Pre-Class Problems 2 for Wednesday, January 24

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem letter.

1. Simplify each expression and write the result in the form $a + bi$.

   a. $3 + \sqrt{-98}$ b. $\dfrac{12 - 7i}{18}$ c. $\dfrac{9 - 15i}{-3}$

   d. $\dfrac{-24 + \sqrt{-48}}{12}$

2. Simplify the power of i.

   a. i^{36} b. i^{27} c. i^{93} d. i^{58} e. i^{-19}

3. Perform the indicated operations. Write your answer in $a + bi$ form.

   a. $(3 - 5i) + (-7 + 2i)$ b. $(12 + 17i) - (5 - 8i)$

   c. $\left(\dfrac{2}{3} - \dfrac{3}{4}i\right) - \left(\dfrac{1}{2} + \dfrac{5}{8}i\right)$ d. $(2 - 9i) - (3 - i) + (7 + 4i)$

   e. $4i(5 - 12i)$ f. $(4 - 7i)(2 + 3i)$ g. $(3 - 4i)(1 - 6i)$

   h. $(3 + 5i)^2$ i. $(7 - 4i)^2$ j. $(2 + 9i)^2 - (2 - 9i)^2$

   k. $(5 - 8i)(5 + 8i)$ l. $(1 + 3i)(1 - 3i)$ m. $\dfrac{2 - 3i}{4 + 7i}$

n. $\dfrac{3+8i}{5-2i}$ o. $\dfrac{9-5i}{9+5i}$ p. $(6-\sqrt{5}\,i)^{-1}$ q. $\dfrac{4}{11i}$

Theorem If a and b are real numbers and $ab = 0$, then either $a = 0$ or $b = 0$.

4. Solve the following equations by factoring.

a. $(x+3)(x-7) = 0$ b. $6t(t-2) = 7t - 15$

c. $4y(y+2) = y + 30$ d. $98n^2 - 50 = 0$

e. $5x^2 = 180$ f. $(m+2)(m-6) = 20$

Theorem If a is any real number, then $\sqrt{a^2} = |a|$

5. Solve the following equations using square roots.

a. $7x^2 - 42 = 0$ b. $3y^2 + 48 = 0$

c. $4(w-6)^2 - 25 = 87$ d. $(t+8)^2 + 18 = 15$

Additional problems available in the textbook: Page 111 … 29 – 88 and Examples 3 – 8 starting on page 106. Page 123 … 7 – 30 and Examples 1 and starting on page 114.

SOLUTIONS:

1a. $3 + \sqrt{-98} = 3 + i\sqrt{49 \cdot 2} = 3 + 7i\sqrt{2}$

Answer: $3 + 7i\sqrt{2}$ **or** $3 + 7\sqrt{2}\,i$ Back to Problem 1.

1b. $\dfrac{12 - 7i}{18} = \dfrac{12}{18} - \dfrac{7i}{18} = \dfrac{2}{3} - \dfrac{7}{18}i$

Answer: $\dfrac{2}{3} - \dfrac{7}{18}i$ Back to Problem 1.

1c. $\dfrac{9 - 15i}{-3} = \dfrac{9}{-3} - \dfrac{15i}{-3} = -3 + 5i$

Answer: $-3 + 5i$ Back to Problem 1.

1d. $\dfrac{-24 + \sqrt{-48}}{12} = \dfrac{-24 + i\sqrt{16 \cdot 3}}{12} = \dfrac{-24 + 4i\sqrt{3}}{12} = \dfrac{-24}{12} + \dfrac{4i\sqrt{3}}{12} =$

$-2 + \dfrac{\sqrt{3}}{3}i$

Answer: $-2 + \dfrac{\sqrt{3}}{3}i$ Back to Problem 1.

2a. $i^{36} = (i^4)^9 = 1^9 = 1$

NOTE: 36 is a multiple of 4 and 36 = 4(9).

Answer: 1 Back to Problem 2.

2b. $i^{27} = i^{24+3} = i^{24}i^3 = i^3 = i^2 i = -1i = -i$

NOTE: 24 is the largest multiple of 4 that is less than 27 and 27 = 24 + 3.

NOTE: $i^{24} = (i^4)^6 = 1^6 = 1$

Answer: $-i$ Back to Problem 2.

2c. $i^{93} = i^{92+1} = i^{92}i = i$

NOTE: 92 is the largest multiple of 4 that is less than 93 and 93 = 92 + 1.

NOTE: $i^{92} = (i^4)^{23} = 1^{23} = 1$

Answer: i Back to Problem 2.

2d. $i^{58} = i^{56+2} = i^{56}i^2 = i^2 = -1$

NOTE: 56 is the largest multiple of 4 that is less than 58 and 58 = 56 + 2.

NOTE: $i^{56} = (i^4)^{14} = 1^{14} = 1$

Answer: -1 Back to Problem 2.

2e. $i^{-19} = \dfrac{1}{i^{19}}$

$i^{19} = i^{16+3} = i^{16}i^3 = i^3 = -i$

NOTE: 16 is the largest multiple of 4 that is less than 19 and 19 = 16 + 3.

NOTE: $i^{16} = (i^4)^4 = 1^4 = 1$ and $i^3 = i^2 i = -1i = -i$

$$i^{-19} = \frac{1}{i^{19}} = \frac{1}{-i} = -\frac{1}{i} \cdot \frac{i}{i} = -\frac{i}{-1} = i$$

Answer: i Back to Problem 2.

NOTE: You can look at Example 4d on page 107 of the textbook in order to see how the author worked this problem.

3a. $(3 - 5i) + (-7 + 2i) = 3 - 5i - 7 + 2i = -4 - 3i$

Answer: $-4 - 3i$ Back to Problem 3.

3b. $(12 + 17i) - (5 - 8i) = 12 + 17i - 5 + 8i = 7 + 25i$

Answer: $7 + 25i$ Back to Problem 3.

3c. $\left(\frac{2}{3} - \frac{3}{4}i\right) - \left(\frac{1}{2} + \frac{5}{8}i\right) = \frac{2}{3} - \frac{3}{4}i - \frac{1}{2} - \frac{5}{8}i = \frac{4}{6} - \frac{6}{8}i - \frac{3}{6} - \frac{5}{8}i =$

$\frac{1}{6} - \frac{11}{8}i$

Answer: $\frac{1}{6} - \frac{11}{8}i$ Back to Problem 3.

3d. $(2 - 9i) - (3 - i) + (7 + 4i) = 2 - 9i - 3 + i + 7 + 4i = 6 - 4i$

NOTE: $2 - 3 + 7 = 6$ and $-9i + i + 4i = -4i$

Answer: $6 - 4i$ Back to Problem 3.

3e. $4i(5 - 12i) = 20i - 48i^2 = 20i - (-48) = 48 + 20i$

NOTE: $i^2 = -1$

Answer: $48 + 20i$ Back to Problem 3.

3f. $(4 - 7i)(2 + 3i) = 8 + 12i - 14i - 21i^2 = 8 - 2i + 21 = 29 - 2i$

NOTE: $i^2 = -1$ and $-21i^2 = 21$

Answer: $29 - 2i$ Back to Problem 3.

3g. $(3 - 4i)(1 - 6i) = 3 - 18i - 4i + 24i^2 = 3 - 22i - 24 = -21 - 22i$

NOTE: $i^2 = -1$ and $24i^2 = -24$

Answer: $-21 - 22i$ Back to Problem 3.

3h. $(3 + 5i)^2$

NOTE: We will use the following special product formula.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$(3+5i)^2 = 9 + 30i + 25i^2 = 9 + 30i - 25 = -16 + 30i$

NOTE: $i^2 = -1$ and $25i^2 = -25$

Answer: $-16 + 30i$ Back to Problem 3.

3i. $(7-4i)^2$

NOTE: We will use the following special product formula.

$$(a-b)^2 = a^2 - 2ab + b^2$$

$(7-4i)^2 = 49 - 56i + 16i^2 = 33 - 56i$

NOTE: $i^2 = -1$ and $16i^2 = -16$

Answer: $33 - 56i$ Back to Problem 3.

3j. $(2+9i)^2 - (2-9i)^2 = 4 + 36i + 81i^2 - (4 - 36i + 81i^2) =$

$4 + 36i + 81i^2 - 4 + 36i - 81i^2 = 72i$

Answer: $72i$ Back to Problem 3.

3k. $(5-8i)(5+8i)$

NOTE: We will use the following special product formula.

$$(a - b)(a + b) = a^2 - b^2$$

$(5 - 8i)(5 + 8i) = 25 - 64i^2 = 89$

NOTE: $i^2 = -1$ and $-64i^2 = 64$

Answer: 89 Back to Problem 3.

3l. $(1 + 3i)(1 - 3i)$

NOTE: We will use the following special product formula.

$$(a + b)(a - b) = a^2 - b^2$$

$(1 + 3i)(1 - 3i) = 1 - 9i^2 = 10$

NOTE: $i^2 = -1$ and $-9i^2 = 9$

Answer: 10 Back to Problem 3.

3m. $\dfrac{2 - 3i}{4 + 7i}$

To divide these two complex numbers, you will multiply both the numerator and the denominator of the fraction by the conjugate of the denominator.

The conjugate of the complex number $a + bi$ is $a - bi$.

Thus, the conjugate of $4 + 7i$ is $4 - 7i$.

$$\frac{2-3i}{4+7i} = \frac{2-3i}{4+7i} \cdot \frac{4-7i}{4-7i} = \frac{(2-3i)(4-7i)}{16+49} = \frac{8-14i-12i+21i^2}{65} =$$

$$\frac{-13-26i}{65} = -\frac{13}{65} - \frac{26}{65}i$$

Answer: $-\frac{13}{65} - \frac{26}{65}i$ Back to Problem 3.

3n. $\frac{3+8i}{5-2i}$

The conjugate of $5-2i$ is $5+2i$.

$$\frac{3+8i}{5-2i} = \frac{3+8i}{5-2i} \cdot \frac{5+2i}{5+2i} = \frac{15+6i+40i+16i^2}{25+4} = \frac{-1+46i}{29} =$$

$$-\frac{1}{29} + \frac{46}{29}i$$

Answer: $-\frac{1}{29} + \frac{46}{29}i$ Back to Problem 3.

3o. $\frac{9-5i}{9+5i}$

The conjugate of $9+5i$ is $9-5i$.

$$\frac{9-5i}{9+5i} = \frac{9-5i}{9+5i} \cdot \frac{9-5i}{9-5i} = \frac{(9-5i)^2}{81+25} = \frac{81-90i-25}{106} = \frac{56}{106} - \frac{25}{106}i$$

Answer: $\frac{56}{106} - \frac{25}{106}i$ Back to Problem 3.

3p. $(6 - \sqrt{5}\,i)^{-1}$

$$(6 - \sqrt{5}\,i)^{-1} = \frac{1}{6 - \sqrt{5}\,i} = \frac{1}{6 - \sqrt{5}\,i} \cdot \frac{6 + \sqrt{5}\,i}{6 + \sqrt{5}\,i} = \frac{6 + \sqrt{5}\,i}{36 + 5} =$$

$$\frac{6 + \sqrt{5}\,i}{41} = \frac{6}{41} + \frac{\sqrt{5}}{41}i$$

The conjugate of $6 - \sqrt{5}\,i$ is $6 + \sqrt{5}\,i$.

Answer: $\frac{6}{41} + \frac{\sqrt{5}}{41}i$ Back to Problem 3.

3q. $\frac{4}{11i} = \frac{4}{11i} \cdot \frac{i}{i} = \frac{4i}{11i^2} = \frac{4i}{-11} = -\frac{4}{11}i$

Answer: $0 - \frac{4}{11}i$ Back to Problem 3.

NOTE: You can write the answer as $-\frac{4}{11}i$. However, in the ALEKS homework, they might ask you to write the answer as $0 - \frac{4}{11}i$.

4a. $(x + 3)(x - 7) = 0$

$x + 3 = 0 \Rightarrow x = -3$

$x - 7 = 0 \Rightarrow x = 7$

Answer: $x = -3,\ x = 7$ **or** $\{-3, 7\}$ Back to Problem 4.

4b. $6t(t - 2) = 7t - 15 \Rightarrow 6t^2 - 12t = 7t - 15 \Rightarrow 6t^2 - 19t + 15 = 0$

$\Rightarrow (2t - 3)(3t - 5) = 0$

$2t - 3 = 0 \Rightarrow 2t = 3 \Rightarrow t = \frac{3}{2}$

$3t - 5 = 0 \Rightarrow 3t = 5 \Rightarrow t = \frac{5}{3}$

Answer: $t = \frac{3}{2},\ t = \frac{5}{3}$ **or** $\left\{\frac{3}{2}, \frac{5}{3}\right\}$ Back to Problem 4.

4c. $4y(y + 2) = y + 30 \Rightarrow 4y^2 + 8y = y + 30 \Rightarrow 4y^2 + 7y - 30 = 0$

$\Rightarrow (y - 2)(4y + 15) = 0$

$y - 2 = 0 \Rightarrow y = 2$

$4y + 15 = 0 \Rightarrow 4y = -15 \Rightarrow y = -\frac{15}{4}$

Answer: $y = -\frac{15}{4},\ y = 2$ **or** $\left\{-\frac{15}{4}, 2\right\}$ Back to Problem 4.

4d. $98n^2 - 50 = 0 \Rightarrow 2(49n^2 - 25) = 0 \Rightarrow 2(7n + 5)(7n - 5) = 0$

$7n + 5 = 0 \Rightarrow 7n = -5 \Rightarrow n = -\frac{5}{7}$

$7n - 5 = 0 \Rightarrow 7n = 5 \Rightarrow n = \frac{5}{7}$

Answer: $n = -\frac{5}{7},\ n = \frac{5}{7}$ **or** $\left\{-\frac{5}{7}, \frac{5}{7}\right\}$ Back to Problem 4.

4e. $5x^2 = 180 \Rightarrow 5x^2 - 180 = 0 \Rightarrow 5(x^2 - 36) = 5(x + 6)(x - 6) = 0$

$x + 6 = 0 \Rightarrow x = -6$

$x - 6 = 0 \Rightarrow x = 6$

Answer: $x = -6,\ x = 6$ **or** $\{-6, 6\}$ Back to Problem 4.

4f. $(m + 2)(m - 6) = 20 \Rightarrow m^2 - 6m + 2m - 12 = 20 \Rightarrow$

$m^2 - 4m - 12 = 20 \Rightarrow m^2 - 4m - 32 = 0 \Rightarrow (m + 4)(m - 8) = 0$

$m + 4 = 0 \Rightarrow m = -4$

$m - 8 = 0 \Rightarrow m = 8$

Answer: $m = -4,\ x = 8$ **or** $\{-4, 8\}$ Back to Problem 4.

5a. $7x^2 - 42 = 0 \Rightarrow 7x^2 = 42 \Rightarrow x^2 = 6 \Rightarrow \sqrt{x^2} = \sqrt{6} \Rightarrow |x| = \sqrt{6}$

$\Rightarrow x = \pm\sqrt{6}$

Answer: $x = -\sqrt{6}$, $x = \sqrt{6}$ **or** $\{-\sqrt{6}, \sqrt{6}\}$

Back to Problem 5.

5b. $3y^2 + 48 = 0 \Rightarrow 3y^2 = -48 \Rightarrow y^2 = -16 \Rightarrow \sqrt{y^2} = \sqrt{-16} \Rightarrow$

$|y| = 4i \Rightarrow y = \pm 4i$

Answer: $y = -4i$, $y = 4i$ **or** $\{-4i, 4i\}$ Back to Problem 5.

5c. $4(w-6)^2 - 25 = 87 \Rightarrow 4(w-6)^2 = 112 \Rightarrow (w-6)^2 = 28 \Rightarrow$

$\sqrt{(w-6)^2} = \sqrt{28} \Rightarrow |w-6| = 2\sqrt{7} \Rightarrow w - 6 = \pm 2\sqrt{7} \Rightarrow$

$w = 6 \pm 2\sqrt{7}$

Answer: $w = 6 - 2\sqrt{7}$, $w = 6 + 2\sqrt{7}$ **or** $\{6 - 2\sqrt{7}, 6 + 2\sqrt{7}\}$

Back to Problem 5.

5d. $(t+8)^2 + 18 = 15 \Rightarrow (t+8)^2 = -3 \Rightarrow \sqrt{(t+8)^2} = \sqrt{-3} \Rightarrow$

$$\left|t + 8\right| = i\sqrt{3} \;\Rightarrow\; t + 8 = \pm i\sqrt{3} \;\Rightarrow\; t = -8 \pm i\sqrt{3}$$

Answer: $t = -8 - i\sqrt{3}$, $t = -8 + i\sqrt{3}$ **or** $\{-8 - i\sqrt{3}, -8 + i\sqrt{3}\}$

Back to Problem 5.