Pre-Class Problems 22 for Monday, April 23

**These are the type of problems that you will be working on in class.**

Discussion of Sequences

1. Find the first four terms of the sequence.

a. $a_n = \dfrac{n+4}{3n-2}$ b. $b_n = \sqrt[3]{6 - n^2}$ c. $c_n = \left(-\dfrac{2}{5}\right)^n$

d. $b_n = \dfrac{3}{10^n}$ e. $a_n = (-1)^n \dfrac{n^2}{e^{4n}}$ f. $c_n = \sqrt{2}$

g. $a_n = (-1)^{n+1} \dfrac{3^n}{n!}$

**<u>Definition</u>** The expression $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_{n-2} + a_{n-1} + a_n$ is a finite sum of the first *n* terms of the sequence $\{a_i\}$. The expression $\sum_{i=1}^{\infty} a_i = a_1 + a_2 + a_3 + \cdots + a_i + \cdots$ is an infinite sum of all the terms of the sequence $\{a_i\}$. You will study infinite series in a calculus course. We will study the infinite geometric series later in this course.

NOTATION: The *i* in the expression $\sum_{i=1}^{n} a_i$ is the index of summation. You should not confuse it with the *i* used for the complex number $\sqrt{-1}$. The number 1 is called the lower limit of summation and the number *n* is called the upper limit of summation. The other letters used for the index of summation are *j*, *k*, and *n*.

2. Find the following sums.

a. $\sum_{i=1}^{5} (4i + 7)$ b. $\sum_{j=3}^{6} (-2)^{j}$ c. $\sum_{k=2}^{7} (k+2)(k-5)$

**Definition** An arithmetic sequence $\{a_n\}$ is a sequence of the form

$$a_1,\ a_1 + d,\ a_1 + 2d,\ a_1 + 3d, \ldots\ldots, a_1 + (n-1)d, \ldots\ldots$$

NOTE: Each next term in the sequence is obtained by adding a fixed constant *d* to the previous term. This fixed constant *d* is called the common difference of the sequence since $d = a_{n+1} - a_n$ for all *n*. The *n*th term of the sequence is given by $a_n = a_1 + (n-1)d$.

3. Determine if the following sequences are arithmetic. If the sequence is arithmetic, then find the common difference.

a. 5, 8, 11, 14, 17, . . . . b. 12, 7, 2, $-3$, $-8$, . . . .

c. 1, 4, 9, 16, 25, . . . .

4. Write the first five terms of the arithmetic sequence $\{a_n\}$ with the given first term and common difference.

a. $a_1 = -3$ and $d = 7$ b. $a_1 = 10$ and $d = -4$

5. Find the *n*th term of the arithmetic sequence $\{b_n\}$ with the given first term and common difference. Then find the indicated term.

a. $b_1 = 28$ and $d = 12$. Find $b_{15}$.

b. $b_1 = 12$ and $d = -5$. Find $b_{201}$.

6. Find the tenth term of the arithmetic sequence with $c_1 = -8$ and $c_{25} = 136$.

7. Find the thirty-fifth term of the arithmetic sequence with $a_1 = 57$ and $a_{100} = -636$.

8. Determine the number of terms in the finite arithmetic sequence 16, 20, 24, 28, . . . . , 184.

Additional problems available in the textbook: Page 698 … 7 – 12, 21 – 24, 45 – 70. and Examples 1, 4, and 6 starting on page 691. Page 709 … 7 – 14, 15a – 18a, 19 – 24, 31 – 38. Examples 1, 2a, 4, and 5 starting on page 702.

**SOLUTIONS:**

1a. $a_n = \dfrac{n + 4}{3n - 2}$ Back to Problem 1.

$a_1 = \dfrac{5}{1} = 5$ , $a_2 = \dfrac{6}{4} = \dfrac{3}{2}$ , $a_3 = \dfrac{7}{7} = 1$ , $a_4 = \dfrac{8}{10} = \dfrac{4}{5}$

**Answer:** 5, $\dfrac{3}{2}$, 1, $\dfrac{4}{5}$

1b. $b_n = \sqrt[3]{6 - n^2}$ Back to Problem 1.

$b_1 = \sqrt[3]{5}$ , $b_2 = \sqrt[3]{2}$ , $b_3 = \sqrt[3]{-3} = -\sqrt[3]{3}$ , $b_4 = \sqrt[3]{-10} = -\sqrt[3]{10}$

**Answer:** $\sqrt[3]{5}$, $\sqrt[3]{2}$, $-\sqrt[3]{3}$, $-\sqrt[3]{10}$

1c. $c_n = \left(-\dfrac{2}{5}\right)^n$ Back to Problem 1.

$c_1 = -\dfrac{2}{5}$ , $c_2 = \dfrac{4}{25}$ , $c_3 = -\dfrac{8}{125}$ , $c_4 = \dfrac{16}{625}$

**Answer:** $-\dfrac{2}{5}$, $\dfrac{4}{25}$, $-\dfrac{8}{125}$, $\dfrac{16}{625}$

1d. $b_n = \dfrac{3}{10^n}$ Back to Problem 1.

$b_1 = \dfrac{3}{10} = 0.3$ , $b_2 = \dfrac{3}{100} = 0.03$ , $b_3 = \dfrac{3}{1000} = 0.003$ , $b_4 = \dfrac{3}{10000} = 0.0003$

**Answer:** $\dfrac{3}{10}$, $\dfrac{3}{100}$, $\dfrac{3}{1000}$, $\dfrac{3}{10000}$

**or** 0.3, 0.03, 0.003, 0.0003

1e. $a_n = (-1)^n \dfrac{n^2}{e^{4n}}$ Back to Problem 1.

$$a_1 = -\frac{1}{e^4}\ , \quad a_2 = \frac{4}{e^8}\ , \quad a_3 = -\frac{9}{e^{12}}\ , \quad a_4 = \frac{16}{e^{16}}$$

**Answer:** $-\dfrac{1}{e^4},\ \dfrac{4}{e^8},\ -\dfrac{9}{e^{12}},\ \dfrac{16}{e^{16}}$

1f. $c_n = \sqrt{2}$ Back to Problem 1.

$$c_1 = \sqrt{2}\ , \quad c_2 = \sqrt{2}\ , \quad c_3 = \sqrt{2}\ , \quad c_4 = \sqrt{2}$$

**Answer:** $\sqrt{2},\ \sqrt{2},\ \sqrt{2},\ \sqrt{2}$

NOTE: This sequence is called a constant sequence.

1g. $a_n = (-1)^{n+1} \dfrac{3^n}{n!}$ Back to Problem 1.

Recall: $n! = n(n-1)(n-2) \cdot\cdot\cdot\cdot\cdot 3 \cdot 2 \cdot 1$

$$a_1 = \frac{3}{1} = 3\ , \quad a_2 = -\frac{9}{2}\ , \quad a_3 = \frac{27}{6} = \frac{9}{2}\ , \quad a_4 = -\frac{81}{24} = -\frac{27}{8}$$

**Answer:** 3, $-\dfrac{9}{2},\ \dfrac{9}{2},\ -\dfrac{27}{8}$

2a. $\sum_{i=1}^{5} (4i + 7)$ Back to Problem 2.

$$\sum_{i=1}^{5} (4i + 7) = 11 + 15 + 19 + 23 + 27 = 30 + 15 + 50 = 95$$

**Answer:** 95

NOTE: $a_i = 4i + 7$ and $a_1 = 4 + 7 = 11$, $a_2 = 8 + 7 = 15$, $a_3 = 12 + 7 = 19$,

$a_4 = 16 + 7 = 23$, and $a_5 = 20 + 7 = 27$

2b. $\sum_{j=3}^{6} (-2)^j$ Back to Problem 2.

$$\sum_{j=3}^{6} (-2)^j = -8 + 16 + (-32) + 64 = -40 + 80 = 40$$

**Answer:** 40

NOTE: $a_j = (-2)^j$ and $a_3 = (-2)^3 = -8$, $a_4 = (-2)^4 = 16$,

$a_5 = (-2)^5 = -32$, and $a_6 = (-2)^6 = 64$

2c. $\sum_{k=2}^{7} (k + 2)(k - 5)$ Back to Problem 2.

$$\sum_{k=2}^{7}(k+2)(k-5) = 4(-3)+5(-2)+6(-1)+7(0)+8(1)+9(2) =$$

$$-12-10-6+0+8+18 = -28+26 = -2$$

**Answer:** $-2$

3a. 5, 8, 11, 14, 17 . . . . Back to Problem 3.

$a_2 - a_1 = 8 - 5 = 3$ $\qquad$ $a_3 - a_2 = 11 - 8 = 3$

$a_4 - a_3 = 14 - 11 = 3$ $\qquad$ $a_5 - a_4 = 17 - 14 = 3$

NOTE: The difference between each term and its preceding term is 3.

**Answer:** Yes. The common difference is 3.

3b. 12, 7, 2, $-3$, $-8$, . . . . Back to Problem 3.

$a_2 - a_1 = 7 - 12 = -5$ $\qquad$ $a_3 - a_2 = 2 - 7 = -5$

$a_4 - a_3 = -3 - 2 = -5$ $\qquad$ $a_5 - a_4 = -8 - (-3)\ -8 + 3 = -5$

NOTE: The difference between each term and its preceding term is $-5$.

**Answer:** Yes. The common difference is $-5$.

3c. 1, 4, 9, 16, 25, . . . . Back to Problem 3.

$a_2 - a_1 = 4 - 1 = 3$ $a_3 - a_2 = 9 - 4 = 5$

This sequence is not arithmetic. The difference between the second term and the first term is 3. However, the difference between the third term and the second term is 5.

**Answer:** No

4a. $a_1 = -3$ and $d = 7$ Back to Problem 4.

**Answer:** $-3$, 4, 11, 18, 25

4b. $a_1 = 10$ and $d = -4$ Back to Problem 4.

**Answer:** 10, 6, 2, $-2$, $-6$

5a. $b_1 = 28$ and $d = 12$. Find $b_{15}$. Back to Problem 5.

$$b_n = b_1 + (n - 1)d \Rightarrow b_n = 28 + (n - 1)12$$

Simplifying, we have that

$$b_n = 28 + (n - 1)12 = 28 + 12n - 12 = 12n + 16.$$

Thus, $b_{15} = 12(15) + 16 = 180 + 16 = 196$

**Answer:** $b_n = 12n + 16$, $b_{15} = 196$

5b. $b_1 = 12$ and $d = -5$. Find $b_{201}$. Back to Problem 5.

$b_n = b_1 + (n - 1)d \Rightarrow b_n = 12 + (n - 1)(-5)$

Simplifying, we have that

$b_n = 12 + (n - 1)(-5) = 12 - 5n + 5 = 17 - 5n$.

Thus, $b_{201} = 17 - 5(201) = 17 - 1005 = -988$

**Answer:** $b_n = 17 - 5n$, $b_{201} = -988$

NOTE: This is the sequence that we were given in Problem 1b above: 12, 7, 2, $-3$, $-8$, . . . .

6. $c_1 = -8$ and $c_{25} = 136$. Find $c_{10}$. Back to Problem 6.

$c_n = c_1 + (n - 1)d \Rightarrow c_n = -8 + (n - 1)d$ since $c_1 = -8$

Then $c_{25} = -8 + (25 - 1)d = -8 + 24d$. We were given that $c_{25} = 136$.

Thus, $c_{25} = -8 + 24d$ and $c_{25} = 136 \Rightarrow 136 = -8 + 24d \Rightarrow 144 = 24d$

$\Rightarrow d = 6$.

Thus, $c_n = -8 + (n-1)d$ and $d = 6 \Rightarrow c_n = -8 + (n-1)6$

Simplifying, we have that

$c_n = -8 + (n-1)6 = -8 + 6n - 6 = 6n - 14$.

Thus, $c_{10} = 6(10) - 14 = 60 - 14 = 46$

**Answer:** $c_{10} = 46$

7. $a_1 = 57$ and $a_{100} = -636$. Find $a_{35}$. Back to Problem 7.

$a_n = a_1 + (n-1)d \Rightarrow a_n = 57 + (n-1)d$ since $a_1 = 57$

Then $a_{100} = 57 + (100-1)d = 57 + 99d$. We were given that $a_{100} = -636$.

Thus, $a_{100} = 57 + 99d$ and $a_{100} = -636 \Rightarrow -636 = 57 + 99d \Rightarrow$

$-693 = 99d \Rightarrow -231 = 33d \Rightarrow d = -7$.

Thus, $a_n = 57 + (n-1)d$ and $d = -7 \Rightarrow a_n = 57 + (n-1)(-7)$

Simplifying, we have that

$$a_n = 57 + (n - 1)(-7) = 57 - 7n + 7 = 64 - 7n.$$

Thus, $a_{35} = 64 - 7(35) = 64 - 245 = -181$

**Answer:** $a_{35} = -181$

8. 16, 20, 24, 28, . . . . , 184 Back to Problem 8.

NOTE: $a_1 = 16$ and $d = 4$

Find the value of *n* so that $a_n = 184$

$$a_n = a_1 + (n - 1)d \Rightarrow a_n = 16 + (n - 1)4$$

Simplifying, we have that

$$a_n = 16 + (n - 1)4 = 16 + 4n - 4 = 4n + 12$$

$a_n = 4n + 12$ and $a_n = 184 \Rightarrow 184 = 4n + 12 \Rightarrow 172 = 4n \Rightarrow$

$n = 43$

**Answer:** 43

Notation: The symbol $Z^+$ is used to denote the set of positive integers. Thus, $Z^+ = \{1, 2, 3, .....\}$.

**Definition** A sequence is a function whose domain is the set $Z^+ = \{1, 2, 3, .....\}$.

Up to this point, we have worked with functions of a real variable. That is, the domain of the function was either the set of real numbers or was a subset of the set of real numbers.

**Example** Consider the sequence $f(n) = \sqrt{n + 3}$.

The domain of this sequence is $Z^+$, and we have that

$$\begin{aligned} f(1) &= \sqrt{4} = 2 \\ f(2) &= \sqrt{5} \\ f(3) &= \sqrt{6} \\ f(4) &= \sqrt{7} \\ f(5) &= \sqrt{8} = 2\sqrt{2} \\ &\vdots \end{aligned}$$

If $f$ is a sequence, then for each positive integer $n$ in the domain of the sequence, there corresponds a real number $f(n)$ in the range of the sequence. Then the numbers in the range of the sequence would be

$$f(1),\ f(2),\ f(3),\ f(4)\ .\ .\ .\ .\ .\ .\ ,\ f(n),\ .\ .\ .\ .\ .\ .$$

and the number $f(1)$ is called the first term of the sequence, $f(2)$ is called the second term of the sequence, $f(3)$ is called the third term of the sequence, $f(4)$ is called the fourth term of the sequence, and $f(n)$ is called the $n$ th term of the sequence.

NOTATION: When working with a sequence, it is customary to use a subscript notation. Thus, instead of writing $f(n)$, we write $f_n$. Thus, for our example above, instead of having $f(n) = \sqrt{n+3}$, we would have $f_n = \sqrt{n+3}$. Then we would have written the following

$$
\begin{aligned}
&f_1 = \sqrt{4} = 2 \quad \text{instead of} \quad f(1) = \sqrt{4} = 2 \\
&f_2 = \sqrt{5} \quad \text{instead of} \quad f(2) = \sqrt{5} \\
&f_3 = \sqrt{6} \quad \text{instead of} \quad f(3) = \sqrt{6} \\
&f_4 = \sqrt{7} \quad \text{instead of} \quad f(4) = \sqrt{7} \\
&f_5 = \sqrt{8} = 2\sqrt{2} \quad \text{instead of} \quad f(5) = \sqrt{8} = 2\sqrt{2} \\
&\qquad \vdots
\end{aligned}
$$

COMMENT: The most common names for functions of a real variable are *f*, *g*, *h*, and *k*. The most common names for sequences are *a*, *b*, and *c*.

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