Pre-Class Problems 23 for Wednesday, April 25

Earn one bonus point because you checked the Pre-Class problems. Send me an email with PC23 in the Subject line.

These are the type of problems that you will be working on in class.

Definition An arithmetic sequence $\{a_n\}$ is a sequence of the form

$$a_1,\ a_1 + d,\ a_1 + 2d,\ a_1 + 3d, \ldots\ldots, a_1 + (n-1)d, \ldots\ldots$$

NOTE: Each next term in the sequence is obtained by adding a fixed constant *d* to the previous term. This fixed constant *d* is called the common difference of the sequence since $d = a_{n+1} - a_n$ for all *n*. The *n*th term of the sequence is given by $a_n = a_1 + (n-1)d$.

1. If $a_{10} = 120$ and $a_{32} = 428$ in the arithmetic sequence $\{a_n\}$, then find a_1 and *d*.

Theorem The sum S_n of the first *n* terms of the arithmetic sequence $\{a_n\}$ is given by $S_n = \dfrac{n(a_1 + a_n)}{2}$, where a_1 is the first term of the sequence and a_n is the *n*th term of the sequence.

2. Find the sum of the following arithmetic sequences.

   a. The sequence 16, 20, 24, 28, ... , 184.

   b. The first fifteen terms in the sequence with $b_1 = 28$ and $d = 12$.

   c. The first 201 terms in the sequence with $b_1 = 12$ and $d = -5$.

d. The first 25 terms in the sequence with $c_1 = -8$ and $c_{25} = 136$.

e. The first 100 terms in the sequence with $a_1 = 57$ and $a_{100} = -636$.

f. $\sum_{i=1}^{5} (4i + 7)$ g. $\sum_{i=1}^{50} (4i + 7)$ h. $\sum_{j=1}^{100} j$

i. $\sum_{i=1}^{n} i$

<u>Definition</u> An geometric sequence $\{a_n\}$ is a sequence of the form

$$a_1, \ a_1 r, \ a_1 r^2, \ a_1 r^3, \ldots\ldots, \ a_1 r^{n-1}, \ldots\ldots$$

NOTE: Each next term in the sequence is obtained by multiplying a fixed constant *r* to the previous term. This fixed constant *r* is called the common ratio of the sequence since $r = \frac{a_{n+1}}{a_n}$ for all *n*. The *n*th term of the sequence is given by $a_n = a_1 r^{n-1}$.

3. Determine if the following sequences are geometric. If the sequence is geometric, then find the common ratio.

a. 4, 8, 16, 32, 64, ... b. $5, -\frac{10}{3}, \frac{20}{9}, -\frac{40}{27}, \frac{80}{81}, \ldots$

c. 1, 3, 12, 60, 360, ...

4. Write the first five terms of the geometric sequence $\{a_n\}$ with the given first term and common ratio.

a. $a_1 = -2$ and $r = 4$

b. $a_1 = 6$ and $r = -\frac{1}{2}$

5. Find the *n*th term of the geometric sequence $\{b_n\}$ with the given first term and common ratio. Then find the indicated term.

a. $b_1 = 3$ and $r = -2$. Find b_9.

b. $b_1 = -\frac{4}{5}$ and $r = \frac{3}{2}$. Find b_5.

6. Find the seventh term of the geometric sequence with $c_1 = 12$ and $c_4 = -324$.

7. Find the fifth term of the geometric sequence with $a_1 = 24$ and $a_2 = 32$.

Theorem The sum S_n of the first *n* terms of the geometric sequence $\{a_n\}$ is given by $S_n = \dfrac{a_1(r^n - 1)}{r - 1}$, where a_1 is the first term of the sequence and *r* is the common ratio of the sequence, where $r \neq 1$.

8. Find the sum of the following geometric sequences.

a. $\sum_{n=1}^{5} (-2)4^{n-1}$

b. $\sum_{n=1}^{5} 6\left(-\frac{1}{2}\right)^{n-1}$

c. $\sum_{n=1}^{5} 4(2)^{n-1}$

d. $\sum_{n=1}^{10} 4(2)^{n-1}$

e. $\sum_{n=1}^{15} \left(\frac{2}{3}\right)^{n-1}$

Theorem The infinite sum of the geometric sequence $\{a_1 r^{n-1}\}$, denoted by $\sum_{n=1}^{\infty} a_1 r^{n-1}$, is given by $\sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r}$ if $|r| < 1$ and $\sum_{n=1}^{\infty} a_1 r^{n-1}$ does not exist if $|r| \geq 1$. The expression $\sum_{n=1}^{\infty} a_1 r^{n-1}$ is called a geometric series.

9. Find the sum of the following geometric series if possible.

a. $\sum_{n=1}^{\infty} 6\left(-\frac{1}{2}\right)^{n-1}$ b. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1}$ c. $\sum_{n=1}^{\infty} (-12)\left(-\frac{3}{4}\right)^{n-1}$

d. $\sum_{n=1}^{\infty} 18\left(\frac{7}{6}\right)^{n-1}$ e. $\sum_{n=1}^{\infty} (-2)4^{n-1}$ f. $\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^{n-1}$

g. $\sum_{n=1}^{\infty} (-12)\left(\frac{3}{4}\right)^{n-1}$ h. $\sum_{n=1}^{\infty} 16\left(\frac{7}{11}\right)^{n-1}$ i. $\sum_{n=1}^{\infty} \frac{3}{10^n}$

Additional problems available in the textbook: Page 710 … 39 – 70. Examples 3, 6 – 9 starting on page 702. Page 721 … 9 – 24, 35 – 44, 49 – 68, 73, 74, 77, 78. Examples 1, 2, 4, 6 – 9 starting on page 712.

SOLUTIONS:

1. $a_{10} = 120$ and $a_{32} = 428$ Back to Problem 1.

$$a_n = a_1 + (n-1)d \Rightarrow a_{10} = a_1 + (10-1)d = a_1 + 9d$$

$$a_{10} = a_1 + 9d \text{ and } a_{10} = 120 \Rightarrow 120 = a_1 + 9d$$

$a_n = a_1 + (n - 1)d \Rightarrow a_{32} = a_1 + (32 - 1)d = a_1 + 31d$

$a_{32} = a_1 + 31d$ and $a_{32} = 428 \Rightarrow 428 = a_1 + 31d$

In order to find a_1 and d, we need to solve the system of equations:

$$\begin{aligned} a_1 + 9d &= 120 \\ a_1 + 31d &= 428 \end{aligned}$$

Using the addition method, we obtain

$$\begin{aligned} a_1 + 9d &= 120 \\ a_1 + 31d &= 428 \end{aligned} \quad \rightarrow \quad \begin{array}{r} -a_1 - 9d = -120 \\ a_1 + 31d = 428 \\ \hline 22d = 308 \end{array}$$

$22d = 308 \Rightarrow 11d = 154 \Rightarrow d = 14$

$a_1 + 9d = 120$ and $d = 14 \Rightarrow a_1 + 126 = 120 \Rightarrow a_1 = -6$

Answer: $a_1 = -6$ and $d = 14$

2a. 16, 20, 24, 28, ... , 184 Back to Problem 2.

In Problem 8 of Pre-Class Problems 22, we found that there are 43 terms in this finite arithmetic sequence. Thus, $a_1 = 16$, $n = 43$, and $a_{43} = 184$.

Thus, $S_n = \dfrac{n(a_1 + a_n)}{2} \Rightarrow S_{43} = \dfrac{43(16 + 184)}{2} = \dfrac{43(200)}{2} = 43(100)$

$= 4300$

Answer: 4300

2b. $b_1 = 28$ and $d = 12$. Find b_{15}. Back to Problem 2.

In Problem 5a of Pre-Class Problems 22, we found that $b_{15} = 196$. Thus, $b_1 = 28$, $n = 15$, and $b_{15} = 196$.

Thus, $S_n = \dfrac{n(b_1 + b_n)}{2} \Rightarrow S_{15} = \dfrac{15(28 + 196)}{2} = \dfrac{15(224)}{2} = 15(112)$

$= 1680$

Answer: 1680

2c. $b_1 = 12$ and $d = -5$ Back to Problem 2.

In Problem 5b of Pre-Class Problems 22, we found that $b_{201} = -988$. Thus, $b_1 = 12$, $n = 201$, and $b_{201} = -988$.

Thus, $S_n = \dfrac{n(b_1 + b_n)}{2} \Rightarrow S_{201} = \dfrac{201(12 - 988)}{2} = \dfrac{201(-976)}{2} =$

$201(-488) = -98{,}088$

Answer: $-98{,}088$

2d. $c_1 = -8$ and $c_{25} = 136$ Back to Problem 2.

Thus, $S_n = \dfrac{n(c_1 + c_n)}{2} \Rightarrow S_{25} = \dfrac{25(-8 + 136)}{2} = \dfrac{25(128)}{2} = 25(64) =$

$25(4)(16) = 100(16) = 1600$

Answer: 1600

2e. $a_1 = 57$ and $a_{100} = -636$ Back to Problem 2.

Thus, $S_n = \dfrac{n(a_1 + a_n)}{2} \Rightarrow S_{100} = \dfrac{100(57 - 636)}{2} = \dfrac{100(-579)}{2} =$

$50(-579) = -28{,}950$

Answer: $-28{,}950$

2f. $\displaystyle\sum_{i=1}^{5} (4i + 7)$ Back to Problem 2.

This is Problem 2a from Pre-Class Problems 22.

First, let's verify that the sequence $\{a_i\}$, where $a_i = 4i + 7$, is an arithmetic sequence. We just need to show that $a_{i+1} - a_i$ is a constant.

$a_{i+1} = 4(i + 1) + 7 = 4i + 4 + 7 = 4i + 11$

$a_i = 4i + 7$

$a_{i+1} - a_i = 4i + 11 - (4i + 7) = 4i + 11 - 4i - 7 = 4$

Thus, $a_{i+1} - a_i = 4$ for all i. Thus, the sequence $\{a_i\}$ is an arithmetic sequence.

$a_i = 4i + 7 \Rightarrow a_1 = 11$ and $a_5 = 27$

Thus, $S_n = \dfrac{n(a_1 + a_n)}{2} \Rightarrow S_5 = \dfrac{5(11 + 27)}{2} = \dfrac{5(38)}{2} = 5(19) = 95$

Answer: 95

NOTE: Here's how we did this problem in Pre-Class Problems 22:

$$\sum_{i=1}^{5} (4i + 7) = 11 + 15 + 19 + 23 + 27 = 30 + 15 + 50 = 95$$

NOTE: In general, you can show that any sequence $\{a_i\}$, where $a_i = mi + b$, is an arithmetic sequence, where the common difference *d* is *m*.

2g. $\displaystyle\sum_{i=1}^{50} (4i + 7)$ Back to Problem 2.

$a_i = 4i + 7 \Rightarrow a_1 = 11$ and $a_{50} = 207$

Thus, $S_n = \dfrac{n(a_1 + a_n)}{2} \Rightarrow S_{50} = \dfrac{50(11 + 207)}{2} = \dfrac{50(218)}{2} = 50(109)$

$= 5450$

Answer: 5450

2h. $\sum_{j=1}^{100} j$ Back to Problem 2.

The sequence $\{a_j\}$, where $a_j = j$, is an arithmetic sequence with $a_1 = 1$ and $d = 1$.

$a_j = j \Rightarrow a_1 = 1$ and $a_{100} = 100$

Thus, $S_n = \dfrac{n(a_1 + a_n)}{2} \Rightarrow S_{100} = \dfrac{100(1 + 100)}{2} = \dfrac{100(101)}{2} = 50(101)$

$= 5050$

Answer: 5050

2i. $\sum_{i=1}^{n} i$ Back to Problem 2.

The sequence $\{a_i\}$, where $a_i = i$, is an arithmetic sequence with $a_1 = 1$ and $d = 1$.

$a_i = i \Rightarrow a_1 = 1$ and $a_n = n$

Thus, $S_n = \dfrac{n(a_1 + a_n)}{2} \Rightarrow S_n = \dfrac{n(1 + n)}{2} = \dfrac{n(n + 1)}{2}$

Answer: $\displaystyle\sum_{i = 1}^{n} i = \frac{n(n + 1)}{2}$

NOTE: This is a formula that is used in calculus to calculate certain Riemann integrals using the definition of the Riemann integral.

3a. 4, 8, 16, 32, 64, ... Back to Problem 3.

$\dfrac{a_2}{a_1} = \dfrac{8}{4} = 2$ $\dfrac{a_3}{a_2} = \dfrac{16}{8} = 2$

$\dfrac{a_4}{a_3} = \dfrac{32}{16} = 2$ $\dfrac{a_5}{a_4} = \dfrac{64}{32} = 2$

NOTE: The ratio between each term and its preceding term is 2.

Answer: Yes. The common ratio is 2.

3b. $5, -\dfrac{10}{3}, \dfrac{20}{9}, -\dfrac{40}{27}, \dfrac{80}{81}, \ldots$ Back to Problem 3.

$$\frac{a_2}{a_1} = -\frac{10}{3} \cdot \frac{1}{5} = -\frac{2}{3}$$

$$\frac{a_3}{a_2} = \frac{20}{9}\left(-\frac{3}{10}\right) = \frac{2}{3}\left(-\frac{1}{1}\right) = -\frac{2}{3}$$

$$\frac{a_4}{a_3} = -\frac{40}{27} \cdot \frac{9}{20} = -\frac{2}{3} \cdot \frac{1}{1} = -\frac{2}{3}$$

$$\frac{a_4}{a_3} = \frac{80}{81}\left(-\frac{27}{40}\right) = \frac{2}{3}\left(-\frac{1}{1}\right) = -\frac{2}{3}$$

NOTE: The ratio between each term and its preceding term is $-\frac{2}{3}$.

Answer: Yes. The common ratio is $-\frac{2}{3}$.

3c. 1, 3, 12, 60, 360, ... Back to Problem 3.

$$\frac{a_2}{a_1} = \frac{3}{1} = 3 \qquad \frac{a_3}{a_2} = \frac{12}{3} = 4$$

This sequence is not geometric. The ratio between the second term and the first term is 3. However, the ratio between the third term and the second term is 4.

Answer: No

4a. $a_1 = -2$ and $r = 4$ Back to Problem 4.

Answer: $-2,\ -8,\ -32,\ -128,\ -512$

4b. $a_1 = 6$ and $r = -\dfrac{1}{2}$ Back to Problem 4.

Answer: $6,\ -3,\ \dfrac{3}{2},\ -\dfrac{3}{4},\ \dfrac{3}{8}$

5a. $b_1 = 3$ and $r = -2$. Find b_9. Back to Problem 5.

$b_n = b_1 r^{n-1} \Rightarrow b_n = 3(-2)^{n-1}$

Thus, $b_9 = 3(-2)^8 = 3(256) = 768$

Answer: $b_n = 3(-2)^{n-1}$, $b_9 = 768$

5b. $b_1 = -\dfrac{4}{5}$ and $r = \dfrac{3}{2}$. Find b_5. Back to Problem 5.

$b_n = b_1 r^{n-1} \Rightarrow b_n = -\dfrac{4}{5}\left(\dfrac{3}{2}\right)^{n-1}$

Thus, $b_5 = -\frac{4}{5}\left(\frac{3}{2}\right)^4 = -\frac{4}{5}\left(\frac{81}{16}\right) = -\frac{1}{5}\left(\frac{81}{4}\right) = -\frac{81}{20}$

Answer: $b_n = -\frac{4}{5}\left(\frac{3}{2}\right)^{n-1}$, $b_5 = -\frac{81}{20}$

6. $c_1 = 12$ and $c_4 = -324$. Find c_7. Back to Problem 6.

$c_n = c_1 r^{n-1} \Rightarrow c_n = 12r^{n-1}$ since $c_1 = 12$

Then $c_4 = 12r^3$. We were given that $c_4 = -324$.

Thus, $c_4 = 12r^3$ and $c_4 = -324 \Rightarrow 12r^3 = -324 \Rightarrow 6r^3 = -162 \Rightarrow$

$3r^3 = -81 \Rightarrow r^3 = -27 \Rightarrow r = \sqrt[3]{-27} = -3$.

Thus, $c_n = 12r^{n-1}$ and $r = -3 \Rightarrow c_n = 12(-3)^{n-1}$

Thus, $c_7 = 12(-3)^6 = 12(729) = 8748$ (Yes. I used a calculator.)

Answer: $c_7 = 8748$

7. $a_1 = 24$ and $a_2 = 32$. Find a_5. Back to Problem 7.

$a_n = a_1 r^{n-1} \Rightarrow a_n = 24r^{n-1}$ since $a_1 = 24$

Since a_1 and a_2 are consecutive terms in a geometric sequence, then $r = \dfrac{a_2}{a_1}$ $= \dfrac{32}{24} = \dfrac{4}{3}$.

Thus, $a_n = 24r^{n-1}$ and $r = \dfrac{4}{3} \Rightarrow a_n = 24\left(\dfrac{4}{3}\right)^{n-1}$

Thus, $a_5 = 24\left(\dfrac{4}{3}\right)^4 = 24\left(\dfrac{256}{81}\right) = 8\left(\dfrac{256}{27}\right) = \dfrac{2048}{27}$

Answer: $a_5 = \dfrac{2048}{27}$

8a. $\displaystyle\sum_{n=1}^{5} (-2)4^{n-1}$ Back to Problem 8.

$a_1 = -2$, $r = 4$, and $n = 5$

Thus, $S_n = \dfrac{a_1(r^n - 1)}{r - 1} \Rightarrow S_5 = \dfrac{-2(4^5 - 1)}{4 - 1} = \dfrac{-2(1024 - 1)}{3} =$

$\dfrac{-2(1023)}{3} = -2(341) = -682$

Answer: -682

NOTE: The geometric sequence, which we are summing, is the geometric sequence in Problem 2a above. In this problem, we found that the first five terms were -2, -8, -32, -128, -512. Thus,

$$-2 + (-8) + (-32) + (-128) + (-512) = -(42 + 640) = -682$$

8b. $\sum_{n=1}^{5} 6\left(-\frac{1}{2}\right)^{n-1}$ Back to Problem 8.

$a_1 = 6$, $r = -\frac{1}{2}$, and $n = 5$

NOTE: $S_n = \frac{a_1(r^n - 1)}{r - 1} \Rightarrow S_n = \frac{a_1(1 - r^n)}{1 - r}$

Thus, $S_n = \frac{a_1(1 - r^n)}{1 - r} \Rightarrow S_5 = \frac{6\left[1 - \left(-\frac{1}{2}\right)^5\right]}{1 + \frac{1}{2}} = \frac{6\left[1 - \left(-\frac{1}{32}\right)\right]}{\frac{3}{2}} =$

$$\frac{6\left(1 + \frac{1}{32}\right)}{\frac{3}{2}} = \frac{6\left(1 + \frac{1}{32}\right)}{\frac{3}{2}} \cdot \frac{32}{32} = \frac{6(32 + 1)}{48} = \frac{1(33)}{8} = \frac{33}{8}$$

Answer: $\frac{33}{8}$

NOTE: The geometric sequence, which we are summing, is the geometric sequence in Problem 2b above. In this problem, we found that the first five terms were 6, -3, $\frac{3}{2}$, $-\frac{3}{4}$, $\frac{3}{8}$. Thus,

$$6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8} = 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8} = \frac{24}{8} + \frac{12}{8} - \frac{6}{8} + \frac{3}{8} =$$

$$\frac{24 + 12 - 6 + 3}{8} = \frac{39 - 6}{8} = \frac{33}{8}$$

8c. $\sum_{n=1}^{5} 4(2)^{n-1}$ Back to Problem 8.

$a_1 = 4$, $r = 2$, and $n = 5$

Thus, $S_n = \dfrac{a_1(r^n - 1)}{r - 1} \Rightarrow S_5 = \dfrac{4(2^5 - 1)}{2 - 1} = \dfrac{4(32 - 1)}{1} =$

$4(31) = 124$

Answer: 124

NOTE: The geometric sequence, which we are summing, is first five term of the geometric sequence in Problem 1a above. In this problem, we have that the first five terms were 4, 8, 16, 32, 64. Thus,

$$4 + 8 + 16 + 32 + 64 = 20 + 40 + 64 = 124$$

8d. $\sum_{n=1}^{10} 4(2)^{n-1}$ Back to Problem 8.

$a_1 = 4,\ r = 2$, and $n = 10$

Thus, $S_n = \dfrac{a_1(r^n - 1)}{r - 1} \Rightarrow S_{10} = \dfrac{4(2^{10} - 1)}{2 - 1} = \dfrac{4(1024 - 1)}{1} =$

$4(1023) = 4092$

Answer: 4092

8e. $\displaystyle\sum_{n=1}^{15}\left(\frac{2}{3}\right)^{n-1}$ Back to Problem 8.

$a_1 = 1,\ r = \dfrac{2}{3}$, and $n = 15$

NOTE: $S_n = \dfrac{a_1(r^n - 1)}{r - 1} \Rightarrow S_n = \dfrac{a_1(1 - r^n)}{1 - r}$

Thus, $S_n = \dfrac{a_1(1 - r^n)}{1 - r} \Rightarrow S_5 = \dfrac{1\left[1 - \left(\dfrac{2}{3}\right)^{15}\right]}{1 - \dfrac{2}{3}} = \dfrac{1 - \dfrac{32768}{14348907}}{\dfrac{1}{3}} =$

$3\left(\dfrac{14348907}{14348907} - \dfrac{32768}{14348907}\right) = 3\left(\dfrac{14316139}{14348907}\right) = \dfrac{14316139}{4782969}$

Answer: $\dfrac{14316139}{4782969}$

9a. $\displaystyle\sum_{n=1}^{\infty} 6\left(-\frac{1}{2}\right)^{n-1}$

Back to Problem 9.

$r = -\frac{1}{2} \Rightarrow |r| = \frac{1}{2} < 1 \Rightarrow$ the series is summable; $a_1 = 6$

Thus, $S = \dfrac{a_1}{1 - r} \Rightarrow S = \dfrac{6}{1 + \frac{1}{2}} = \dfrac{6}{1 + \frac{1}{2}} \cdot \dfrac{2}{2} = \dfrac{12}{2 + 1} = \dfrac{12}{3} = 4$

Answer: 4

NOTE: In Problem 6b above, we found that $\displaystyle\sum_{n=1}^{5} 6\left(-\frac{1}{2}\right)^{n-1} = \frac{33}{8}$.

9b. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1}$

Back to Problem 9.

$r = \frac{2}{3} \Rightarrow |r| = \frac{2}{3} < 1 \Rightarrow$ the series is summable; $a_1 = 1$

Thus, $S = \dfrac{a_1}{1 - r} \Rightarrow S = \dfrac{1}{1 - \frac{2}{3}} = \dfrac{1}{1 - \frac{2}{3}} \cdot \dfrac{3}{3} = \dfrac{3}{3 - 2} = \dfrac{3}{1} = 3$

Answer: 3

NOTE: In Problem 6e above, we found that $\sum_{n=1}^{15} \left(\frac{2}{3}\right)^{n-1} = \frac{14316139}{4782969} \approx 2.99314$.

9c. $\sum_{n=1}^{\infty} (-12)\left(-\frac{3}{4}\right)^{n-1}$ Back to Problem 9.

$r = -\frac{3}{4} \Rightarrow |r| = \frac{3}{4} < 1 \Rightarrow$ the series is summable; $a_1 = -12$

Thus, $S = \frac{a_1}{1 - r} \Rightarrow S = \frac{-12}{1 + \frac{3}{4}} = -\frac{12}{1 + \frac{3}{4}} \cdot \frac{4}{4} = -\frac{48}{4 + 3} = -\frac{48}{7}$

Answer: $-\frac{48}{7}$

9d. $\sum_{n=1}^{\infty} 18\left(\frac{7}{6}\right)^{n-1}$ Back to Problem 9.

$r = \frac{7}{6} \Rightarrow |r| = \frac{7}{6} > 1 \Rightarrow$ the series is not summable

Answer: Sum does not exist

9e. $\sum_{n=1}^{\infty} (-2)4^{n-1}$ Back to Problem 9.

$r = 4 \Rightarrow |r| = 4 > 1 \Rightarrow$ the series is not summable

Answer: Sum does not exist

NOTE: In Problem 6a above, we found that $\sum_{n=1}^{5} (-2)4^{n-1} = -682$.

9f. $\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^{n-1}$ Back to Problem 9.

$r = -\frac{2}{3} \Rightarrow |r| = \frac{2}{3} < 1 \Rightarrow$ the series is summable; $a_1 = 1$

Thus, $S = \frac{a_1}{1 - r} \Rightarrow S = \frac{1}{1 + \frac{2}{3}} = \frac{1}{1 + \frac{2}{3}} \cdot \frac{3}{3} = \frac{3}{3 + 2} = \frac{3}{5}$

Answer: $\frac{3}{5}$

NOTE: In Problem 7b above, we found that $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1} = 3.$

9g. $\sum_{n=1}^{\infty} (-12)\left(\frac{3}{4}\right)^{n-1}$ Back to Problem 9.

$r = \frac{3}{4} \Rightarrow |r| = \frac{3}{4} < 1 \Rightarrow$ the series is summable; $a_1 = -12$

Thus, $S = \frac{a_1}{1 - r} \Rightarrow S = \frac{-12}{1 - \frac{3}{4}} = -\frac{12}{1 - \frac{3}{4}} \cdot \frac{4}{4} = -\frac{48}{4 - 3} = -\frac{48}{1} =$

-48

Answer: -48

NOTE: In Problem 7c above, we found that $\sum_{n=1}^{\infty} (-12)\left(-\frac{3}{4}\right)^{n-1} = -\frac{48}{7}$.

9h. $\sum_{n=1}^{\infty} 16\left(\frac{7}{11}\right)^{n-1}$ Back to Problem 9.

$r = \frac{7}{11} \Rightarrow |r| = \frac{7}{11} < 1 \Rightarrow$ the series is summable; $a_1 = 16$

Thus, $S = \frac{a_1}{1 - r} \Rightarrow S = \frac{16}{1 - \frac{7}{11}} = \frac{16}{1 - \frac{7}{11}} \cdot \frac{11}{11} = \frac{176}{11 - 7} = \frac{176}{4} = 44$

Answer: 44

9i. $\sum_{n=1}^{\infty} \frac{3}{10^n}$ Back to Problem 9.

NOTE: $\sum_{n=1}^{\infty} \frac{3}{10^n} = \sum_{n=1}^{\infty} 3\left(\frac{1}{10}\right)^n = \sum_{n=1}^{\infty} \frac{3}{10}\left(\frac{1}{10}\right)^{n-1}$

$r = \frac{1}{10} \Rightarrow |r| = \frac{1}{10} < 1 \Rightarrow$ the series is summable; $a_1 = \frac{3}{10}$

NOTE: This geometric series is summing all the terms in the sequence $b_n = \frac{3}{10^n}$ in Problem 1d of Pre-Class Problems 22.

In that problem, we found that $b_1 = \frac{3}{10} = 0.3$, $b_2 = \frac{3}{100} = 0.03$, $b_3 = \frac{3}{1000} = 0.003$, $b_4 = \frac{3}{10000} = 0.0003$

Thus, $\sum_{n=1}^{1} \frac{3}{10^n} = \frac{3}{10} = 0.3$

$\sum_{n=1}^{2} \frac{3}{10^n} = \frac{3}{10} + \frac{3}{100} = 0.3 + 0.03 = 0.33$

$\sum_{n=1}^{3} \frac{3}{10^n} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} = 0.3 + 0.03 + 0.003 = 0.333$

$\sum_{n=1}^{4} \frac{3}{10^n} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} = 0.3 + 0.03 + 0.003 + 0.0003 =$

0.3333

It appears that $\sum_{n=1}^{\infty} \frac{3}{10^n} = 0.\bar{3} = \frac{1}{3}$. Let's see if this is true.

$$S = \frac{a_1}{1 - r} \Rightarrow S = \frac{\frac{3}{10}}{1 - \frac{1}{10}} = \frac{\frac{3}{10}}{1 - \frac{1}{10}} \cdot \frac{10}{10} = \frac{3}{10 - 1} = \frac{3}{9} = \frac{1}{3}$$

Answer: $\frac{1}{3}$