equation containing only the variables of *x* and *y*. We haven't used the information that the volume of the box is to be 288 cubic inches. Since the volume of a box is given by the formula V = lwh, then the volume of our box is $V = (3x)xy = 3x^2y$. Thus, in order for the volume of our box to be 288 cubic inches, we need that $3x^2y = 288$. This is our relationship between *x* and *y*.

Now, we can solve the equation $3x^2y = 288$ for x or y. It is easier to solve for y in terms of x. Thus, $3x^2y = 288 \implies y = \frac{288}{3x^2} \implies y = \frac{96}{x^2}$.

Since $C = 24x^2 + 40xy$ and $y = \frac{96}{x^2}$, then $C = 24x^2 + 40xy =$

$$4\left[6x^{2} + 10x\left(\frac{96}{x^{2}}\right)\right] = 24\left[x^{2} + 10x\left(\frac{16}{x^{2}}\right)\right] = 24\left(x^{2} + \frac{160}{x}\right).$$

Answer: $C = 24\left(x^2 + \frac{160}{x}\right)$ (in cents) Back to <u>Problem 6</u>.

NOTE: In calculus, you will find the dimensions of the box cheapest to construct and will have a volume of 288 cubic inches.

6d. Bill can only afford to buy 100 yards of fencing. He uses the fencing to enclose his rectangular garden. Express the area A of the rectangular enclosure as a function of one variable.



Area of the enclosure: A = xy

Since the amount of the fencing is 100 yards, we have that $2x + 2y = 100 \implies x + y = 50$. Solving for y, we have that y = 50 - x.

Since A = xy and y = 50 - x, then $A = x(50 - x) = 50x - x^{2}$.

Answer: $A = 50x - x^2$ (in square yards) Back to Problem 6.