Pre-Class Problems 4 for Wednesday, January 31

Earn one bonus point because you checked the Pre-Class problems. Send me an email with PC4 in the Subject line.

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem number or letter.

1. A motorboat travels 75 miles with a current of 5 mph. The return trip against the current takes 2 hours longer. Find the average rate of the motorboat in still water.

2. Solve the following equations.

   a. $4|3x - 5| - 7 = 17$

   b. $-11 = -2|y + 9| - 5$

   c. $|7t - 4| + 9 = 6$

   d. $|11 - 4x| = |5x + 8|$

3. Solve the following equations.

   a. $\sqrt{2x - 7} = 3$

   b. $\sqrt{6y + 4} = y - 6$

   c. $\sqrt{t + 33} - 3 = t$

   d. $\sqrt{3w - 5} + \sqrt{w + 6} = 5$

   e. $\sqrt{3x + 16} - \sqrt{x + 13} = -1$

   f. $\sqrt{3y - 1} + \sqrt{6y - 3} = 2$

   g. $\sqrt{4t - 5} + \sqrt{8t - 3} = 4$

   h. $\sqrt{2w + 5} - \sqrt{w + 2} = 1$

<u>Definition</u>

1. $(a, b) = \{x : a < x < b\}$
2. $[a, b] = \{x : a \le x \le b\}$
5. $(a, \infty) = \{x : x > a\}$
6. $[a, \infty) = \{x : x \ge a\}$

3. $[a, b) = \{x : a \le x < b\}$

4. $(a, b] = \{x : a < x \le b\}$

7. $(-\infty, a) = \{x : x < a\}$

8. $(-\infty, a] = \{x : x \le a\}$

NOTE: The x in the sets above is representing a real number; x could not be a complex number. The symbol : in the sets above means the phrase "such that." You can also use the symbol | for such that.

4. Solve the following inequalities. Graph the solution set. Write the solution set in interval notation.

a. $3x + 10 > -2$

b. $-\frac{3}{8}y + \frac{1}{2} \ge \frac{4}{3}$

c. $\frac{t-3}{3} - \frac{4t+7}{6} > -\frac{11}{4}$

d. $13 - 3[7 + 2(x - 4)] \ge 4[6 - 5(2x - 3)]$

5. Solve the following inequalities. Graph the solution set. Write the solution set in interval notation.

a. $-8 \le 7 - 3x < 34$

b. $4 < \frac{6y + 5}{3} < \frac{17}{3}$

c. $3 \le \frac{4w - 9}{5} < 11$

<u>Theorem</u>

1. $|x| < a$ if and only if $-a < x < a$
2. $|x| \le a$ if and only if $-a \le x \le a$
3. $|x| > a$ if and only if $x > a$ or $x < -a$
4. $|x| \ge a$ if and only if $x \ge a$ or $x \le -a$

6. Solve the following inequalities. Graph the solution set. Write the solution set in interval notation.

a. $|x| < 2$ b. $|y| \geq 8$ c. $|t - 3| \leq 4$ d. $|w + 2| > 5$

e. $|3x + 6| < 9$ f. $|6y - 5| \geq 7$ g. $5|4t - 7| - 9 \leq 21$

h. $-11 > -2|w + 9| - 5$ i. $|7x - 4| + 9 < 6$

j. $|7x - 4| + 9 \geq 6$

7. A car travels 70 mph and passes a van traveling 65 mph. How long will it take the car to be more than 10 miles ahead of the van?

8. A rectangular garden is to be constructed so that the width is 75 feet. What are the possible values for the length of the garden if at most 500 feet of fencing are to be used to enclose the garden?

Additional problems available in the textbook: Page 141 … 29 – 118 and Examples 4 – 11 starting on page 134. Page 153 … 9 – 26, 37 – 68, 77 – 80 and Examples 1 – 9 starting on page 145.

SOLUTIONS:

1. A motorboat travels 75 miles with a current of 5 mph. The return trip against the current takes 2 hours longer. Find the average rate of the motorboat in still water.

Let m = the rate of the motorboat in still water

R $T = \dfrac{D}{R}$ D

With the current	$m + 5$	$\dfrac{75}{m + 5}$	75
Against the current	$m - 5$	$\dfrac{75}{m - 5}$	75

NOTE: The difference in the time for the return trip traveling against the current, which is $\dfrac{75}{m - 5}$, and the time for the trip traveling with the current, which is $\dfrac{75}{m + 5}$, is 2 hours. The time to travel against the current is greater than the time to travel with the current. Thus, $\dfrac{75}{m - 5} - \dfrac{75}{m + 5} = 2$.

$$\frac{75}{m - 5} - \frac{75}{m + 5} = 2$$

$$\text{LCD} = (m - 5)(m + 5)$$

$$\frac{75}{m - 5} - \frac{75}{m + 5} = 2 \Rightarrow$$

$$(m - 5)(m + 5)\left(\frac{75}{m - 5} - \frac{75}{m + 5}\right) = 2(m - 5)(m + 5) \Rightarrow$$

$$75(m + 5) - 75(m - 5) = 2(m^2 - 25) \Rightarrow$$

$$75m + 375 - 75m + 375 = 2m^2 - 50 \Rightarrow 750 = 2m^2 - 50 \Rightarrow$$

$$2m^2 = 800 \Rightarrow m^2 = 400 \Rightarrow m = \pm 20$$

Speed can't be negative. Thus, $m = 20$.

Answer: 20 mph Back to Problem 1.

The following is another way that the problem can be work.

NOTE: The time for the return trip against the current, which is $\frac{75}{m-5}$, is 2 hours longer than the time for the trip with the current, which is $\frac{75}{m+5}$. Thus, $\frac{75}{m+5} + 2 = \frac{75}{m-5}$.

$$\frac{75}{m+5} + 2 = \frac{75}{m-5}$$

$$\text{LCD} = (m+5)(m-5)$$

$$\frac{75}{m+5} + 2 = \frac{75}{m-5} \Rightarrow$$

$$(m+5)(m-5)\left(\frac{75}{m+5} + 2\right) = \left(\frac{75}{m-5}\right)(m+5)(m-5) \Rightarrow$$

$$75(m-5) + 2(m+5)(m-5) = 75(m+5) \Rightarrow$$

$$75m - 375 + 2(m^2 - 25) = 75m + 375 \Rightarrow$$

$$-375 + 2m^2 - 50 = 375 \Rightarrow -425 + 2m^2 = 375 \Rightarrow 2m^2 = 800 \Rightarrow$$

$$m^2 = 400 \Rightarrow m = \pm 20$$

Answer: 20 mph Back to Problem 1.

2a. $4|3x - 5| - 7 = 17$ Back to Problem 2.

$$4|3x - 5| - 7 = 17 \Rightarrow 4|3x - 5| = 24 \Rightarrow |3x - 5| = 6 \Rightarrow$$

$$3x - 5 = \pm 6 \Rightarrow 3x = 5 \pm 6 \Rightarrow x = \frac{5 \pm 6}{3}$$

$$x = \frac{5 - 6}{3} = -\frac{1}{3}, \quad x = \frac{5 + 6}{3} = \frac{11}{3}$$

Answer: $x = -\frac{1}{3}, \frac{11}{3}$ **or** $\left\{-\frac{1}{3}, \frac{11}{3}\right\}$

2b. $-11 = -2|y + 9| - 5$ Back to Problem 2.

$$-11 = -2|y + 9| - 5 \Rightarrow -6 = -2|y + 9| \Rightarrow |y + 9| = 3 \Rightarrow$$

$$y + 9 = \pm 3 \Rightarrow y = -9 \pm 3$$

$$y = -9 - 3 = -12, \quad y = -9 + 3 = -6$$

Answer: $y = -12, -6$ **or** $\{-12, -6\}$

2c. $|7t - 4| + 9 = 6$ Back to Problem 2.

$|7t - 4| + 9 = 6 \Rightarrow |7t - 4| = -3$

The absolute value of a real number is nonnegative (zero or positive).

Answer: No solution **or** The empty set

2d. $|11 - 4x| = |5x + 8|$ Back to Problem 2.

$|11 - 4x| = |5x + 8| \Rightarrow 11 - 4x = \pm(5x + 8)$

$11 - 4x = 5x + 8$ or $11 - 4x = -(5x + 8)$

$3 = 9x$ $\quad$ $11 - 4x = -5x - 8$

$x = \frac{3}{9} = \frac{1}{3}$ $\quad$ $x = -19$

Answer: $x = -19, \frac{1}{3}$ **or** $\left\{-19, \frac{1}{3}\right\}$

3a. $\sqrt{2x - 7} = 3$ Back to Problem 3.

$\sqrt{2x - 7} = 3 \Rightarrow (\sqrt{2x - 7})^2 = 3^2 \Rightarrow 2x - 7 = 9 \Rightarrow$

$2x = 16 \Rightarrow x = 8$

Check: $\sqrt{16-7} \overset{?}{=} 3 \Rightarrow \sqrt{9} \overset{?}{=} 3 \Rightarrow 3 \overset{?}{=} 3$ True

Answer: $x = 8$ **or** $\{8\}$

3b. $\sqrt{6y+4} = y - 6$ Back to Problem 3.

$\sqrt{6y+4} = y - 6 \Rightarrow (\sqrt{6y+4})^2 = (y-6)^2 \Rightarrow$

$6y + 4 = y^2 - 12y + 36 \Rightarrow 0 = y^2 - 18y + 32 \Rightarrow$

$0 = (y-2)(y-16) \Rightarrow y = 2,\ y = 16$

Check for $y = 2$: $\sqrt{12+4} \overset{?}{=} 2 - 6 \Rightarrow \sqrt{16} \overset{?}{=} -4 \Rightarrow 4 \overset{?}{=} -4$ False

Check for $y = 16$: $\sqrt{96+4} \overset{?}{=} 16 - 6 \Rightarrow \sqrt{100} \overset{?}{=} 10 \Rightarrow 10 \overset{?}{=} 10$ True

Answer: $y = 16$ **or** $\{16\}$

3c. $\sqrt{t+33} - 3 = t$ Back to Problem 3.

$\sqrt{t+33} - 3 = t \Rightarrow \sqrt{t+33} = t + 3 \Rightarrow (\sqrt{t+33})^2 = (t+3)^2 \Rightarrow$

$t + 33 = t^2 + 6t + 9 \Rightarrow 0 = t^2 + 5t - 24 \Rightarrow 0 = (t+8)(t-3) \Rightarrow$

$t = -8,\ t = 3$

Check for $t = -8$: $\sqrt{-8 + 33} - 3 \overset{?}{=} -8 \Rightarrow \sqrt{25} - 3 \overset{?}{=} -8 \Rightarrow$

$5 - 3 \overset{?}{=} -8 \Rightarrow 2 \overset{?}{=} -8$ False

Check for $t = 3$: $\sqrt{3 + 33} - 3 \overset{?}{=} 3 \Rightarrow \sqrt{36} - 3 \overset{?}{=} 3 \Rightarrow 6 - 3 \overset{?}{=} 3 \Rightarrow$

$3 \overset{?}{=} 3$ True

Answer: $t = 3$ **or** $\{3\}$

3d. $\sqrt{3w - 5} + \sqrt{w + 6} = 5$ Back to Problem 3.

$\sqrt{3w - 5} + \sqrt{w + 6} = 5 \Rightarrow \sqrt{3w - 5} = 5 - \sqrt{w + 6} \Rightarrow$

$(\sqrt{3w - 5})^2 = (5 - \sqrt{w + 6})^2 \Rightarrow 3w - 5 = 25 - 10\sqrt{w + 6} + w + 6$

$\Rightarrow 3w - 5 = 31 - 10\sqrt{w + 6} + w \Rightarrow 2w - 36 = -10\sqrt{w + 6} \Rightarrow$

$2(w - 18) = -10\sqrt{w + 6} \Rightarrow w - 18 = -5\sqrt{w + 6} \Rightarrow$

$(w - 18)^2 = (-5\sqrt{w + 6})^2 \Rightarrow w^2 - 36w + 324 = 25(w + 6) \Rightarrow$

$w^2 - 36w + 324 = 25w + 150 \Rightarrow w^2 - 61w + 174 = 0 \Rightarrow$

$(w - 3)(w - 58) = 0 \Rightarrow w = 3,\ w = 58$

NOTE: The prime factorization of 174 is $2 \cdot 3 \cdot 29$.

Factors of 174: 1, 174 ; 2, 87 ; 3, 58 ; and 6, 29

Check for $w = 3$: $\sqrt{9-5} + \sqrt{3+6} \overset{?}{=} 5 \Rightarrow \sqrt{4} + \sqrt{9} \overset{?}{=} 5 \Rightarrow$

$2 + 3 \overset{?}{=} 5 \Rightarrow 5 \overset{?}{=} 5$ True

Check for $w = 58$: $\sqrt{174-5} + \sqrt{58+6} \overset{?}{=} 5 \Rightarrow \sqrt{169} + \sqrt{64} \overset{?}{=} 5$

$\Rightarrow 13 + 8 \overset{?}{=} 5 \Rightarrow 21 \overset{?}{=} 5$ False

Answer: $w = 3$ **or** $\{3\}$

3e. $\sqrt{3x+16} - \sqrt{x+13} = -1$ Back to Problem 3.

$\sqrt{3x+16} - \sqrt{x+13} = -1 \Rightarrow \sqrt{3x+16} = \sqrt{x+13} - 1 \Rightarrow$

$(\sqrt{3x+16})^2 = (\sqrt{x+13} - 1)^2 \Rightarrow 3x + 16 = x + 13 - 2\sqrt{x+13} + 1$

$\Rightarrow 3x + 16 = x + 14 - 2\sqrt{x+13} \Rightarrow 2x + 2 = -2\sqrt{x+13} \Rightarrow$

$2(x+1) = -2\sqrt{x+13} \Rightarrow x + 1 = -\sqrt{x+13} \Rightarrow$

$(x+1)^2 = (-\sqrt{x+13})^2 \Rightarrow x^2 + 2x + 1 = x + 13 \Rightarrow$

$x^2 + x - 12 = 0 \Rightarrow (x+4)(x-3) = 0 \Rightarrow x = -4,\ x = 3$

Check for $x = -4$: $\sqrt{-12+16} - \sqrt{-4+13} \overset{?}{=} -1 \Rightarrow$

$\sqrt{4} - \sqrt{9} \overset{?}{=} -1 \Rightarrow 2 - 3 \overset{?}{=} -1 \Rightarrow -1 \overset{?}{=} -1$ True

Check for $x = 3$: $\sqrt{9+16} - \sqrt{3+13} \overset{?}{=} -1 \Rightarrow$

$$\sqrt{25} - \sqrt{16} \overset{?}{=} -1 \Rightarrow 5 - 4 \overset{?}{=} -1 \Rightarrow 1 \overset{?}{=} -1 \quad \text{False}$$

Answer: $x = -4$ **or** $\{-4\}$

3f. $\sqrt{3y-1} + \sqrt{6y-3} = 2$ Back to Problem 3.

$$\sqrt{3y-1} + \sqrt{6y-3} = 2 \Rightarrow \sqrt{6y-3} = 2 - \sqrt{3y-1} \Rightarrow$$

$$(\sqrt{6y-3})^2 = (2 - \sqrt{3y-1})^2 \Rightarrow 6y - 3 = 4 - 4\sqrt{3y-1} + 3y - 1$$

$$\Rightarrow 6y - 3 = 3 - 4\sqrt{3y-1} + 3y \Rightarrow 3y - 6 = -4\sqrt{3y-1} \Rightarrow$$

$$(3y-6)^2 = (-4\sqrt{3y-1})^2 \Rightarrow 9y^2 - 36y + 36 = 16(3y-1) \Rightarrow$$

$$9y^2 - 36y + 36 = 48y - 16 \Rightarrow 9y^2 - 84y + 52 = 0 \Rightarrow$$

$$(3y-2)(3y-26) = 0 \Rightarrow y = \frac{2}{3},\ y = \frac{26}{3}$$

Check for $y = \frac{2}{3}$: $\sqrt{2-1} + \sqrt{4-3} \overset{?}{=} 2 \Rightarrow \sqrt{1} + \sqrt{1} \overset{?}{=} 2 \Rightarrow$

$$1 + 1 \overset{?}{=} 2 \Rightarrow 2 \overset{?}{=} 2 \quad \text{True}$$

Check for $y = \frac{26}{3}$: $\sqrt{26-1} + \sqrt{52-3} \overset{?}{=} 2 \Rightarrow \sqrt{25} + \sqrt{49} \overset{?}{=} 2 \Rightarrow$

$$5 + 7 \overset{?}{=} 2 \Rightarrow 12 \overset{?}{=} 2 \quad \text{False}$$

Answer: $y = \frac{2}{3}$ **or** $\left\{\frac{2}{3}\right\}$

3g. $\sqrt{4t-5} + \sqrt{8t-3} = 4$ Back to Problem 3.

$$\sqrt{4t-5} + \sqrt{8t-3} = 4 \Rightarrow \sqrt{8t-3} = 4 - \sqrt{4t-5} \Rightarrow$$

$$(\sqrt{8t-3})^2 = (4 - \sqrt{4t-5})^2 \Rightarrow 8t - 3 = 16 - 8\sqrt{4t-5} + 4t - 5$$

$$\Rightarrow 8t - 3 = 11 - 8\sqrt{4t-5} + 4t \Rightarrow 4t - 14 = -8\sqrt{4t-5} \Rightarrow$$

$$2(2t-7) = -8\sqrt{4t-5} \Rightarrow 2t - 7 = -4\sqrt{4t-5} \Rightarrow$$

$$(2t-7)^2 = (-4\sqrt{4t-5})^2 \Rightarrow 4t^2 - 28t + 49 = 16(4t-5) \Rightarrow$$

$$4t^2 - 28t + 49 = 64t - 80 \Rightarrow 4t^2 - 92t + 129 = 0 \Rightarrow$$

$$(2t-3)(2t-43) = 0 \Rightarrow t = \frac{3}{2},\ t = \frac{43}{2}$$

Check for $t = \frac{3}{2}$: $\sqrt{6-5} + \sqrt{12-3} \stackrel{?}{=} 4 \Rightarrow \sqrt{1} + \sqrt{9} \stackrel{?}{=} 4 \Rightarrow$

$1 + 3 \stackrel{?}{=} 4 \Rightarrow 4 \stackrel{?}{=} 4$ True

Check for $t = \frac{43}{2}$: $\sqrt{86-5} + \sqrt{172-3} \stackrel{?}{=} 4 \Rightarrow \sqrt{81} + \sqrt{169} \stackrel{?}{=} 4$

$\Rightarrow 9 + 13 \stackrel{?}{=} 4 \Rightarrow 22 \stackrel{?}{=} 4$ False

Answer: $t = \frac{3}{2}$ **or** $\left\{\frac{3}{2}\right\}$

3h. $\sqrt{2w+5} - \sqrt{w+2} = 1$ Back to Problem 3.

$\sqrt{2w+5} - \sqrt{w+2} = 1 \Rightarrow \sqrt{2w+5} = 1 + \sqrt{w+2} \Rightarrow$

$(\sqrt{2w+5})^2 = (1 + \sqrt{w+2})^2 \Rightarrow 2w + 5 = 1 + 2\sqrt{w+2} + w + 2 \Rightarrow$

$w + 2 = 2\sqrt{w+2} \Rightarrow (w+2)^2 = (2\sqrt{w+2})^2 \Rightarrow$

$w^2 + 4w + 4 = 4(w+2) \Rightarrow w^2 + 4w + 4 = 4w + 8 \Rightarrow w^2 + 4 = 8 \Rightarrow$

$w^2 = 4 \Rightarrow w = \pm 2$

Check for $w = -2$: $\sqrt{-4+5} - \sqrt{-2+2} \stackrel{?}{=} 1 \Rightarrow \sqrt{1} - \sqrt{0} \stackrel{?}{=} 1 \Rightarrow$
$1 - 0 \stackrel{?}{=} 1 \Rightarrow 1 \stackrel{?}{=} 1$ True

Check for $w = 2$: $\sqrt{4+5} - \sqrt{2+2} \stackrel{?}{=} 1 \Rightarrow \sqrt{9} - \sqrt{4} \stackrel{?}{=} 1 \Rightarrow$
$3 - 2 \stackrel{?}{=} 1 \Rightarrow 1 \stackrel{?}{=} 1$ True

Answer: $w = -2,\ w = 2$ **or** $\{-2, 2\}$

4a. $3x + 10 > -2$ Back to Problem 4.

$3x + 10 > -2 \Rightarrow 3x > -12 \Rightarrow x > -4$

Answer: 

Answer: $(-4, \infty)$

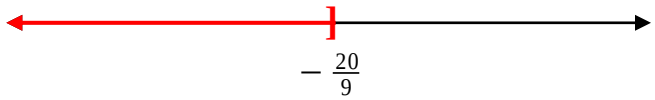
4b. $-\frac{3}{8}y + \frac{1}{2} \geq \frac{4}{3}$ Back to Problem 4.

LCD = 24

$$24\left(-\frac{3}{8}y + \frac{1}{2}\right) \geq \left(\frac{4}{3}\right)24 \Rightarrow -9y + 12 \geq 32 \Rightarrow -9y \geq 20 \Rightarrow$$

$$y \leq -\frac{20}{9}$$

RECALL: When you multiply or divide both sides of an inequality by a negative number, the direction of the inequality sign changes.

Answer: 

Answer: $\left(-\infty, -\frac{20}{9}\right]$

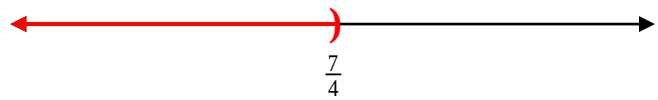
4c. $\frac{t-3}{3} - \frac{4t+7}{6} > -\frac{11}{4}$ Back to Problem 4.

LCD = 12

$$12\left(\frac{t-3}{3}-\frac{4t+7}{6}\right) > \left(-\frac{11}{4}\right)12 \Rightarrow 4(t-3)-2(4t+7) > -33 \Rightarrow$$

$$4t-12-8t-14 > -33 \Rightarrow -4t-26 > -33 \Rightarrow -4t > -7 \Rightarrow t < \frac{7}{4}$$

Answer:



Answer: $\left(-\infty, \frac{7}{4}\right)$

4d. $13-3[7+2(x-4)] \geq 4[6-5(2x-3)]$ Back to Problem 4.

$$13-3[7+2(x-4)] \geq 4[6-5(2x-3)] \Rightarrow$$

$$13-3(7+2x-8) \geq 4(6-10x+15) \Rightarrow$$

$$13-3(2x-1) \geq 4(21-10x) \Rightarrow 13-6x+3 \geq 84-40x \Rightarrow$$

$$16-6x \geq 84-40x \Rightarrow 34x \geq 68 \Rightarrow x \geq 2$$

Answer:

Answer: $[2, \infty)$

5a. $-8 \leq 7-3x < 34$ Back to Problem 5.

$$-8 \leq 7-3x < 34 \Rightarrow -15 \leq -3x < 27 \Rightarrow 5 \geq x > -9 \Rightarrow -9 < x \leq 5$$

Answer: 

Answer: $(-9,\ 5]$

5b. $4 < \dfrac{6y + 5}{3} < \dfrac{17}{3}$ Back to Problem 5.

$$4 < \frac{6y + 5}{3} < \frac{17}{3} \Rightarrow 12 < 6y + 5 < 17 \Rightarrow 7 < 6y < 12 \Rightarrow$$

$$\frac{7}{6} < y < 2$$

Answer: 

Answer: $\left(\dfrac{7}{6},\ 2\right)$

5c. $3 \le \dfrac{4w - 9}{5} < 11$ Back to Problem 5.

$$3 \le \frac{4w - 9}{5} < 11 \Rightarrow 15 \le 4w - 9 < 55 \Rightarrow 24 \le 4w < 64 \Rightarrow$$

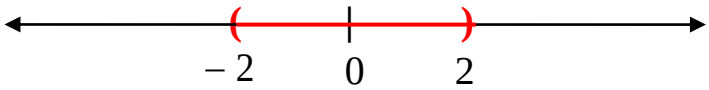
$$6 \le w < 16$$

Answer: 

Answer: $[6, 16)$

6a. $|x| < 2$ Back to Problem 6.

$|x| < 2 \Rightarrow -2 < x < 2$ (using the Theorem)

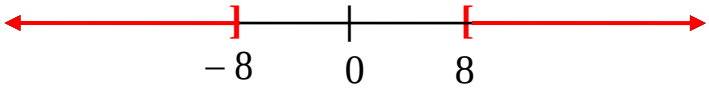
Answer: 

Answer: $(-2, 2)$

NOTE: The interval is a one-dimensional circle where the center of the circle is 0 and the radius is 2.

6b. $|y| \geq 8$ Back to Problem 6.

$|y| \geq 8 \Rightarrow y \geq 8$ or $y \leq -8$ (using the Theorem)

Answer: 

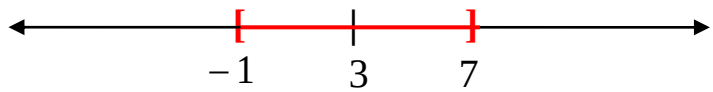
Answer: $(-\infty, -8] \cup [8, \infty)$

NOTE: The complement of the intervals is a one-dimensional circle where the center of the circle is 0 and the radius is 8.

6c. $|t - 3| \le 4$ Back to Problem 6.

$$|t - 3| \le 4 \Rightarrow -4 \le t - 3 \le 4 \Rightarrow -1 \le t \le 7$$

Answer:



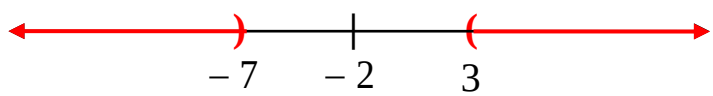
Answer: $[-1, 7]$

NOTE: The interval is like a one-dimensional circle where the center of the circle is 3 and the radius is 4.

6d. $|w + 2| > 5$ Back to Problem 6.

$$|w + 2| > 5 \Rightarrow \begin{array}{lcl} w + 2 > 5 & \text{or} & w + 2 < -5 \\ w > 3 & & w < -7 \end{array}$$

Answer:

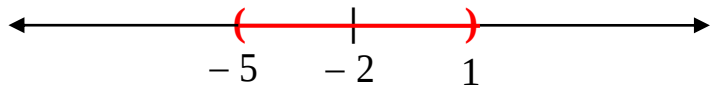


Answer: $(-\infty, -7) \cup (3, \infty)$

NOTE: The complement of the intervals is a one-dimensional circle where the center of the circle is -2 and the radius is 5.

6e. $|3x + 6| < 9$ Back to Problem 6.

$$|3x + 6| < 9 \Rightarrow -9 < 3x + 6 < 9 \Rightarrow -15 < 3x < 3 \Rightarrow -5 < x < 1$$

Answer: 

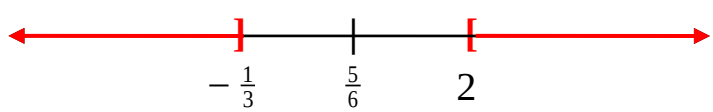
Answer: $(-5,\ 1)$

NOTE: The interval is like a one-dimensional circle where the center of the circle is -2 and the radius is 3.

NOTE: $|3x + 6| < 9 \Rightarrow |3(x + 2)| < 9 \Rightarrow 3|x + 2| < 9 \Rightarrow$
$|x + 2| < 3 \Rightarrow -3 < x + 2 < 3 \Rightarrow -5 < x < 1$

6f. $|6y - 5| \geq 7$ Back to Problem 6.

$$|6y - 5| \geq 7 \Rightarrow 6y - 5 \geq 7 \quad \text{or} \quad 6y - 5 \leq -7$$
$$6y \geq 12 \qquad\qquad 6y \leq -2$$
$$y \geq 2 \qquad\qquad y \leq -\frac{1}{3}$$

Answer: 

Answer: $\left(-\infty,\ -\frac{1}{3}\right] \cup [2,\ \infty)$

NOTE: The complement of the intervals is a one-dimensional circle where the center of the circle is $\frac{5}{6}$ and the radius is $\frac{7}{6}$.

NOTE: $|6y - 5| \geq 7 \Rightarrow \left|6\left(y - \frac{5}{6}\right)\right| \geq 7 \Rightarrow 6\left|y - \frac{5}{6}\right| \geq 7 \Rightarrow$

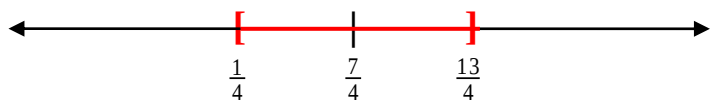
$\left|y - \frac{5}{6}\right| \geq \frac{7}{6}$

6g. $5|4t - 7| - 9 \leq 21$ Back to Problem 6.

$5|4t - 7| - 9 \leq 21 \Rightarrow 5|4t - 7| \leq 30 \Rightarrow |4t - 7| \leq 6 \Rightarrow$

$-6 \leq 4t - 7 \leq 6 \Rightarrow 1 \leq 4t \leq 13 \Rightarrow \frac{1}{4} \leq t \leq \frac{13}{4}$

Answer:



Answer: $\left[\frac{1}{4}, \frac{13}{4}\right]$

NOTE: The interval is like a one-dimensional circle where the center of the circle is $\frac{7}{4}$ and the radius is $\frac{6}{4} = \frac{3}{2}$.

NOTE: $|4t - 7| \leq 6 \Rightarrow \left|4\left(t - \frac{7}{4}\right)\right| \leq 6 \Rightarrow 4\left|t - \frac{7}{4}\right| \leq 6 \Rightarrow$

$\left|t - \frac{7}{4}\right| \leq \frac{3}{2}$

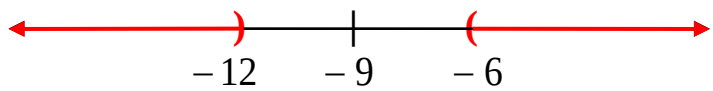
6h. $-11 > -2|w + 9| - 5$ Back to Problem 6.

$$-11 > -2|w+9| - 5 \Rightarrow -6 > -2|w+9| \Rightarrow 3 < |w+9| \Rightarrow$$

$$|w+9| > 3$$

$$|w+9| > 3 \Rightarrow \begin{array}{ccc} w+9>3 & \text{or} & w+9<-3 \\ w>-6 & & w<-12 \end{array}$$

Answer:



Answer: $(-\infty, -12) \cup (-6, \infty)$

NOTE: The complement of the intervals is a one-dimensional circle where the center of the circle is -9 and the radius is 3.

6i. $|7x-4| + 9 < 6$ Back to Problem 6.

$$|7x-4| + 9 < 6 \Rightarrow |7x-4| < -3$$

For any real number x, $|7x-4|$ is nonnegative (either zero or positive.) Thus, the inequality $|7x-4| < -3$ has no solution.

Answer: The empty set

6j. $|7x-4| + 9 \geq 6$ Back to Problem 6.

$$|7x - 4| + 9 \geq 6 \Rightarrow |7x - 4| \geq -3$$

For any real number x, $|7x - 4|$ is nonnegative (either zero or positive.)
Thus, every real number will satisfy the inequality $|7x - 4| \geq -3$.

Answer: 

Answer: $(-\infty, \infty)$

7. A car travels 70 mph and passes a van traveling 65 mph. How long will it take the car to be more than 7 miles ahead of the van? Back to Problem 7.

	R	T	$D = RT$
Car	70	t	$70t$
Van	65	t	$65t$

$$70t - 65t > 7 \Rightarrow 5t > 7 \Rightarrow t > \frac{7}{5} \Rightarrow t > 1.4$$

Answer: It will take more than 1.4 hours, which is 1 hour and 24 minutes.

8. A rectangular garden is to be constructed so that the width is 75 feet. What are the possible values for the length of the garden if at most 500 feet of fencing are to be used to enclose the garden? Back to Problem 8.

Let l = the length of the garden

NOTE: The perimeter of the garden is $2l + 2(50) = 2l + 100$. Since the fencing is to enclose the garden and at most 500 feet of fencing is be used, then $2l + 100 \leq 500$.

$$2l + 100 \leq 500 \Rightarrow 2l \leq 400 \Rightarrow l \leq 200$$

Answer: The length of the garden must be 200 feet or less.