Pre-Class Problems 3 for Monday, January 29

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem letter or problem number.

Theorem (Quadratic Formula) If $ax^2 + bx + c = 0$, where $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

1. Solve the following equations using the quadratic formula.

a. $x^2 - 7x - 5 = 0$ b. $3t^2 = 7 - 4t$ c. $4w(w + 3) = 5$

d. $9 - 3y = -4y^2$ e. $(3x + 2)(x - 4) = 8x(x - 2) + 4$

f. $36z^2 + 49 = 0$ g. $\frac{1}{2}y^2 + \frac{2}{3}y - \frac{3}{4} = 0$

h. $t^2 - 8t + 16 = 0$

Definition The expression $b^2 - 4ac$ in the Quadratic Formula is called the discriminant.

Theorem Given the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$. Then

1. If $b^2 - 4ac < 0$, the quadratic equation has two complex solutions. These two solutions are conjugates on one another.

2. If $b^2 - 4ac = 0$, the quadratic equation has one real solution.

3. If $b^2 - 4ac > 0$, the quadratic equation has two real solutions.

2. Determine the type of solutions for the following quadratic equations by calculating the discriminant.

a. $3x^2 - 2x + 8 = 0$

b. $15 - 7t - 2t^2 = 0$

c. $4y^2 + 12y + 9 = 0$

3. Solve $s = \frac{1}{2}gt^2 + v_0 t + s_0$ for t.

4. Solve the following problems.

a. The product of two numbers is 20. If one of the numbers is three less than twice the other number, then find the numbers.

b. The product of two numbers is p. If one of the numbers is three less than twice the other number, then find the numbers.

c. The length of a rectangular garden is three times the width. If the area of the garden is 450 square feet, then find the dimensions of the garden.

d. Find two consecutive integers whose sum of their squares is 85.

e. The sum of the first n positive integers is given by $\frac{n(n+1)}{2}$. That is, $1 + 2 + 3 + \cdots\cdots + n = \frac{n(n+1)}{2}$. If the sum of the first n positive integers is 78, then find n.

f. The base of a triangle is four meters more than three times the height of the triangle. If the area of the triangle is 80 square meters, then find the base and height of the triangle.

Theorem If $a^r = 0$ and r is a positive rational number, then $a = 0$.

5. Solve the following equations.

a. $3x(7 - 4x)(x + 5)^2 = 0$

b. $8(t^2 + 6)(t^2 - 16) = 0$

c. $45w^3 - 27w^2 - 20w + 12 = 0$

d. $x^3 + 36x = 2(x^2 + 36)$

e. $2y^4 = 128$

f. $y^4 = 64y$

g. $5t^6 = -135t^3$

6. Solve the following equations.

a. $\dfrac{6x}{x - 3} - \dfrac{4}{x + 6} = \dfrac{5x^2 + 39x}{x^2 + 3x - 18}$

b. $\dfrac{32}{y^2 + 4y} + 7 = \dfrac{2y}{y + 4}$

c. $8 - \dfrac{5}{w} = \dfrac{7}{w^2}$

Additional problems available in the textbook: Page 123 … 55 – 70, 79 – 118 and Examples 6 – 10 starting on page 119. Page 130 … 5 – 12, 17 – 31 and Examples 1 and 2 starting on page 126. Page 141 … 5 – 28 and Examples 1 – 3 starting on page 133.

SOLUTIONS:

1a. $x^2 - 7x - 5 = 0$

$a = 1,\ b = -7,\ c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{49 - 4(1)(-5)}}{2} = \frac{7 \pm \sqrt{49 + 20}}{2} =$$

$$\frac{7 \pm \sqrt{69}}{2}$$

Answer: $x = \dfrac{7 \pm \sqrt{69}}{2}$ **or** $\left\{ \dfrac{7 \pm \sqrt{69}}{2} \right\}$ Back to Problem 1.

1b. $3t^2 = 7 - 4t \Rightarrow 3t^2 + 4t - 7 = 0$

$a = 3,\ b = 4,\ c = -7$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(3)(-7)}}{6} = \frac{-4 \pm \sqrt{16 + 84}}{6} =$$

$$\frac{-4 \pm \sqrt{100}}{6} = \frac{-4 \pm 10}{6}$$

$t = \dfrac{-4 - 10}{6} = \dfrac{-14}{6} = -\dfrac{7}{3}$ $t = \dfrac{-4 + 10}{6} = \dfrac{6}{6} = 1$

Answer: $t = -\dfrac{7}{3},\ t = 1$ **or** $\left\{ -\dfrac{7}{3},\ 1 \right\}$ Back to Problem 1.

1c. $4w(w + 3) = 5 \Rightarrow 4w^2 + 12w - 5 = 0$

$a = 4,\ b = 12,\ c = -5$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{144 - 4(4)(-5)}}{8} =$$

$$\frac{-12 \pm \sqrt{144 + 80}}{8} = \frac{-12 \pm \sqrt{224}}{8} = \frac{-12 \pm \sqrt{16 \cdot 14}}{8} = \frac{-12 \pm 4\sqrt{14}}{8}$$

$$= \frac{4(-3 \pm \sqrt{14})}{8} = \frac{-3 \pm \sqrt{14}}{2}$$

NOTE: Since $144 = 12 \cdot 12 = (4 \cdot 3)(4 \cdot 3) = 16 \cdot 9$, then we could have simplified our answer in the following manner:

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{144 - 4(4)(-5)}}{8} =$$

$$\frac{-12 \pm \sqrt{16(9 + 5)}}{8} = \frac{-12 \pm \sqrt{16(14)}}{8} = \frac{-12 \pm 4\sqrt{14}}{8} =$$

$$= \frac{4(-3 \pm \sqrt{14})}{8} = \frac{-3 \pm \sqrt{14}}{2}$$

Answer: $w = \dfrac{-3 \pm \sqrt{14}}{2}$ **or** $\left\{ \dfrac{-3 \pm \sqrt{14}}{2} \right\}$ Back to Problem 1.

1d. $9 - 3y = -4y^2 \Rightarrow 4y^2 - 3y + 9 = 0$

$a = 4,\ b = -3,\ c = 9$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(4)(9)}}{8} = \frac{3 \pm \sqrt{9[1 - 4(4)]}}{8} =$$

$$\frac{3 \pm \sqrt{9(1-16)}}{8} = \frac{3 \pm \sqrt{9(-15)}}{8} = \frac{3 \pm 3\sqrt{-15}}{8} = \frac{3 \pm 3i\sqrt{15}}{8}$$

NOTE: We could have combined some steps together to save some writing:

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(4)(9)}}{8} = \frac{3 \pm \sqrt{9(1-16)}}{8} =$$

$$\frac{3 \pm 3\sqrt{-15}}{8} = \frac{3 \pm 3i\sqrt{15}}{8}$$

Answer: $y = \dfrac{3 \pm 3i\sqrt{15}}{8}$ **or** $\left\{ \dfrac{3 \pm 3i\sqrt{15}}{8} \right\}$ Back to Problem 1.

1e. $(3x+2)(x-4) = 8x(x-2) + 4$

$3x^2 - 12x + 2x - 8 = 8x^2 - 16x + 4$

$3x^2 - 10x - 8 = 8x^2 - 16x + 4$

$0 = 5x^2 - 6x + 12$

$a = 5,\ b = -6,\ c = 12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 4(5)(12)}}{10} = \frac{6 \pm \sqrt{4[9 - (5)(12)]}}{10} =$$

$$\frac{6 \pm \sqrt{4(9-60)}}{10} = \frac{6 \pm \sqrt{4(-51)}}{10} = \frac{6 \pm 2\sqrt{-51}}{10} = \frac{6 \pm 2i\sqrt{51}}{10} =$$

$$\frac{2(3 \pm i\sqrt{51})}{10} = \frac{3 \pm i\sqrt{51}}{5}$$

NOTE: We could have combined some steps together to save some writing:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 4(5)(12)}}{10} = \frac{6 \pm \sqrt{4(9 - 60)}}{10} =$$

$$\frac{6 \pm 2\sqrt{-51}}{10} = \frac{6 \pm 2i\sqrt{51}}{10} = \frac{3 \pm i\sqrt{51}}{5}$$

Answer: $x = \dfrac{3 \pm i\sqrt{51}}{5}$ **or** $\left\{ \dfrac{3 \pm i\sqrt{51}}{5} \right\}$ Back to Problem 1.

1f. $36z^2 + 49 = 0$

$a = 36,\ b = 0,\ c = 49$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{0 - 4(36)(49)}}{72} = \frac{\pm \sqrt{-4(36)(49)}}{72} =$$

$$\pm \frac{i\sqrt{4(36)(49)}}{72} = \pm \frac{i\sqrt{4}\sqrt{36}\sqrt{49}}{72} = \pm \frac{i(2)(6)(7)}{72} = \pm \frac{i(1)(6)(7)}{36} =$$

$$\pm \frac{i(1)(1)(7)}{6} = \pm \frac{7i}{6}$$

Answer: $z = \pm \dfrac{7i}{6}$ **or** $\left\{ \pm \dfrac{7i}{6} \right\}$ Back to Problem 1.

1g. $\frac{1}{2}y^2 + \frac{2}{3}y - \frac{3}{4} = 0$

LCD(2, 3, 4) = 12

$$\frac{1}{2}y^2 + \frac{2}{3}y - \frac{3}{4} = 0 \Rightarrow 12\left(\frac{1}{2}y^2 + \frac{2}{3}y - \frac{3}{4}\right) = 0(12) \Rightarrow$$

$$6y^2 + 8y - 9 = 0$$

$$a = 6,\ b = 8,\ c = 9$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{64 - 4(6)(9)}}{12} = \frac{-8 \pm \sqrt{4(16 - 54)}}{12} =$$

$$\frac{-8 \pm \sqrt{4(-38)}}{12} = \frac{-8 \pm 2i\sqrt{38}}{12} = \frac{2(-4 \pm i\sqrt{38})}{12} = \frac{-4 \pm i\sqrt{38}}{6}$$

Answer: $y = \frac{-4 \pm i\sqrt{38}}{6}$ **or** $\left\{\frac{-4 \pm i\sqrt{38}}{6}\right\}$ Back to Problem 1.

1h. $t^2 - 8t + 16 = 0$

$$a = 1,\ b = -8,\ c = 16$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{64 - 4(1)(16)}}{2} = \frac{8 \pm \sqrt{64 - 64}}{2} =$$

$$\frac{8 \pm \sqrt{0}}{2} = \frac{8 \pm 0}{2} = \frac{8}{2} = 4$$

Answer: $t = 4$ **or** $\{4\}$ Back to Problem 1.

2a. $3x^2 - 2x + 8 = 0$ Back to Problem 2.

$a = 3,\ b = -2,\ c = 8$

$b^2 - 4ac = 4 - 4(3)(8) = 4 - 96 < 0$

Answer: Two complex solutions

2b. $15 - 7t - 2t^2 = 0$ Back to Problem 2.

$a = -2,\ b = -7,\ c = 15$

$b^2 - 4ac = 49 - 4(-2)(15) = 49 + 120 > 0$

Answer: Two real solutions

2c. $4y^2 + 12y + 9 = 0$ Back to Problem 2.

$a = 4,\ b = 12,\ c = 9$

$b^2 - 4ac = 144 - 4(4)(9) = 144 - 144 = 0$

Answer: One real solution

3. $s = \frac{1}{2}gt^2 + v_0 t + s_0 \Rightarrow 2s = gt^2 + 2v_0 t + 2s_0 \Rightarrow$

$$0 = gt^2 + 2v_0 t + 2s_0 - 2s \Rightarrow 0 = gt^2 + 2v_0 t + 2(s_0 - s)$$

$$a = g,\ b = 2v_0,\ c = 2(s_0 - s)$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2v_0 \pm \sqrt{4v_0^2 - 4g[2(s_0 - s)]}}{2g} =$$

$$t = \frac{-2v_0 \pm \sqrt{4v_0^2 - 8g(s_0 - s)}}{2g} = \frac{-2v_0 \pm \sqrt{4[v_0^2 - 2g(s_0 - s)]}}{2g} =$$

$$\frac{-2v_0 \pm 2\sqrt{v_0^2 - 2g(s_0 - s)}}{2g} = \frac{2[-v_0 \pm \sqrt{v_0^2 - 2g(s_0 - s)}\,]}{2g} =$$

$$\frac{-v_0 \pm \sqrt{v_0^2 - 2g(s_0 - s)}}{g} = \frac{-v_0 \pm \sqrt{v_0^2 - 2gs_0 + 2gs}}{g} =$$

$$\frac{-v_0 \pm \sqrt{v_0^2 + 2gs - 2gs_0}}{g}$$

Answer: $t = \dfrac{-v_0 \pm \sqrt{v_0^2 + 2gs - 2gs_0}}{g}$

Back to Problem 3.

4a. Find two numbers whose product is 20 and where one number is three less than twice the other number.

Let x = one of the numbers
Then $2x - 3$ = the other number

The product of these two numbers is $x(2x - 3)$. Since the product is to be 20, then $x(2x - 3) = 20$.

$$x(2x - 3) = 20 \Rightarrow 2x^2 - 3x - 20 = 0 \Rightarrow (x - 4)(2x + 5) = 0$$

$$x - 4 = 0 \Rightarrow x = 4$$

$$2x + 5 = 0 \Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$$

When $x = 4$, then $2x - 3 = 8 - 3 = 5$.

When $x = -\frac{5}{2}$, then $2x - 3 = -5 - 3 = -8$.

There are two answers to this problem.

Answer 1: 4, 5

Answer 2: $-\frac{5}{2}, -8$

Back to Problem 4.

4b. Find two numbers whose product is *p* and where one number is three less than twice the other number.

Let x = one of the numbers
Then $2x - 3$ = the other number

The product of these two numbers is $x(2x - 3)$. Since the product is to be *p*, then $x(2x - 3) = p$.

$$x(2x - 3) = p \Rightarrow 2x^2 - 3x - p = 0$$

We will solve this quadratic equation using the Quadratic Formula with $a = 2,\ b = -3,\ c = -p$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(2)(-p)}}{4} = \frac{3 \pm \sqrt{9 + 8p}}{4}$$

When $x = \dfrac{3 - \sqrt{9 + 8p}}{4}$, then $2x - 3 = \dfrac{3 - \sqrt{9 + 8p}}{2} - \dfrac{6}{2} =$
$\dfrac{3 - \sqrt{9 + 8p} - 6}{2} = \dfrac{-3 - \sqrt{9 + 8p}}{2} = -\dfrac{3 + \sqrt{9 + 8p}}{2}$.

Let's see if the product is *p*:

$$\frac{3 - \sqrt{9 + 8p}}{4}\left(-\frac{3 + \sqrt{9 + 8p}}{2}\right) = -\frac{9 - (9 + 8p)}{8} = -\frac{9 - 9 - 8p}{8} =$$
$$-\frac{-8p}{8} = -(-p) = p$$

NOTE: You use the special product formula $(a + b)(a - b) = a^2 - b^2$ to multiply $(3 - \sqrt{9 + 8p})(3 + \sqrt{9 + 8p})$.

$$(3 - \sqrt{9 + 8p})(3 + \sqrt{9 + 8p}) = 3^2 - (\sqrt{9 + 8p})^2 = 9 - (9 + 8p)$$

When $x = \dfrac{3 + \sqrt{9 + 8p}}{4}$, then $2x - 3 = \dfrac{3 + \sqrt{9 + 8p}}{2} - \dfrac{6}{2} =$
$\dfrac{3 + \sqrt{9 + 8p} - 6}{2} = \dfrac{-3 + \sqrt{9 + 8p}}{2} = \dfrac{-3 + \sqrt{9 + 8p}}{2}$.

Let's see if the product is *p*:

$$\frac{3 + \sqrt{9 + 8p}}{4} \cdot \frac{-3 + \sqrt{9 + 8p}}{2} = \frac{\sqrt{9 + 8p} + 3}{4} \cdot \frac{\sqrt{9 + 8p} - 3}{2} =$$

$$\frac{9 + 8p - 9}{8} = \frac{8p}{8} = p$$

NOTE: You use the special product formula $(a + b)(a - b) = a^2 - b^2$ to multiply $(\sqrt{9 + 8p} + 3)(\sqrt{9 + 8p} - 3)$.

$$(\sqrt{9 + 8p} + 3)(\sqrt{9 + 8p} - 3) = (\sqrt{9 + 8p})^2 - 3^2 = 9 + 8p - 9$$

There are two answers to this problem.

Answer 1: $\dfrac{3 - \sqrt{9 + 8p}}{4}, -\dfrac{3 + \sqrt{9 + 8p}}{2}$

Answer 2: $\dfrac{3 + \sqrt{9 + 8p}}{4}, \dfrac{-3 + \sqrt{9 + 8p}}{2}$ Back to Problem 4.

4c. The length of a rectangular garden is three times the width. If the area of the garden is 450 square feet, then find the dimensions of the garden.

Let w = the width of the garden
Then $3w$ = the length of the garden

The area of the rectangle is length times width = $3w \cdot w = 3w^2$. Since the area of the rectangular garden is given as 450 square feet, then $3w^2 = 450$.

$$3w^2 = 450 \Rightarrow w^2 = 150 \Rightarrow \sqrt{w^2} = \sqrt{150} \Rightarrow |w| = 5\sqrt{6} \Rightarrow w = \pm 5\sqrt{6}$$

NOTE: $\sqrt{150} = \sqrt{25 \cdot 6} = 5\sqrt{6}$

Since the width is a positive number, then $w = 5\sqrt{6}$.

Length = $3w = 15\sqrt{6}$

Answer: Length = $15\sqrt{6}$ feet
Width = $5\sqrt{6}$ feet

Back to Problem 4.

4d. Find two consecutive integers whose sum of their squares is 85.

Let n = the first integer
Then $n + 1$ = the second integer

The square of the first integer is n^2. The square of the second integer is $(n+1)^2$. The sum of these squares is $n^2 + (n+1)^2$. This sum is to be 85. Thus, $n^2 + (n+1)^2 = 85$.

$$n^2 + (n+1)^2 = 85 \Rightarrow n^2 + n^2 + 2n + 1 = 85 \Rightarrow 2n^2 + 2n - 84 = 0 \Rightarrow$$

$$2(n^2 + n - 42) = 0 \Rightarrow 2(n+7)(n-6) = 0$$

$$n + 7 = 0 \Rightarrow n = -7$$
$$n - 6 = 0 \Rightarrow n = 6$$

There are two answers to this problem.

Answer 1: -7, -6

Answer 2: 6, 7

Back to Problem 4.

4e. The sum of the first n positive integers is given by $\dfrac{n(n+1)}{2}$. That is,

$1 + 2 + 3 + \cdots\cdots + n = \dfrac{n(n+1)}{2}$. If the sum of the first *n* positive integers is 78, then find *n*.

We want to solve the equation $\dfrac{n(n+1)}{2} = 78$.

$$\frac{n(n+1)}{2} = 78 \Rightarrow n(n+1) = 156 \Rightarrow n^2 + n - 156 = 0 \Rightarrow$$

$$(n+13)(n-12) = 0$$

NOTE: Use the prime factorization of 156 to help you find the factors of 156. $156 = 2 \cdot 78 = 2 \cdot 2 \cdot 39 = 2 \cdot 2 \cdot 3 \cdot 13$. Thus, prime factorization of 156 is $2 \cdot 2 \cdot 3 \cdot 13$. Thus, the factors of 156 are 1, 156; 2, 78; 3, 52; 4, 39; 6, 26; and 12, 13.

$$n + 13 = 0 \Rightarrow n = -13$$

$$n - 12 = 0 \Rightarrow n = 12$$

Since *n* is to be a positive integer, then $n = 12$.

Answer: 12 Back to Problem 4.

4f. The base of a triangle is four meters more than three times the height of the triangle. If the area of the triangle is 80 square meters, then find the base and height of the triangle.

Let *h* = the height of the triangle
Then $3h + 4$ = the base of the triangle

The area of a triangle is given by the formula $A = \dfrac{1}{2}bh$, where *b* is the base of the triangle and *h* is the height.

Thus, the area of our triangle is $A = \frac{1}{2}(3h + 4)h$. Since the area of the triangle is to be 80 square meters, then we have that $\frac{1}{2}(3h + 4)h = 80$.

$\frac{1}{2}(3h + 4)h = 80 \Rightarrow (3h + 4)h = 160 \Rightarrow 3h^2 + 4h - 160 = 0 \Rightarrow$

$(h + 8)(3h - 20) = 0$

$h + 8 = 0 \Rightarrow h = -8$

$3h - 20 = 0 \Rightarrow 3h = 20 \Rightarrow h = \frac{20}{3}$

Since h is a dimension, it is a positive number. Thus, h is $\frac{20}{3}$ meters.

The base is equal to $3h + 4$. Thus, when $h = \frac{20}{3}$, $3h + 4 = 20 + 4 = 24$ meters.

Answer: Base = 24 meters

Height = $\frac{20}{3}$ meters

Back to Problem 4.

5a. $3x(7 - 4x)(x + 5)^2 = 0$

Back to Problem 5.

$x = 0$

$$7 - 4x = 0 \Rightarrow 7 = 4x \Rightarrow x = \frac{7}{4}$$

$$(x + 5)^2 = 0 \Rightarrow x + 5 = 0 \Rightarrow x = -5$$

Answer: $x = -5,\ 0,\ \frac{7}{4}$ **or** $\left\{-5,\ 0,\ \frac{7}{4}\right\}$

5b. $8(t^2 + 6)(t^2 - 16) = 0$ Back to Problem 5.

$$t^2 + 6 = 0 \Rightarrow t^2 = -6 \Rightarrow t = \pm i\sqrt{6}$$

$$t^2 - 16 = 0 \Rightarrow t^2 = 16 \Rightarrow t = \pm 4$$

Answer: $t = \pm 4,\ \pm i\sqrt{6}$ **or** $\left\{\pm 4,\ \pm i\sqrt{6}\right\}$

5c. $45w^3 - 27w^2 - 20w + 12 = 0$ Back to Problem 5.

We will try to factor the expression $45w^3 - 27w^2 - 20w + 12$ by grouping.

If we look at the first two terms in the expression, we can factor out the common factor of $9w^2$. Thus, $45w^3 - 27w^2 = 9w^2(5w - 3)$.

Now, in order for the factoring to continue, we need to be able to factor out a common factor from the last two terms and obtain $5w - 3$. Notice that we can do this if we factor out -4. Thus, $-20w + 12 = -4(5w - 3)$.

Thus, we can factor the expression $45w^3 - 27w^2 - 20w + 12$ by grouping:

$$45w^3 - 27w^2 - 20w + 12 = 9w^2(5w - 3) - 4(5w - 3) = (5w - 3)(9w^2 - 4)$$

$$45w^3 - 27w^2 - 20w + 12 = 0 \Rightarrow 9w^2(5w - 3) - 4(5w - 3) = 0 \Rightarrow$$

$$(5w - 3)(9w^2 - 4) = 0$$

$$5w - 3 = 0 \Rightarrow w = \frac{3}{5}$$

$$9w^2 - 4 = 0 \Rightarrow w^2 = \frac{4}{9} \Rightarrow w = \pm\frac{2}{3}$$

Answer: $w = \pm\frac{2}{3}, \frac{3}{5}$ **or** $\left\{\pm\frac{2}{3}, \frac{3}{5}\right\}$

5d. $x^3 + 36x = 2(x^2 + 36)$

Back to Problem 5.

$$x^3 + 36x = 2(x^2 + 36) \Rightarrow x^3 + 36x = 2x^2 + 72 \Rightarrow$$

$$x^3 - 2x^2 + 36x - 72 = 0 \Rightarrow x^2(x - 2) + 36(x - 2) = 0 \Rightarrow$$

$$(x - 2)(x^2 + 36) = 0$$

$$x - 2 = 0 \Rightarrow x = 2$$

$$x^2 + 36 = 0 \Rightarrow x^2 = -36 \Rightarrow x = \pm 6i$$

Answer: $x = 2, \pm 6i$ **or** $\{2, \pm 6i\}$

5e. $2y^4 = 128$ Back to Problem 5.

$2y^4 = 128 \Rightarrow 2y^4 - 128 = 0 \Rightarrow 2(y^4 - 64) = 0 \Rightarrow$

$2(y^2 + 8)(y^2 - 8) = 0$

NOTE: $y^4 - 64 = (y^2)^2 - 8^2$ is a difference of squares

$y^2 + 8 = 0 \Rightarrow y^2 = -8 \Rightarrow y = \pm i\sqrt{8} = \pm 2i\sqrt{2}$

$y^2 - 8 = 0 \Rightarrow y^2 = 8 \Rightarrow y = \pm\sqrt{8} = \pm 2\sqrt{2}$

Answer: $y = \pm 2\sqrt{2}, \pm 2i\sqrt{2}$ **or** $\{\pm 2\sqrt{2}, \pm 2i\sqrt{2}\}$

5f. $y^4 = 64y$ Back to Problem 5.

$y^4 = 64y \Rightarrow y^4 - 64y = 0 \Rightarrow y(y^3 - 64) = 0$

NOTE: In order to factor the expression $y^3 - 64$, which is a difference of cubes, you will need the following factoring formula:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Thus, $y^3 - 64 = y^3 - 4^3 = (y - 4)(y^2 + 4y + 16)$

$y^4 = 64y \Rightarrow y^4 - 64y = 0 \Rightarrow y(y^3 - 64) = 0 \Rightarrow$

$y(y - 4)(y^2 + 4y + 16) = 0$

$y = 0$

$y - 4 = 0 \Rightarrow y = 4$

$y^2 + 4y + 16 = 0$

$a = 1, \; b = 4, \; c = 16$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(1)(16)}}{2} = \frac{-4 \pm \sqrt{16(1 - 4)}}{2} =$$

$$\frac{-4 \pm 4\sqrt{-3}}{2} = \frac{-4 \pm 4i\sqrt{3}}{2} = -2 \pm 2i\sqrt{3}$$

Answer: $y = 0, \; 4, \; -2 \pm 2i\sqrt{3}$ **or** $\left\{0, \; 4, \; -2 \pm 2i\sqrt{3}\right\}$

5g. $5t^6 = -135\,t^3$

Back to Problem 5.

$5t^6 = -135\,t^3 \Rightarrow 5t^6 + 135\,t^3 = 0 \Rightarrow 5t^3(t^3 + 27) = 0$

NOTE: In order to factor the expression $t^3 + 27$, which is a sum of cubes, you will need the following factoring formula:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Thus, $t^3 + 27 = t^3 + 3^3 = (t + 3)(t^2 - 3t + 9)$

$$5t^6 = -135t^3 \Rightarrow 5t^6 + 135t^3 = 0 \Rightarrow 5t^3(t^3 + 27) = 0 \Rightarrow$$

$$5t^3(t + 3)(t^2 - 3t + 9) = 0$$

$$t^3 = 0 \Rightarrow t = 0$$

$$t + 3 = 0 \Rightarrow t = -3$$

$$t^2 - 3t + 9 = 0$$

$$a = 1,\ b = -3,\ c = 9$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(1)(9)}}{2} = \frac{3 \pm \sqrt{9(1 - 4)}}{2} =$$

$$\frac{3 \pm 3\sqrt{-3}}{2} = \frac{3 \pm 3i\sqrt{3}}{2}$$

Answer: $t = -3,\ 0,\ \dfrac{3 \pm 3i\sqrt{3}}{2}$ **or** $\left\{-3,\ 0,\ \dfrac{3 \pm 3i\sqrt{3}}{2}\right\}$

6a. $$\frac{6x}{x - 3} - \frac{4}{x + 6} = \frac{5x^2 + 39x}{x^2 + 3x - 18}$$

Back to Problem 6.

$$x^2 + 3x - 18 = (x + 6)(x - 3)$$

$$\frac{6x}{x - 3} - \frac{4}{x + 6} = \frac{5x^2 + 39x}{x^2 + 3x - 18} \Rightarrow \frac{6x}{x - 3} - \frac{4}{x + 6} = \frac{5x^2 + 39x}{(x + 6)(x - 3)}$$

NOTE: $x \neq -6,\ x \neq 3$

LCD $= (x+6)(x-3)$

$$\frac{6x}{x-3} - \frac{4}{x+6} = \frac{5x^2 + 39x}{(x+6)(x-3)} \Rightarrow$$

$$(x+6)(x-3)\left(\frac{6x}{x-3} - \frac{4}{x+6}\right) = \left[\frac{5x^2 + 39x}{(x+6)(x-3)}\right](x+6)(x-3)$$

$$\Rightarrow 6x(x+6) - 4(x-3) = 5x^2 + 39x \Rightarrow$$

$$6x^2 + 36x - 4x + 12 = 5x^2 + 39x \Rightarrow 6x^2 + 32x + 12 = 5x^2 + 39x \Rightarrow$$

$$x^2 - 7x + 12 = 0 \Rightarrow (x-3)(x-4) = 0 \Rightarrow x = 3,\ 4$$

If $x = 3$, then two of the fractions in the equation are undefined because you would have division by zero. Thus, $x = 4$ is the only solution.

Answer: $x = 4$ **or** $\{4\}$

6b. $$\frac{32}{y^2 + 4y} + 7 = \frac{2y}{y+4}$$

Back to Problem 6.

$$y^2 + 4y = y(y+4)$$

$$\frac{32}{y^2 + 4y} + 7 = \frac{2y}{y+4} \Rightarrow \frac{32}{y(y+4)} + 7 = \frac{2y}{y+4}$$

NOTE: $y \neq 0,\ y \neq -4$

LCD $= y(y+4)$

$$\frac{32}{y(y+4)} + 7 = \frac{2y}{y+4} \Rightarrow y(y+4)\left[\frac{32}{y(y+4)} + 7\right] = \left(\frac{2y}{y+4}\right)y(y+4)$$

$$\Rightarrow 32 + 7y(y+4) = 2y^2 \Rightarrow 32 + 7y^2 + 28y = 2y^2 \Rightarrow$$

$$5y^2 + 28y + 32 = 0 \Rightarrow (y+4)(5y+8) = 0 \Rightarrow y = -4,\ \frac{8}{5}$$

If $y = -4$, then two of the fractions in the equation are undefined because you would have division by zero. Thus, $y = \frac{8}{5}$ is the only solution.

Answer: $y = \frac{8}{5}$ **or** $\left\{\frac{8}{5}\right\}$

6c. $8 - \frac{5}{w} = \frac{7}{w^2}$

Back to Problem 6.

NOTE: $w \neq 0$

LCD $= w^2$

$$8 - \frac{5}{w} = \frac{7}{w^2} \Rightarrow w^2\left(8 - \frac{5}{w}\right) = \left(\frac{7}{w^2}\right)w^2 \Rightarrow 8w^2 - 5w = 7 \Rightarrow$$

$$8w^2 - 5w - 7 = 0$$

$$a = 8,\ b = -5,\ c = -7$$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4(8)(-7)}}{16} = \frac{5 \pm \sqrt{25 + 224}}{16} =$$

$$\frac{5 \pm \sqrt{249}}{16}$$

Answer: $w = \dfrac{5 \pm \sqrt{249}}{16}$ **or** $\left\{ \dfrac{5 \pm \sqrt{249}}{16} \right\}$