Pre-Class Problems 21 for Monday, April 16, and Wednesday, April 18

These are the type of problems that you will be working on in class on Wednesday.

The standard form for the equation of a hyperbola, whose center is at the origin, is given by either $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, where a > 0 and b > 0

For the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the vertices of the hyperbola, are given by $(\pm a, 0)$. The vertices of this hyperbola are also the *x*-intercepts of the graph of the hyperbola. The graph of the hyperbola has no *y*-intercepts. The two fixed points, which are called foci, of the hyperbola are given by $(\pm c, 0)$, where $c^2 = a^2 + b^2$. Note that the vertices and the foci of the hyperbola lie on the *x*-axis. The asymptotes of the hyperbola are given by $y = \pm \frac{b}{a}x$.

For the equation $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, the vertices of the hyperbola, are given by $(0, \pm a)$. The vertices of this hyperbola are also the *y*-intercepts of the graph of the hyperbola. The graph of the hyperbola has no *x*-intercepts. The two fixed points, which are called foci, of the hyperbola are given by $(0, \pm c)$, where $c^2 = a^2 + b^2$. Note that the vertices and the foci of the hyperbola lie on the *y*-axis. The asymptotes of the hyperbola are given by $y = \pm \frac{a}{b}x$.

NOTE: The slope of a line is the change in y divided by the change in x. That is, $m = \frac{\Delta y}{\Delta x}$. The numerical slope of the asymptotes in both cases is the square root of the number under the y^2 term divided by the square root of the number under the x^2 term. Use the slope formula to help you remember the order of y over x.

The standard form for the equation of a hyperbola, whose center is at the point

$$(h, k)$$
, is given by either $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or
 $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, where $a > 0$ and $b > 0$

For the equation $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, the center of the hyperbola is the point (h, k). The vertices of the hyperbola, are given by $(\pm a, 0) + (h, k) = (h \pm a, k)$. The two fixed points, which are called foci, of the hyperbola are given by $(\pm c, 0) + (h, k) = (h \pm c, k)$, where $c^2 = a^2 + b^2$. Note that the vertices and the foci of the hyperbola lie on the line y = k. The asymptotes of the hyperbola are given by $y - k = \pm \frac{b}{a}(x-h)$.

For the equation $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, the center of the hyperbola is the point (h, k). The vertices of the hyperbola, are given by $(0, \pm a) + (h, k) = (h, k \pm a)$. The two fixed points, which are called foci, of the hyperbola are given by $(0, \pm c) + (h, k) = (h, k \pm c)$, where $c^2 = a^2 + b^2$. Note that the vertices and the foci of the hyperbola lie on the line x = h. The asymptotes of the hyperbola are given by $y - k = \pm \frac{a}{b}(x - h)$.

Summary

1. For the following hyperbolas, identify the center, the vertices, the foci, and the asymptotes. Sketch the graph of the hyperbola.

a.
$$\frac{x^2}{36} - \frac{y^2}{64} = 1$$

b. $25y^2 - 4x^2 = 100$
c. $9x^2 - 16y^2 - 36x - 160y - 508 = 0$

2. Write the standard form of the equation of the hyperbola with vertices of (-4, 0) and (4, 0) and foci of $(-\sqrt{19}, 0)$ and $(\sqrt{19}, 0)$.

The standard form for the equation of a parabola, whose vertex is at the origin, is given by either $x^2 = 4py$ or $y^2 = 4px$, where $p \neq 0$.

For the equation $x^2 = 4py$, the vertex of the parabola is the origin (0, 0). The fixed point, which is called the focus, of the parabola is given by (0, p). The equation of the fixed line, which is called the directrix, is given by the equation y = -p. The axis of symmetry of the parabola is the line x = 0, which is the *y*-axis. Note that the vertex and the focus of the parabola lie on the axis of symmetry. If p > 0, then the graph of the parabola opens upward, and if p < 0, then the graph of the parabola opens downward.

For the equation $y^2 = 4 px$, the vertex of the parabola is the origin (0, 0). The fixed point, which is called the focus, of the parabola is given by (p, 0). The equation of the fixed line, which is called the directrix, is given by the equation x = -p. The axis of symmetry of the parabola is the line y = 0, which is the *x*-axis. Note that the vertex and the focus of the parabola lie on the axis of symmetry. If p > 0, then the graph of the parabola opens to the right, and if p < 0, then the graph of the left.

The standard form for the equation of a hyperbola, whose center is at the point (h, k), is given by either $(x - h)^2 = 4p(y - k)$ or $(y - k)^2 = 4p(x - h)$, where $p \neq 0$.

For the equation $(x - h)^2 = 4p(y - k)$, the vertex of the parabola is the point (h, k). The fixed point, which is called the focus, of the parabola is given by (0, p) + (h, k) = (h, p + k). The equation of the fixed line, which is called the directrix, is given by the equation $y - k = -p \Rightarrow y = k - p$. The axis of symmetry of the parabola is the line $x - h = 0 \Rightarrow x = h$. Note that the vertex and the focus of the parabola lie on the axis of symmetry. If p > 0, then the graph of the parabola opens upward, and if p < 0, then the graph of the parabola opens

For the equation $(y - k)^2 = 4p(x - h)$, the vertex of the parabola is the point (h, k). The fixed point, which is called the focus, of the parabola is given by (p, 0) + (h, k) = (p + h, k). The equation of the fixed line, which is called the directrix, is given by the equation $x - h = -p \Rightarrow x = h - p$. The axis of symmetry of the parabola is the line $y - k = 0 \Rightarrow y = k$. Note that the vertex and the focus of the parabola lie on the axis of symmetry. If p > 0, then the graph of the parabola opens to the right, and if p < 0, then the graph of the parabola opens to the left.

Summary

- 3. For the following parabolas, identify the vertex, the focus, the directrix, and the axis of symmetry. Sketch the graph of the parabola.
 - a. $x^2 = 12y$ b. $(x + 4)^2 = -3(y - 2)$

c.
$$y^2 = -6x$$
 d. $(y + 1)^2 = 16(x - 7)$

Problems available in the textbook: Page 662 ... 13 - 50 and Examples 1 - 5 starting on page 653. Page 676 ... 15 - 26, 27cd - 44cd, 45 - 52 (don't identify the focal diameter), 61 - 68 and Examples 1 - 6 starting on page 669.

SOLUTIONS:

1a. $\frac{x^2}{36} - \frac{y^2}{64} = 1$ Back to <u>Problem 1</u>.

Center: (0, 0)

Vertices: $(\pm a, 0) = (\pm 6, 0)$

NOTE:
$$a^2 = 36 \implies a = 6$$
 since $a > 0$

Foci: $(\pm c, 0) = (\pm 10, 0)$

 $c^{2} = a^{2} + b^{2} = 36 + 64 = 100 \implies c = 10$ since c > 0

NOTE: $a^2 = 36$ and $b^2 = 64$

Asymptotes: $y = \pm \frac{8}{6}x = \pm \frac{4}{3}x$

NOTE: The slope of a line is the change in y divided by the change in x. That is, $m = \frac{\Delta y}{\Delta x}$. The numerical slope of the asymptotes is the square root of the number under the y^2 term divided by the square root of the number under the x^2 term. Use the slope formula to help you remember y over x. Sketch: Will be shown in class.

1b.
$$25y^2 - 4x^2 = 100$$
 Back to Problem 1.

The equation of this hyperbola is not in standard form. To obtain the standard form of the hyperbola, we need to divide both sides of the equation by 100 in order to get 1 on the right side of the equation.

$$25y^{2} - 4x^{2} = 100 \implies \frac{25y^{2}}{100} - \frac{4x^{2}}{100} = \frac{100}{100} \implies \frac{y^{2}}{4} - \frac{x^{2}}{25} = 1$$

Center: (0, 0)

Vertices: $(0, \pm a) = (0, \pm 2)$ NOTE: $a^2 = 4 \implies a = 2$ since a > 0

Foci: $(0, \pm c) = (0, \pm \sqrt{29})$

 $c^{2} = a^{2} + b^{2} = 4 + 25 = 29 \implies c = \sqrt{29}$ since c > 0

NOTE: $a^2 = 4$ and $b^2 = 100$

Asymptotes: $y = \pm \frac{2}{5}x$

NOTE: The slope of a line is the change in y divided by the change in x. That is, $m = \frac{\Delta y}{\Delta x}$. The numerical slope of the asymptotes is the square root of the number under the y^2 term divided by the square root of the number under the x^2 term. Use the slope formula to help you remember y over x.

Sketch: Will be shown in class.

1c.
$$9x^2 - 16y^2 - 36x - 160y - 508 = 0$$

 $9x^2 - 16y^2 - 36x - 160y - 508 = 0$
 $9x^2 - 36x + __ - 16y^2 - 160y + __ = 508$
 $9(x^2 - 4x + __) - 16(y^2 + 10y + __) = 508$
 $9(x^2 - 4x + __) - 16(y^2 + 10y + __) = 508 + 36 - 400$
 $\downarrow Half$ $\downarrow Half$
 $2 \qquad 5$
 $\downarrow Square$ $\downarrow Square$
 $4 \qquad 25$
 $9(x - 2)^2 - 16(y + 5)^2 = 144 \Rightarrow \frac{9(x - 2)^2}{144} - \frac{16(y + 5)^2}{144} = \frac{144}{144} \Rightarrow$

$$\frac{(x-2)^2}{16} - \frac{(y+5)^2}{9} = 1$$

Center: (2, -5)

Vertices: (-2, -5), (6, -5)

 $(\pm a, 0) + (h, k) = (\pm 4, 0) + (2, -5) = (2 \pm 4, -5)$ NOTE: $a^2 = 16 \implies a = 4$ since a > 0

Foci: (-3, -5), (7, -5) $(\pm c, 0) + (h, k) = (\pm 5, 0) + (2, -5) = (2 \pm 5, -5)$ $c^2 = a^2 + b^2 = 16 + 9 = 25 \implies c = 5$ since c > 0NOTE: $a^2 = 16$ and $b^2 = 9$

Asymptotes: $y + 5 = \pm \frac{3}{4}(x - 2)$

NOTE: The slope of a line is the change in y divided by the change in x. That is, $m = \frac{\Delta y}{\Delta x}$. The numerical slope of the asymptotes is the square root of the number under the y^2 term divided by the square root of the number under the x^2 term. Use the slope formula to help you remember y over x.

Sketch: Will be shown in class.

2. Vertices: (-4, 0) and (4, 0)Foci: $(-\sqrt{19}, 0)$ and $(\sqrt{19}, 0)$ Back to Problem 2.

Since the vertices are given by the points $(\pm a, 0)$, then a = 4.

Since the foci are given by the points $(\pm c, 0)$, then $c = \sqrt{19}$.

Since
$$c^2 = a^2 + b^2$$
 and $a = 4$ and $c = \sqrt{19}$, then
 $c^2 = a^2 + b^2 \implies 19 = 16 + b^2 \implies b^2 = 3$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \implies \frac{x^2}{16} - \frac{y^2}{3} = 1$$

Answer:
$$\frac{x^2}{16} - \frac{y^2}{3} = 1$$

3a.
$$x^2 = 12 y$$

Back to Problem 3.

Vertex: (0, 0)

Opens: Up

NOTE: $4p = 12 \implies p = 3 > 0$

Axis of Symmetry: x = 0 (y-axis)

Focus: (0, p) = (0, 3)

NOTE: $4p = 12 \implies p = 3$

Directrix: y = -3

NOTE:
$$y = -p = -3$$

Sketch: Will be shown in class.

3b.
$$(x + 4)^2 = -3(y - 2)$$
 Back to Problem 3.

Vertex: (-4, 2)

Opens: Down

NOTE:
$$4p = -3 \implies p = -\frac{3}{4} < 0$$

Axis of Symmetry: x = -4

$$x + 4 = 0 \implies x = -4$$

Focus: $\left(-4,\frac{5}{4}\right)$

$$(0, p) + (h, k) = \left(0, -\frac{3}{4}\right) + (-4, 2) = \left(-4, -\frac{3}{4} + \frac{8}{4}\right)$$

NOTE:
$$4p = -3 \implies p = -\frac{3}{4}$$

Directrix: $y = \frac{11}{4}$

$$y - 2 = -p = \frac{3}{4} \implies y = \frac{3}{4} + \frac{8}{4}$$

NOTE: $p = -\frac{3}{4}$

Sketch: Will be shown in class.

3c. $y^2 = -6x$

Vertex: (0, 0)

Opens: Left

NOTE:
$$4p = -6 \implies p = -\frac{3}{2} < 0$$

Axis of Symmetry: y = 0 (*x*-axis)

Focus:
$$(p, 0) = \left(-\frac{3}{2}, 0\right)$$

NOTE:
$$4p = -6 \implies p = -\frac{6}{4} = -\frac{3}{2}$$

Directrix: $x = \frac{3}{2}$

NOTE:
$$x = -p = \frac{3}{2}$$

Sketch: Will be shown in class.

3d.
$$(y + 1)^2 = 16(x - 7)$$
 Back to Problem 3.

Vertex: (7, -1)

Opens: Right

NOTE:
$$4p = 16 \implies p = 4 > 0$$

Axis of Symmetry: y = -1

$$y + 1 = 0 \implies y = -1$$

Focus: (11, -1)

(p, 0) + (h, k) = (4, 0) + (7, -1) = (11, -1)

NOTE: $4p = 16 \implies p = 4$

Directrix: x = 3

$$x - 7 = -p = -4 \implies x = 3$$

NOTE: p = 4

Sketch: Will be shown in class.

For the following, a > 0 and b > 0

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Hyperbola:	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Center:	(0,0)	(h, k)	(0,0)	(h, k)
Vertices:	$(\pm a, 0)$	$(\pm a, 0) + (h, k) = (h \pm a, k)$	$(0, \pm a)$	$(0, \pm a) + (h, k) = (h, k \pm a)$
F ·	$(\pm c, 0)$	$(\pm c, 0) + (h, k) = (h \pm c, k)$	$(0, \pm c)$	$(0, \pm c) + (h, k) = (h, k \pm c)$
Foci:	$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$	$c^2 = a^2 + b^2$
Asymptotes:	$y = \pm \frac{b}{a}x$	$y - k = \pm \frac{b}{a}(x - h)$	$y = \pm \frac{a}{b}x$	$y - k = \pm \frac{a}{b}(x - h)$

For the following, $p \neq 0$

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Parabola:	$x^2 = 4 p y$	$(x-h)^2 = 4p(y-k)$	$y^2 = 4 p x$	$(y-k)^2 = 4p(x-h)$
Vertex:	(0,0)	(h, k)	(0,0)	(h, k)
Opens:	Up if $p > 0$	Up if $p > 0$	Right if $p > 0$	Right if $p > 0$
	Down if $p < 0$	Down if $p < 0$	Left if $p < 0$	Left if $p < 0$
Focus:	(0, <i>p</i>)	(0, p) + (h, k) = (h, p + k)	(<i>p</i> , 0)	(p, 0) + (h, k) = (p + h, k)
Directrix:	y = -p	$y - k = -p \implies y = k - p$	x = -p	$x - h = -p \implies x = h - p$
Axis of Symmetry:	y-axis $(x = 0)$	$x - h = 0 \implies x = h$	x-axis ($y = 0$)	$y - k = 0 \implies y = k$

NOTE: The vertex and the focus of a parabola lie on the axis of symmetry. The directrix of a parabola is perpendicular to the axis of symmetry.