Pre-Class Problems 5 for Monday, February 5, and Wednesday, February 7

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem number or letter.

Theorem The distance between the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the *xy*-plane is given by $d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $d(P_1, P_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Theorem The midpoint of the line segment joining the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the *xy*-plane is given by $M(P_1, P_2) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$.

1. Find the exact distance between the following two points and the exact midpoint of the line segment joining the points.

   a. $(-3, 7)$ and $(5, -11)$ b. $(-2, -5)$ and $(1, 4)$

   c. $(\sqrt{3}, -2\sqrt{6})$ and $(-5\sqrt{3}, \sqrt{6})$

2. Identify the set of values of *x* for which *y* will be a real number. Use interval notation to write your answer.

   a. $y = \dfrac{3}{x + 6}$ b. $y = \sqrt{x + 4}$ c. $y = \dfrac{7}{11 - 5x}$

   d. $y = \sqrt{16 - 9x}$

Definition An *x*-intercept of the graph of an equation in the variables of *x* and/or *y* is a point on the *x*-axis where the graph of the equation intersects the *x*-axis.

NOTE: Since any point on the *x*-axis has a *y*-coordinate of zero, then an *x*-intercept of a graph is a point of the form $(a,\ 0)$, where *a* is a real number.

To find a *x*-intercept, set $y = 0$ since any point on the *x*-axis has a *y*-coordinate of zero.

Definition An *y*-intercept of the graph of an equation in the variables of *x* and/or *y* is a point on the *y*-axis where the graph of the equation intersects the *y*-axis.

NOTE: Since any point on the *y*-axis has a *x*-coordinate of zero, then an *y*-intercept of a graph is a point of the form $(0,\ a)$, where *a* is a real number.

To find a *y*-intercept, set $x = 0$ since any point on the *y*-axis has a *x*-coordinate of zero.

3. Find the *x*-intercept(s) and the *y*-intercept(s) of the graph of the following equations.

a. $3x - 7y = 15$ b. $6x^2 + y^2 = 36$ c. $4x^2 - 25y^2 = 100$

d. $x^2 = 12 - 5y$ e. $x = 3y^2 - 8$ f. $y = 4|6x - 5| + 9$

g. $\dfrac{(x + 5)^2}{9} + \dfrac{(y - 2)^2}{16} = 1$

Definition A circle is the set of all points in a plane that are equidistant from fixed point, called the center of the circle. The fixed distant from the center of the circle to a point on the circle is called the radius of the circle.

NOTE: Since the radius of a circle is a distant, then it is a positive real number.

Theorem If the point $C(h,k)$ is the center of a circle and r is the radius of the circle, then the equation of the circle in the *xy*-plane is given by $(x-h)^2+(y-k)^2=r^2$. This equation is called the standard form of the equation for a circle.

4. Write the equation of the circle in standard form given the following information.

   a. Center: $(4,-7)$; Radius: 6

   b. Center: $(-1,3)$; Radius: $\sqrt{5}$

   c. The endpoints of a diameter are $(-4,\ 7)$ and $(8,\ -9)$.

   d. The endpoints of a diameter are $(-6,\ -2)$ and $(-5,\ 3)$.

   e. The center is $(-7,-3)$ and the point $(2,-8)$ is a point on the circle.

   f. The center is $(4,0)$ and the point $(-3,\ 6)$ is a point on the circle.

   g. Write an equation that represents the set of points that are 5 units from the point $(0,-3)$.

   h. The center is $(7,-4)$ and the circle is tangent to the *y*-axis.

   i. The center is $(7,-4)$ and the circle is tangent to the *x*-axis.

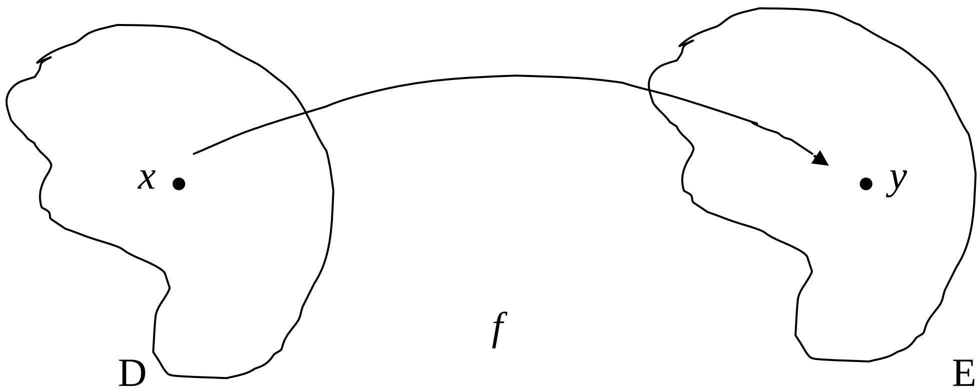
5. Find the *x*-intercept(s) and *y*-intercept(s) of the equation of the circle in Problem 4h.

6. Find the *x*-intercept(s) and *y*-intercept(s) of the equation of the circle in Problem 4i.

7. Find the center and radius of the circle whose equation is given by the following.

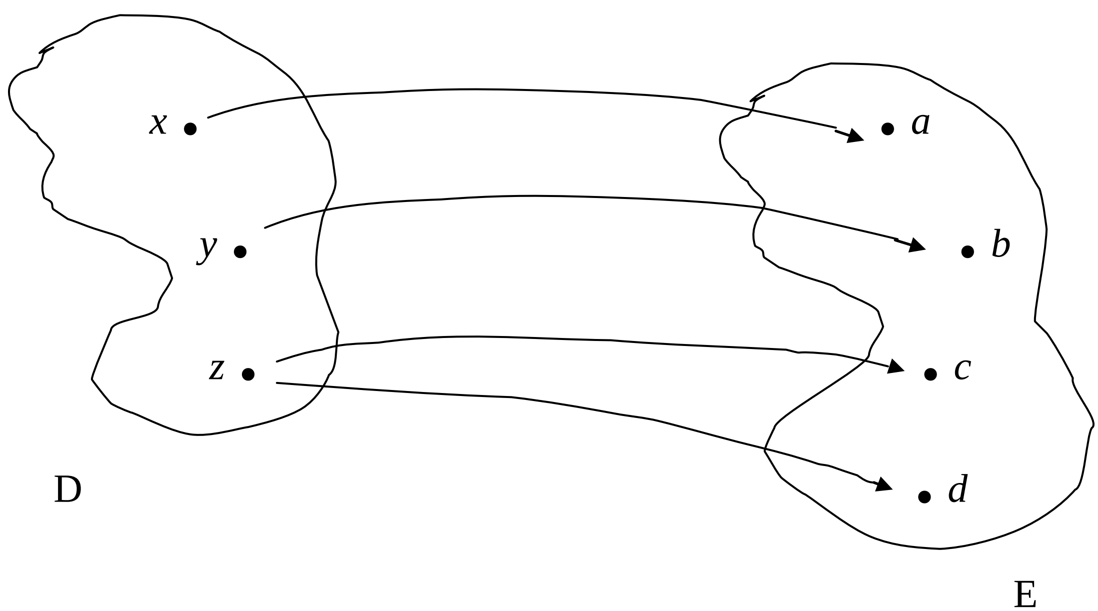
a. $x^2 + y^2 - 10x + 18y + 20 = 0$

Definiton A function f from a set D to a set E is a correspondence that assigns to each element x of D a unique element y of E .

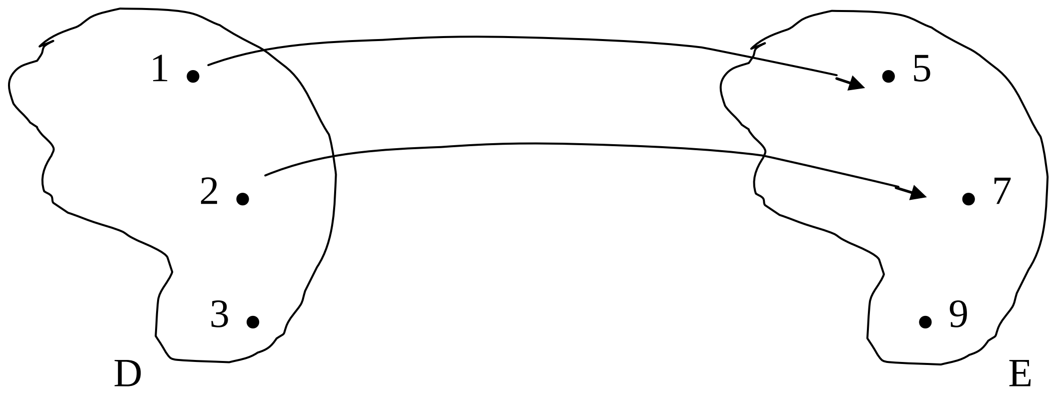


8. Determine if the following are functions or not.

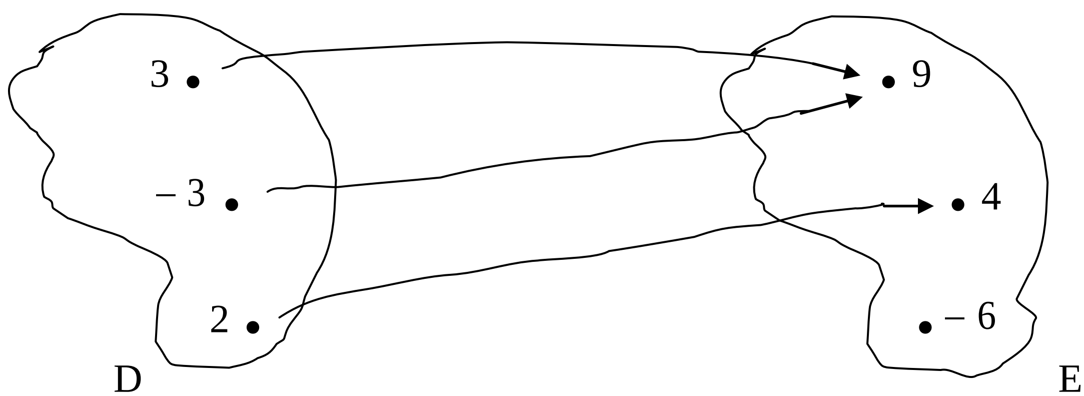
a.



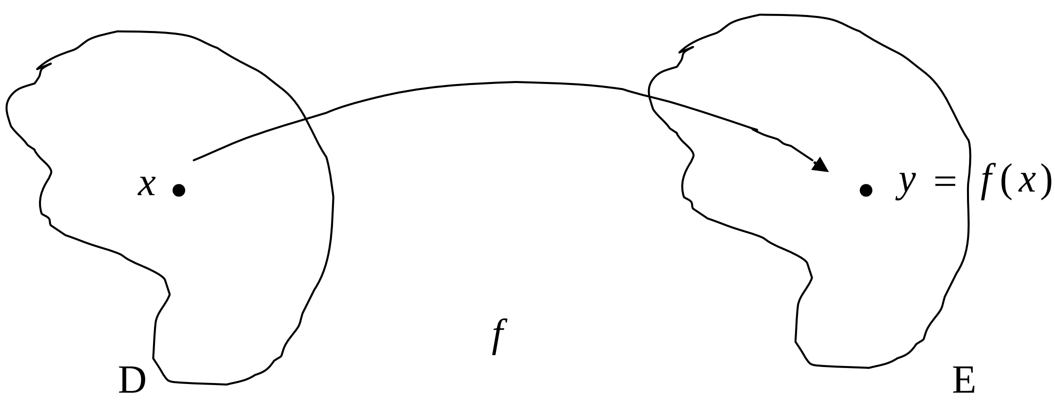
b.



c.

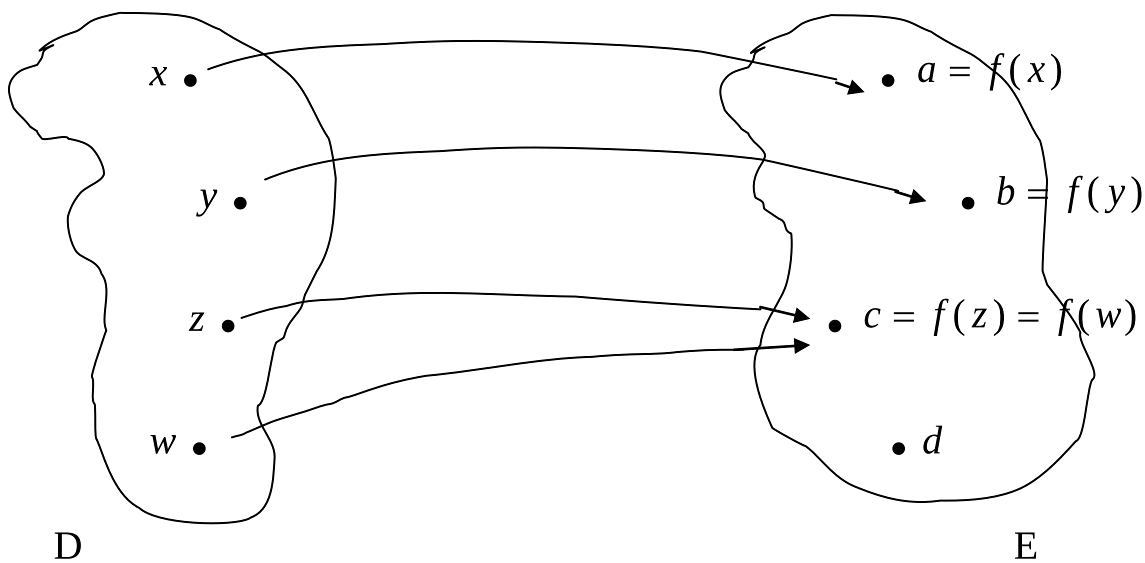


Let's develop some notation and terminology using the following picture for the function f.



The element y of the set E is called the value of the function f at x and is denoted by $f(x)$, read "f of x." A common mistake, which is made by students, is saying that $f(x)$ is the function. The name of the function is f and $f(x)$ is a functional value, namely the value of the function at *x*. The set D is called the domain of the function. The range of the function f is the set consisting of all possible functional values $f(x)$, where *x* is in the domain D.

Example



Domain of f = { x, y, z, w }

Range of f = { a, b, c }

9. If $f(x) = 6x^2 - 17x - 14$, then find

a. $f(1)$ b. $f(-2)$ c. $f(0)$ d. $f(x+h)$

10. If $g(x) = \sqrt{9 - 4x}$, then find

a. $g(1)$ b. $g(0)$ c. $g(-9)$ d. $g(8)$

Additional problems available in the textbook: Page 173 … 11 – 14, 17, 18, 25 – 30, 51 – 62, 71, 72 and Page 181 … 5 – 36, 41 – 54 and Examples 1, 3, 6 starting on page 167 and Examples 1 – 4 starting on page 178. Page 192 … 15 – 26, 35 – 60 and Examples 2 – 6 starting on page 184.

SOLUTIONS:

1a. $(-3,\ 7)$ and $(5,\ -11)$ Back to Problem 1.

$$d = \sqrt{(-3-5)^2 + (7+11)^2} = \sqrt{(-8)^2 + 18^2} = \sqrt{64+324}$$

$$\sqrt{4(16+81)} = 2\sqrt{97}$$

$$M = \left(\frac{-3+5}{2},\ \frac{7-11}{2}\right) = \left(\frac{2}{2},\ \frac{-4}{2}\right) = (1,\ -2)$$

Answers: $2\sqrt{97}$; $(1,\ -2)$

1b. $(-2,\ -5)$ and $(1,\ 4)$ Back to Problem 1.

$$d = \sqrt{(-2-1)^2 + (-5-4)^2} = \sqrt{(-3)^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$

$$M = \left(\frac{-2+1}{2},\ \frac{-5+4}{2}\right) = \left(-\frac{1}{2},\ -\frac{1}{2}\right)$$

Answers: $3\sqrt{10}$; $\left(-\frac{1}{2},\ -\frac{1}{2}\right)$

1c. $(\sqrt{3}\,,\ -2\sqrt{6}\,)$ and $(-5\sqrt{3}\,,\ \sqrt{6}\,)$ Back to Problem 1.

$$d = \sqrt{(\sqrt{3} + 5\sqrt{3}\,)^2 + (-2\sqrt{6} - \sqrt{6}\,)^2} = \sqrt{(6\sqrt{3}\,)^2 + (-3\sqrt{6}\,)^2} =$$

$$\sqrt{108 + 54} = \sqrt{9(12 + 6)} = 3\sqrt{18} = 3 \cdot 3\sqrt{2} = 9\sqrt{2}$$

$$M = \left(\frac{\sqrt{3} - 5\sqrt{3}}{2}\,,\ \frac{-2\sqrt{6} + \sqrt{6}}{2} \right) = \left(\frac{-4\sqrt{3}}{2}\,,\ \frac{-\sqrt{6}}{2} \right) = \left(-2\sqrt{3}\,,\ -\frac{\sqrt{6}}{2} \right)$$

Answers: $9\sqrt{2}$; $\left(-2\sqrt{3}\,,\ -\dfrac{\sqrt{6}}{2} \right)$

2a. $y = \dfrac{3}{x + 6}$ Back to Problem 2.

The value of y is determined by dividing 3 by $x + 6$. Since division by zero is undefined, then $x + 6 \neq 0$.

$$x + 6 \neq 0 \Rightarrow x \neq -6$$

Answer: $(-\infty, -6) \cup (-6, \infty)$

2b. $y = \sqrt{x + 4}$ Back to Problem 2.

The value of y is determined by taking the square root of $x + 4$. If $x + 4 < 0$, then $\sqrt{x + 4}$ will be a complex number. If $x + 4 \geq 0$, then $\sqrt{x + 4}$ will be a real number.

$x + 4 \geq 0 \Rightarrow x \geq -4$

Answer: $[-4, \infty)$

2c. $y = \dfrac{7}{11 - 5x}$ Back to Problem 2.

The value of y is determined by dividing 7 by $11 - 5x$. Since division by zero is undefined, then $11 - 5x \neq 0$.

$11 - 5x \neq 0 \Rightarrow 11 \neq 5x \Rightarrow x \neq \dfrac{11}{5}$

Answer: $\left(-\infty, \dfrac{11}{5}\right) \cup \left(\dfrac{11}{5}, \infty\right)$

2d. $y = \sqrt{16 - 9x}$ Back to Problem 2.

The value of y is determined by taking the square root of $16 - 9x$. If $16 - 9x < 0$, then $\sqrt{16 - 9x}$ will be a complex number. If $16 - 9x \geq 0$, then $\sqrt{16 - 9x}$ will be a real number.

$$16 - 9x \geq 0 \Rightarrow 16 \geq 9x \Rightarrow \frac{16}{9} \geq x \Rightarrow x \leq \frac{16}{9}$$

Answer: $\left(-\infty, \frac{16}{9} \right]$

3a. $3x - 7y = 15$ Back to Problem 3.

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$3x - 7y = 15$ and $y = 0 \Rightarrow 3x = 15 \Rightarrow x = 5$

***x*-intercept(s):** $(5, 0)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$3x - 7y = 15$ and $x = 0 \Rightarrow -7y = 15 \Rightarrow y = -\frac{15}{7}$

***y*-intercept(s):** $\left(0, -\frac{15}{7} \right)$

NOTE: The graph of the equation $3x - 7y = 15$ is a straight line.

3b. $6x^2 + y^2 = 36$ Back to Problem 3.

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$6x^2 + y^2 = 36$ and $y = 0 \Rightarrow 6x^2 = 36 \Rightarrow x^2 = 6 \Rightarrow x = \pm\sqrt{6}$

***x*-intercept(s):** $(-\sqrt{6}, 0)$; $(\sqrt{6}, 0)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$6x^2 + y^2 = 36$ and $x = 0 \Rightarrow y^2 = 36 \Rightarrow y = \pm 6$

***y*-intercept(s):** $(0, -6)$; $(0, 6)$

NOTE: The graph of the equation $6x^2 + y^2 = 36$ is an ellipse. We will study ellipses later in the course.

3c. $4x^2 - 25y^2 = 100$ Back to Problem 3.

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$4x^2 - 25y^2 = 100$ and $y = 0 \Rightarrow 4x^2 = 100 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

***x*-intercept(s):** $(-5, 0)$; $(5, 0)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$4x^2 - 25y^2 = 100$ and $x = 0 \Rightarrow -25y^2 = 100 \Rightarrow y^2 = -4 \Rightarrow y = \pm 2i$

NOTE: Since the solutions to this equation are complex numbers, then the graph of the equation does not have any *y*-intercepts.

***y*-intercept(s):** None

NOTE: The graph of the equation $4x^2 - 25y^2 = 100$ is an hyperbola. We will study hyperbolas later in the course.

3d. $x^2 = 12 - 5y$ Back to Problem 3.

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$x^2 = 12 - 5y$ and $y = 0 \Rightarrow x^2 = 12 \Rightarrow x = \pm\sqrt{12} \Rightarrow x = \pm 2\sqrt{3}$

***x*-intercept(s):** $(-2\sqrt{3}, 0)$; $(2\sqrt{3}, 0)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$x^2 = 12 - 5y$ and $x = 0 \Rightarrow 0 = 12 - 5y \Rightarrow 5y = 12 \Rightarrow y = \dfrac{12}{5}$

***y*-intercept(s):** $\left(0, \dfrac{12}{5}\right)$

NOTE: The graph of the equation $x^2 = 12 - 5y$ is a parabola. We will study parabolas later in the course.

3e. $x = 3y^2 - 8$ Back to Problem 3.

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$x = 3y^2 - 8$ and $y = 0 \Rightarrow x = -8$

***x*-intercept(s):** $(-8,\ 0)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$x = 3y^2 - 8$ and $x = 0 \Rightarrow 0 = 3y^2 - 8 \Rightarrow 8 = 3y^2 \Rightarrow y^2 = \dfrac{8}{3} \Rightarrow$

$y = \pm\sqrt{\dfrac{8}{3}} = \pm\dfrac{\sqrt{24}}{3} = \pm\dfrac{2\sqrt{6}}{3}$

***y*-intercept(s):** $\left(0,\ -\dfrac{2\sqrt{6}}{3}\right)$; $\left(0,\ \dfrac{2\sqrt{6}}{3}\right)$

NOTE: The graph of the equation $x = 3y^2 - 8$ is a parabola. We will study parabolas later in the course.

3f. $y = 4|6x - 5| + 9$ Back to Problem 3.

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$y = 4|6x - 5| + 9$ and $y = 0 \Rightarrow 0 = 4|6x - 5| + 9 \Rightarrow -9 = 4|6x - 5|$

$|6x - 5| = -\dfrac{9}{4}$

Since this equation does not have any solutions, then the graph of the equation does not have any *x*-intercepts.

***x*-intercept(s):** None

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$y = 4|6x - 5| + 9$ and $x = 0 \Rightarrow y = 4|-5| + 9 = 4(5) + 9 = 20 + 9 = 29$

***y*-intercept(s):** $(0, 29)$

3g. $\dfrac{(x + 5)^2}{9} + \dfrac{(y - 2)^2}{16} = 1$ Back to Problem 3.

$\text{LCD} = 144$

$$\frac{(x + 5)^2}{9} + \frac{(y - 2)^2}{16} = 1 \Rightarrow 144\left(\frac{(x + 5)^2}{9} + \frac{(y - 2)^2}{16}\right) = 144 \Rightarrow$$

$$16(x + 5)^2 + 9(y - 2)^2 = 144$$

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$16(x + 5)^2 + 9(y - 2)^2 = 144$ and $y = 0 \Rightarrow$

$$16(x + 5)^2 + 9(-2)^2 = 144 \Rightarrow 16(x + 5)^2 + 9(4) = 144 \Rightarrow$$

$$16(x + 5)^2 + 36 = 144 \Rightarrow 16(x + 5)^2 = 108 \Rightarrow (x + 5)^2 = \frac{108}{16} \Rightarrow$$

$$(x+5)^2 = \frac{27}{4} \Rightarrow x+5 = \pm\frac{\sqrt{27}}{2} \Rightarrow x = -5 \pm \frac{3\sqrt{3}}{2} = \frac{-10 \pm 3\sqrt{3}}{2}$$

***x*-intercept(s):** $\left(\frac{-10-3\sqrt{3}}{2}, 0\right); \left(\frac{-10+3\sqrt{3}}{2}, 0\right)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$16(x+5)^2 + 9(y-2)^2 = 144$ and $x = 0 \Rightarrow$

$16(5)^2 + 9(y-2)^2 = 144 \Rightarrow 16(25) + 9(y-2)^2 = 144 \Rightarrow$

$400 + 9(y-2)^2 = 144 \Rightarrow 9(y-2)^2 = -256 \Rightarrow (y-2)^2 = -\frac{256}{9}$

NOTE: Since the solutions to this equation are complex numbers, then the graph of the equation does not have any *y*-intercepts.

***y*-intercept(s):** None

NOTE: The graph of the equation $\frac{(x+5)^2}{9} + \frac{(y-2)^2}{16} = 1$ is an ellipse. We will study ellipses later in the course.

4a. Center: $(4, -7)$; Radius: 6 Back to Problem 4.

The standard form of the equation for a circle: $(x-h)^2 + (y-k)^2 = r^2$, where the point (h, k) is the center of the circle and *r* is the radius.

$h = 4,\ k = -7,\ r = 6 \Rightarrow (x - 4)^2 + (y + 7)^2 = 36$

Answer: $(x - 4)^2 + (y + 7)^2 = 36$

4b. Center: $(-1, 3)$; Radius: $\sqrt{5}$ Back to Problem 4.

The standard form of the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$, where the point (h, k) is the center of the circle and *r* is the radius.

$h = -1,\ k = 3,\ r = \sqrt{5} \Rightarrow (x + 1)^2 + (y - 3)^2 = 5$

Answer: $(x + 1)^2 + (y - 3)^2 = 5$

4c. The endpoints of a diameter are $(-4,\ 7)$ and $(8,\ -9)$. Back to Problem 4.

NOTE: The diameter of a circle is a line segment passing through the center of the circle and whose endpoints lie on the circle. The midpoint of the diameter is the center of the circle and the length of the diameter is twice the radius of the circle.

Use the midpoint formula to find the center of the circle:

$$\left(\frac{-4 + 8}{2}, \frac{7 - 9}{2} \right) = \left(\frac{4}{2}, \frac{-2}{2} \right) = (2, -1)$$

Use the distance formula to find the length of the diameter in order to find the radius:

$d = \sqrt{(-4-8)^2 + (7+9)^2} = \sqrt{(-12)^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400}$

$= 20$

NOTE: $\sqrt{(-12)^2 + 16^2} = \sqrt{12^2 + 16^2} = \sqrt{(4 \cdot 3)^2 + (4 \cdot 4)^2} =$

$\sqrt{16 \cdot 9 + 16 \cdot 16} = \sqrt{16(9 + 16)} = 4\sqrt{25} = 4(5) = 20$

Since the diameter is 20, then the radius is 10.

Center: $(2, -1)$; Radius: 10

The standard form of the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$, where the point (h, k) is the center of the circle and r is the radius.

$h = 2,\ k = -1,\ r = 10 \Rightarrow (x - 2)^2 + (y + 1)^2 = 100$

Answer: $(x - 2)^2 + (y + 1)^2 = 100$

4d. The endpoints of a diameter are $(-6, -2)$ and $(-5, 3)$. Back to Problem 4.

NOTE: The diameter of a circle is a line segment passing through the center of the circle and whose endpoints lie on the circle. The midpoint of the diameter is the center of the circle and the length of the diameter is twice the radius of the circle.

Use the midpoint formula to find the center of the circle:

$$\left(\frac{-6-5}{2}, \frac{-2+3}{2}\right) = \left(-\frac{11}{2}, \frac{1}{2}\right)$$

Use the distance formula to find the length of the diameter in order to find the radius:

$$d = \sqrt{(-6+5)^2 + (-2-3)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1+25} = \sqrt{26}$$

Since the diameter is $\sqrt{26}$, then the radius is $\frac{\sqrt{26}}{2}$.

Center: $\left(-\frac{11}{2}, \frac{1}{2}\right)$; Radius: $\frac{\sqrt{26}}{2}$

The standard form of the equation for a circle: $(x-h)^2 + (y-k)^2 = r^2$, where the point (h, k) is the center of the circle and *r* is the radius.

$$h = -\frac{11}{2},\ k = \frac{1}{2},\ r = \frac{\sqrt{26}}{2} \Rightarrow \left(x + \frac{11}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{13}{2}$$

NOTE: $\left(\frac{\sqrt{26}}{2}\right)^2 = \frac{26}{4} = \frac{13}{2}$

Answer: $\left(x + \frac{11}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{13}{2}$

4e. The center is $(-7, -3)$ and the point $(2, -8)$ is a point on the circle.
Back to Problem 4.

The standard form of the equation for a circle: $(x-h)^2+(y-k)^2=r^2$, where the point (h,k) is the center of the circle and r is the radius.

$$h=-7,\ k=-3,\ r=?\ \Rightarrow\ (x+7)^2+(y+3)^2=r^2$$

In order for the point $(2,-8)$ to be on the circle, it must satisfy the equation for the circle. Thus,

$$(2+7)^2+(-8+3)^2=r^2\ \Rightarrow\ r^2=9^2+(-5)^2=81+25=106$$

$$r^2=106\ \Rightarrow\ (x+7)^2+(y+3)^2=106$$

Answer: $(x+7)^2+(y+3)^2=106$

4f. The center is $(4,0)$ and the point $(-3,\ 6)$ is a point on the circle.
Back to Problem 4.

The standard form of the equation for a circle: $(x-h)^2+(y-k)^2=r^2$, where the point (h,k) is the center of the circle and r is the radius.

$$h=4,\ k=0,\ r=?\ \Rightarrow\ (x-4)^2+(y-0)^2=r^2\ \Rightarrow\ (x-4)^2+y^2=r^2$$

In order for the point $(-3,\ 6)$ to be on the circle, it must satisfy the equation for the circle. Thus,

$$(-3-4)^2+6^2=r^2\ \Rightarrow\ r^2=(-7)^2+36=49+36=85$$

$$r^2=85\ \Rightarrow\ (x-4)^2+y^2=85$$

Answer: $(x - 4)^2 + y^2 = 85$

4g. Write an equation that represents the set of points that are 5 units from the point $(0, -3)$. Back to Problem 4.

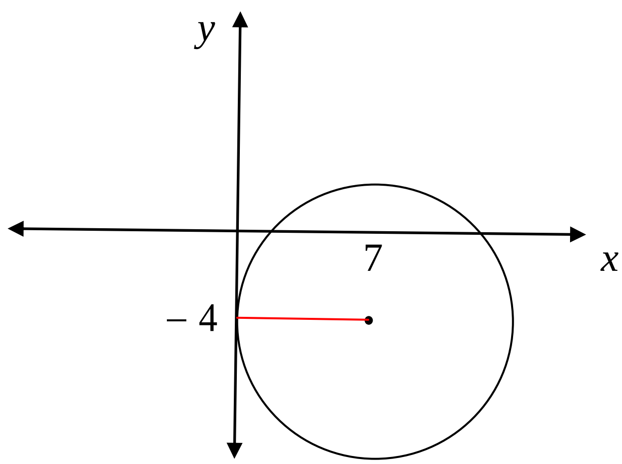
Center: $(0, -3)$; Radius: 5

The standard form of the equation for a circle: $(x - h)^2 + (y - k)^2 = r^2$, where the point (h, k) is the center of the circle and r is the radius.

$$h = 0,\ k = -3,\ r = 5 \Rightarrow (x - 0)^2 + (y + 3)^2 = 25 \Rightarrow x^2 + (y + 3)^2 = 25$$

Answer: $x^2 + (y + 3)^2 = 25$

4h. The center is $(7, -4)$ and the circle is tangent to the y-axis.
Back to Problem 4.



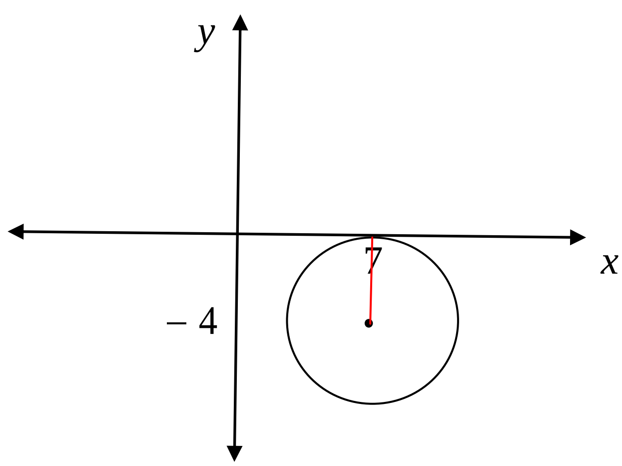
NOTE: The radius of the center is the length of the red line segment shown above. This length is 7.

The standard form of the equation for a circle: $(x-h)^2 + (y-k)^2 = r^2$, where the point (h, k) is the center of the circle and r is the radius.

$h = 7,\ k = -4,\ r = 7 \Rightarrow (x-7)^2 + (y+4)^2 = 49$

Answer: $(x-7)^2 + (y+4)^2 = 49$

4i. The center is $(7, -4)$ and the circle is tangent to the x-axis.
Back to Problem 4.



NOTE: The radius of the center is the length of the red line segment shown above. This length is 4.

The standard form of the equation for a circle: $(x-h)^2 + (y-k)^2 = r^2$, where the point (h, k) is the center of the circle and r is the radius.

$h = 7,\ k = -4,\ r = 4 \Rightarrow (x-7)^2 + (y+4)^2 = 16$

Answer: $(x - 7)^2 + (y + 4)^2 = 16$

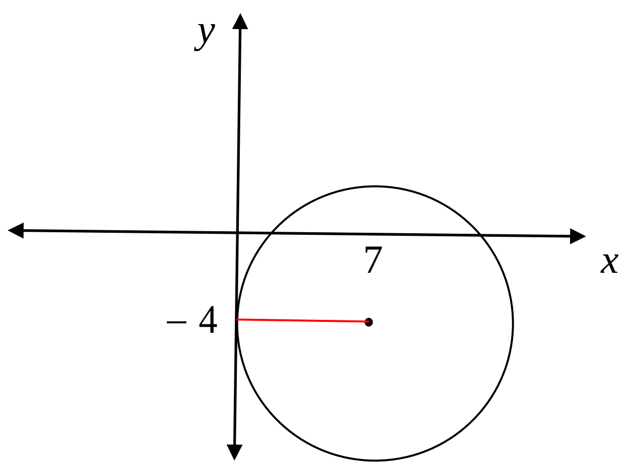
NOTE: The x-intercept for the circle $(x - 7)^2 + (y + 4)^2 = 49$ shown above should be the point $(7, 0)$. Let's check to see that this is true.

x-intercept(s): To find the x-intercept(s), set $y = 0$.

$(x - 7)^2 + (y + 4)^2 = 16$ and $y = 0 \Rightarrow (x - 7)^2 + 4^2 = 16 \Rightarrow$
$(x - 7)^2 + 16 = 16 \Rightarrow (x - 7)^2 = 0 \Rightarrow x - 7 = 0 \Rightarrow x = 7$

It checks. The x-intercept is $(7, 0)$.

5. In Problem 4h, we found the equation of the circle to be
$(x - 7)^2 + (y + 4)^2 = 49$



x-intercept(s): To find the x-intercept(s), set $y = 0$.

$(x - 7)^2 + (y + 4)^2 = 49$ and $y = 0 \Rightarrow (x - 7)^2 + 4^2 = 49 \Rightarrow$

$(x - 7)^2 + 16 = 49 \Rightarrow (x - 7)^2 = 33 \Rightarrow x - 7 = \pm\sqrt{33} \Rightarrow$

$x = 7 \pm \sqrt{33}$

***x*-intercept(s):** $(7 - \sqrt{33}, 0)$; $(7 + \sqrt{33}, 0)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

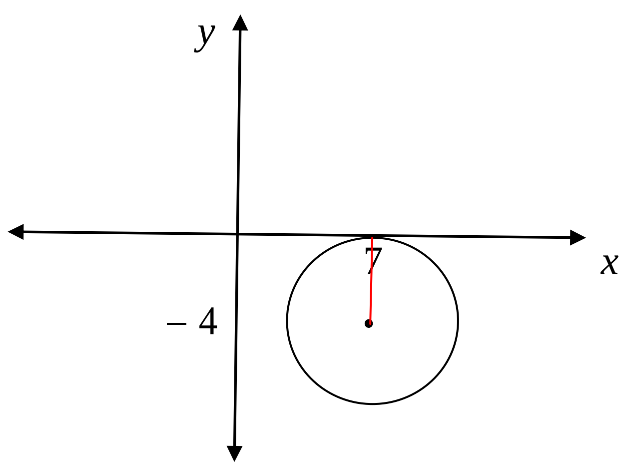
$(x - 7)^2 + (y + 4)^2 = 49$ and $x = 0 \Rightarrow (-7)^2 + (y + 4)^2 = 49 \Rightarrow$

$49 + (y + 4)^2 = 49 \Rightarrow (y + 4)^2 = 0 \Rightarrow y + 4 = 0 \Rightarrow y = -4$

***y*-intercept(s):** $(0, -4)$

Back to Problem 5.

6. In Problem 4i, we found the equation of the circle to be $(x - 7)^2 + (y + 4)^2 = 16$



x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$(x - 7)^2 + (y + 4)^2 = 16$ and $y = 0 \Rightarrow (x - 7)^2 + 4^2 = 16 \Rightarrow$

$(x - 7)^2 + 16 = 16 \Rightarrow (x - 7)^2 = 0 \Rightarrow x - 7 = 0 \Rightarrow x = 7$

***x*-intercept(s):** $(7, 0)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$(x - 7)^2 + (y + 4)^2 = 16$ and $x = 0 \Rightarrow (-7)^2 + (y + 4)^2 = 16 \Rightarrow$

$49 + (y + 4)^2 = 16 \Rightarrow (y + 4)^2 = -33 \Rightarrow y + 4 = \pm i\sqrt{33} \Rightarrow$

$y = -4 \pm i\sqrt{33}$

Since the solutions to this equation are complex numbers, then the graph of the equation does not have any *y*-intercepts. We can also see this in the graph of the circle above.

***y*-intercept(s):** None

Back to Problem 6.

7a. $x^2 + y^2 - 10x + 18y + 20 = 0$

We will need to complete the squares for both *x* and *y* in order to put the equation in standard form.

$x^2 + y^2 - 10x + 18y + 20 = 0$

$$x^2 - 10x + \underline{25} + y^2 + 18y + \underline{81} = -20 + 25 + 81$$

$$\begin{array}{ll} \downarrow \textit{Half} & \downarrow \textit{Half} \\ 5 & 9 \\ \downarrow \textit{Square} & \downarrow \textit{Square} \\ 25 & 81 \end{array}$$

$$(x - 5)^2 + (y + 9)^2 = 86$$

Center: $(5, -9)$ **Radius:** $\sqrt{86}$ Back to Problem 7.

8a. All the elements in D have been corresponded with an element if E. Now, we need to see if it has been done uniquely. x has been uniquely corresponded with a. y has been uniquely corresponded with b. z has **not** been uniquely corresponded. z has been corresponded with c and d. Thus, this is not a function.

Back to Problem 8.

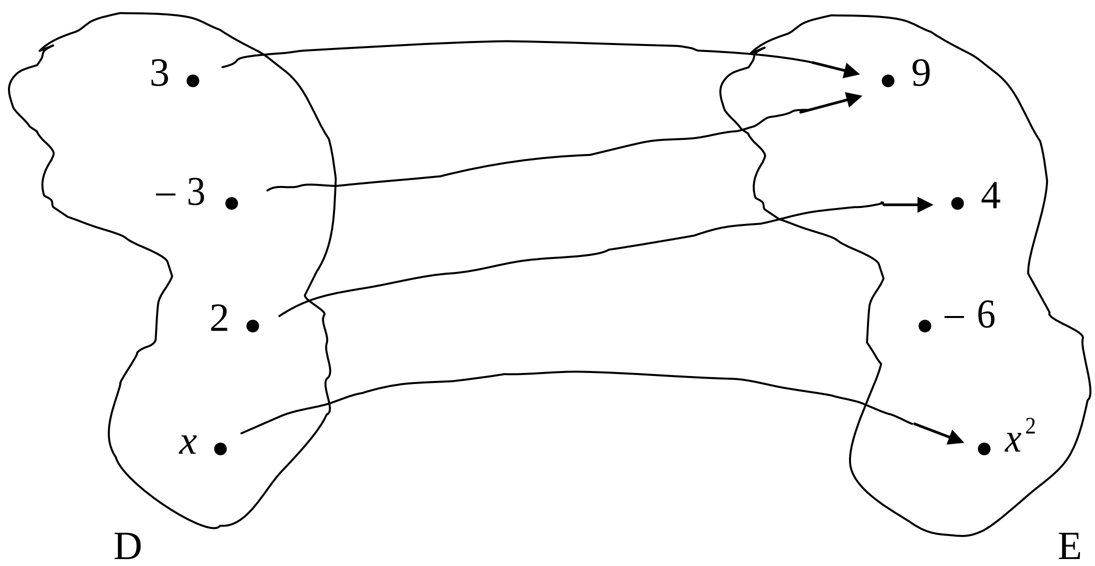
8b. Not all the elements in D have been corresponded with an element if E. 3 has **not** been corresponded with anything in E. Thus, this is not a function.

Back to Problem 8.

8c. All the elements in D have been corresponded with an element if E. Now, we need to see if it has been done uniquely. 3 has been uniquely corresponded with 9. -3 has been uniquely corresponded with 9. 2 has been uniquely corresponded with 4. Thus, this is a function.

NOTE: This function is the function that corresponds each real number with its square.

Since any real number can be squared, we could not show the picture for all real numbers. However, if we let *x* represent any real number, we could do the following:



Back to Problem 8.

9a. To find $f(1)$, replace **all** the *x*'s in $f(x) = 6x^2 - 17x - 14$ by 1.

$f(1) = 6(1)^2 - 17(1) - 14$

Simplifying the exponential expression and the multiplication as we go, we have that

$f(1) = 6 - 17 - 14 = -25$

Answer: -25 Back to Problem 9.

NOTE: The function f corresponds 1 with -25.

9b. To find $f(-2)$, replace **all** the x's in $f(x) = 6x^2 - 17x - 14$ by -2.

$f(-2) = 6(-2)^2 - 17(-2) - 14$

Simplifying the exponential expression and the multiplication as we go, we have that

$f(-2) = 24 + 34 - 14 = 44$

Answer: 44 Back to Problem 9.

NOTE: The function f corresponds -2 with 44.

9c. To find $f(0)$, replace **all** the x's in $f(x) = 6x^2 - 17x - 14$ by 0.

$f(0) = 6(0)^2 - 17(0) - 14$

Simplifying the exponential expression and the multiplication as we go, we have that

$f(0) = 0 + 0 - 14 = -14$

Answer: -14 Back to Problem 9.

NOTE: The function f corresponds 0 with -14.

9d. To find $f(x+h)$, first replace **all** the x's in $f(x) = 6x^2 - 17x - 14$ by $x + h$:

$f(x+h) = 6(x+h)^2 - 17(x+h) - 14$

Now, simplify the algebraic expression of the right side:

$f(x+h) = 6(x^2 + 2xh + h^2) - 17(x+h) - 14 =$

$6x^2 + 12xh + 6h^2 - 17x - 17h - 14$

Answer: $6x^2 + 12xh + 6h^2 - 17x - 17h - 14$ Back to Problem 9.

10a. $g(x) = \sqrt{9 - 4x}$ Back to Problem 10.

$g(1) = \sqrt{9 - 4(1)} = \sqrt{9 - 4} = \sqrt{5}$

Answer: $\sqrt{5}$

10b. $g(x) = \sqrt{9 - 4x}$ Back to Problem 10.

$g(0) = \sqrt{9 - 4(0)} = \sqrt{9 - 0} = \sqrt{9} = 3$

Answer: 3

10c. $g(x) = \sqrt{9 - 4x}$ Back to Problem 10.

$g(-9) = \sqrt{9 - 4(-9)} = \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$

Answer: $3\sqrt{5}$

10d. $g(x) = \sqrt{9 - 4x}$ Back to Problem 10.

$$g(8) = \sqrt{9 - 4(8)} = \sqrt{9 - 32} = \sqrt{-23} = i\sqrt{23}$$

Answer: $i\sqrt{23}$

COMMENT: We want to be able to graph the function *g* in the *xy*-plane. The points that we plot are of the form $(x, g(x))$.

NOTE: The *y*-coordinate of the point, which is plotted in the *xy*-plane, is the value of the function *g* at the value of *x*. Thus, $g(x)$ can **<u>NOT</u>** be a complex number. We will see that 8 is not in the domain of the function *g*.