Pre-Class Problems 20 for Wednesday, April 11

These are the type of problems that you will be working on in class.

The standard form for the equation of an ellipse, whose center is at the origin, is given by either $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$, where $a > b > 0$.

For the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, the center of the ellipse is $(0,0)$. The bigger number of a^2 is in the denominator of the first fraction whose numerator is x^2. Thus, the major axis of the ellipse is the *x*-axis and its endpoints, which are called the vertices of the ellipse, are given by $(\pm a,\ 0)$. The vertices of the ellipse are also the *x*-intercepts of the ellipse. The minor axis is the *y*-axis and its endpoints are given by $(0,\ \pm b)$. The endpoints of the minor axis are the *y*-intercepts of the ellipse. The two fixed points, which are called foci, lie on the major axis of the ellipse and are given by $(\pm c,\ 0)$, where $c^2 = a^2 - b^2$. Note that the vertices and the foci of the ellipse lie on the major axis, which is the *x*-axis. Recall that the equation for the horizontal *x*-axis is given by $y = 0$ and the equation for the vertical *y*-axis is given by $x = 0$.

For the equation $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$, the center of the ellipse is $(0,0)$. The bigger number of a^2 is in the denominator of the second fraction whose numerator is y^2. Thus, the major axis of the ellipse is the *y*-axis and its endpoints, which are called the vertices of the ellipse, are given by $(0,\ \pm a)$. The vertices of the ellipse are also the *y*-intercepts of the ellipse. The minor axis is the *x*-axis and its endpoints are given by $(\pm b,\ 0)$. The endpoints of the minor axis are the *x*-intercepts of the ellipse. The two fixed points, which are called foci, lie on the major axis of the ellipse and are given by $(0,\ \pm c)$, where $c^2 = a^2 - b^2$. Note that the vertices and the foci of the ellipse lie on the major axis, which is the *y*-axis. Recall that the equation for the vertical y-axis is given by $x = 0$ and the equation for the horizontal x-axis is given by $y = 0$.

The standard form for the equation of an ellipse, whose center is the point (h,k), is given by either $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ or $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$, where $a > b > 0$.

For the equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, the center of the ellipse is (h,k). The bigger number of a^2 is in the denominator of the first fraction whose numerator is $(x-h)^2$. Thus, the major axis of the ellipse is the horizontal line $y = k$ and its endpoints, which are called the vertices of the ellipse, are given by $(\pm a,0) + (h,k) = (h \pm a,k)$. The minor axis is the vertical line $x = h$ and its endpoints are given by $(0, \pm b) + (h,k) = (h, k \pm b)$. The two fixed points, which are called foci, lie on the major axis of the ellipse and are given by $(\pm c,0) + (h,k) = (h \pm c,k)$, where $c^2 = a^2 - b^2$. Note that the vertices and the foci of the ellipse lie on the major axis, which is the horizontal line $y = k$.

For the equation $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$, the center of the ellipse is (h,k). The bigger number of a^2 is in the denominator of the second fraction whose numerator is $(y-k)^2$. Thus, the major axis of the ellipse is the vertical line $x = h$ and its endpoints, which are called the vertices of the ellipse, are given by $(0, \pm a) + (h,k) = (h, k \pm a)$. The minor axis is the horizontal line $y = k$ and its endpoints are given by $(\pm b,0) + (h,k) = (h \pm b,k)$. The two fixed points, which are called foci, lie on the major axis of the ellipse and are given by $(0, \pm c) + (h,k) = (h, k \pm c)$, where $c^2 = a^2 - b^2$. Note that the vertices and the foci of the ellipse lie on the major axis, which is the vertical line $x = h$.

Summary

1. For the following ellipses, identify the center, the major axis, the vertices, the length of major axis, the foci, the minor axis, the endpoints of minor axis, and the length of the minor axis. Sketch the graph of the ellipse.

a. $\frac{x^2}{49} + \frac{y^2}{25} = 1$ b. $9x^2 + 4y^2 = 36$

c. $\frac{4x^2}{9} + \frac{25y^2}{7} = 1$ d. $36(x - 3)^2 + 16(y + 5)^2 = 576$

e. $\frac{(x + 2)^2}{64} + (y - 6)^2 = 1$

2. Write the equation of the ellipse $25x^2 + 9y^2 + 150x - 18y + 9 = 0$ in standard form.

3. Write the standard form of the equation of the ellipse with vertices of $(0, -6)$ and $(0, 6)$ and foci of $(0, -4)$ and $(0, 4)$.

Problems available in the textbook: Page 645 … 11 – 55 and Examples 1 – 5 starting on page 637.

SOLUTIONS:

1a. $\frac{x^2}{49} + \frac{y^2}{25} = 1$ Back to Problem 1.

Center: $(0, 0)$

Major Axis: x-axis

Vertices: $(\pm 7,\ 0)$

NOTE: $a^2 = 49 \Rightarrow a = 7$ since $a > 0$

Length of the Major Axis: 14

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = 14$ since $a = 7$.

Foci: $(\pm 2\sqrt{6},\ 0)$

$$c^2 = a^2 - b^2 = 49 - 25 = 24 \Rightarrow c = \sqrt{24} = 2\sqrt{6} \text{ since } c > 0$$

NOTE: $a^2 = 49$ and $b^2 = 25$

Minor Axis: y-axis

Endpoints of the Minor Axis: $(0,\ \pm 5)$

NOTE: $b^2 = 25 \Rightarrow b = 5$ since $b > 0$

Length of the Minor Axis: 10

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = 10$ since $b = 5$.

Sketch: Will be shown in class.

1b. $9x^2 + 4y^2 = 36$ Back to Problem 1.

The equation of this ellipse is not in standard form. To obtain the standard form of the ellipse, we need to divide both sides of the equation by 36 in order to get 1 on the right side of the equation.

$$9x^2 + 4y^2 = 36 \;\Rightarrow\; \frac{9x^2}{36} + \frac{4y^2}{36} = \frac{36}{36} \;\Rightarrow\; \frac{x^2}{4} + \frac{y^2}{9} = 1$$

Center: $(0, 0)$

Major Axis: y-axis

Vertices: $(0, \pm 3)$

NOTE: $a^2 = 9 \Rightarrow a = 3$ since $a > 0$

Length of the Major Axis: 6

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = 6$ since $a = 3$.

Foci: $(0, \pm\sqrt{5})$

$$c^2 = a^2 - b^2 = 9 - 4 = 5 \Rightarrow c = \sqrt{5} \text{ since } c > 0$$

NOTE: $a^2 = 9$ and $b^2 = 4$

Minor Axis: x-axis

Endpoints of the Minor Axis: $(\pm 2,\ 0)$

NOTE: $b^2 = 4 \Rightarrow b = 2$ since $b > 0$

Length of the Minor Axis: 4

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = 4$ since $b = 2$.

Sketch: Will be shown in class.

1c. $\dfrac{4x^2}{9} + \dfrac{25y^2}{7} = 1$ Back to Problem 1.

Even though we have a 1 on the right side of the equation, the equation of this ellipse is not in standard form since the coefficients of the x^2 and y^2 terms are not 1.

$\dfrac{4x^2}{9} + \dfrac{25y^2}{7} = 1 \Rightarrow \dfrac{x^2}{\frac{9}{4}} + \dfrac{y^2}{\frac{7}{25}} = 1$ since $\dfrac{1}{\frac{9}{4}}$ is $\dfrac{4}{9}$ and $\dfrac{1}{\frac{7}{25}}$ is $\dfrac{25}{7}$.

Center: $(0, 0)$

Major Axis: x-axis

Vertices: $\left(\pm \frac{3}{2}, \ 0 \right)$

NOTE: $a^2 = \frac{9}{4} \Rightarrow a = \frac{3}{2}$ since $a > 0$

Length of the Major Axis: 3

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = 3$ since $a = \frac{3}{2}$.

Foci: $\left(\pm \frac{\sqrt{197}}{10}, \ 0 \right)$

$c^2 = a^2 - b^2 = \frac{9}{4} - \frac{7}{25} = \frac{225}{100} - \frac{28}{100} = \frac{197}{100} \Rightarrow c = \frac{\sqrt{197}}{10}$ since $c > 0$

NOTE: $a^2 = \frac{9}{4}$ and $b^2 = \frac{7}{25}$

Minor Axis: y-axis

Endpoints of the Minor Axis: $\left(0, \ \pm \frac{\sqrt{7}}{5} \right)$

NOTE: $b^2 = \frac{7}{25} \Rightarrow b = \frac{\sqrt{7}}{5}$ since $b > 0$

Length of the Minor Axis: $\dfrac{2\sqrt{7}}{5}$

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = \dfrac{2\sqrt{7}}{5}$ since $b = \dfrac{\sqrt{7}}{5}$.

Sketch: I don't think that I will sketch this one.

1d. $36(x - 3)^2 + 16(y + 5)^2 = 576$ Back to Problem 1.

The equation of this ellipse is not in standard form. To obtain the standard form of the ellipse, we need to divide both sides of the equation by 576 in order to get 1 on the right side of the equation.

$$36(x - 3)^2 + 16(y + 5)^2 = 576 \Rightarrow \frac{36(x - 3)^2}{576} + \frac{16(y + 5)^2}{576} = \frac{576}{576} \Rightarrow$$

$$\frac{(x - 3)^2}{16} + \frac{(y + 5)^2}{36} = 1$$ (Yes. I used a calculator.)

Center: $(3, -5)$

Major Axis: Vertical: $x = 0 + 3 \Rightarrow x = 3$

NOTE: The equation $x = 0$ is the equation of the *y*-axis.

Vertices: $(3, -11)$, $(3, 1)$

$$(0, \pm 6) + (3, -5) = (3, -5 \pm 6)$$

NOTE: $a^2 = 36 \Rightarrow a = 6$ since $a > 0$

Length of the Major Axis: 12

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = 12$ since $a = 6$.

Foci: $(3, -5 - 2\sqrt{5})$, $(3, -5 + 2\sqrt{5})$

$$(0, \pm 2\sqrt{5}) + (3, -5) = (3, -5 \pm 2\sqrt{5})$$

$$c^2 = a^2 - b^2 = 36 - 16 = 20 \Rightarrow c = \sqrt{20} = 2\sqrt{5} \text{ since } c > 0$$

NOTE: $a^2 = 36$ and $b^2 = 16$

Minor Axis: Horizontal: $y = 0 + (-5) \Rightarrow y = -5$

NOTE: The equation $y = 0$ is the equation of the *x*-axis.

Endpoints of the Minor Axis: $(-1, -5)$, $(7, -5)$

$$(\pm 4, 0) + (3, -5) = (3 \pm 4, -5)$$

NOTE: $b^2 = 16 \Rightarrow b = 4$ since $b > 0$

Length of the Minor Axis: 8

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = 8$ since $b = 4$.

Sketch: Will be shown in class.

1e. $\dfrac{(x + 2)^2}{64} + (y - 6)^2 = 1$ Back to Problem 1.

Center: $(-2, 6)$

Major Axis: Horizontal: $y = 0 + 6 \Rightarrow y = 6$

NOTE: The equation $y = 0$ is the equation of the *x*-axis.

Vertices: $(-10, 6)$, $(6, 6)$

$$(\pm 8, 0) + (-2, 6) = (\pm 8 - 2, 6)$$

NOTE: $a^2 = 64 \Rightarrow a = 8$ since $a > 0$

Length of the Major Axis: 16

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = 16$ since $a = 8$.

Foci: $(-2 - 3\sqrt{7}, 6)$, $(-2 + 3\sqrt{7}, 6)$

$(\pm 3\sqrt{7}, 0) + (-2, 6) = (-2 \pm 3\sqrt{7}, 6)$

$c^2 = a^2 - b^2 = 64 - 1 = 63 \Rightarrow c = \sqrt{63} = 3\sqrt{7}$ since $c > 0$

NOTE: $a^2 = 64$ and $b^2 = 1$

Minor Axis: Vertical: $x = 0 + (-2) \Rightarrow x = -2$

NOTE: The equation $x = 0$ is the equation of the *y*-axis.

Endpoints of the Minor Axis: $(-2, 5)$, $(-2, 7)$

$(0, \pm 1) + (-2, 6) = (-2, 6 \pm 1)$

NOTE: $b^2 = 1 \Rightarrow b = 1$ since $b > 0$

Length of the Minor Axis: 2

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = 2$ since $b = 1$.

Sketch: Will be shown in class if there is time.

2. $25x^2 + 9y^2 + 150x - 18y + 9 = 0$ Back to Problem 2.

$25x^2 + 9y^2 + 150x - 18y + 9 = 0$

$25x^2 + 150x + \_\_\_\_ + 9y^2 - 18y + \_\_\_\_ = -9$

$$25(x^2 + 6x + \_\_\_) + 9(y^2 - 2y + \_\_\_) = -9$$

$$25(x^2 + 6x + \underline{\ 9\ }) + 9(y^2 - 2y + \underline{\ 1\ }) = -9 + 225 + 9$$

$\downarrow$ *Half* $\qquad\qquad$ $\downarrow$ *Half*

3 $\qquad\qquad$ 1

$\downarrow$ *Square* $\qquad\qquad$ $\downarrow$ *Square*

9 $\qquad\qquad$ 1

$$25(x + 3)^2 + 9(y - 1)^2 = 225 \Rightarrow \frac{25(x + 3)^2}{225} + \frac{9(y - 1)^2}{225} = \frac{225}{225} \Rightarrow$$

$$\frac{(x + 3)^2}{9} + \frac{(y - 1)^2}{25} = 1$$

Answer: $\dfrac{(x + 3)^2}{9} + \dfrac{(y - 1)^2}{25} = 1$

3. Vertices: $(0, -6)$ and $(0, 6)$

Back to Problem 3.

Foci: $(0, -4)$ and $(0, 4)$

Since the vertices of an ellipse lie on the major axis and the vertices are $(0, \pm 6)$, then the major axis is the *y*-axis. Also, since the vertices are given by the points $(0, \pm a)$, then $a = 6$.

The foci of an ellipse, where the major axis is the *y*-axis, are given by $(0, \pm c)$. Since the foci are the points $(0, \pm 4)$, then $c = 4$.

Since $c^2 = a^2 - b^2$ and $a = 6$ and $c = 4$, then

$$c^2 = a^2 - b^2 \Rightarrow 16 = 36 - b^2 \Rightarrow b^2 = 20$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \Rightarrow \frac{x^2}{20} + \frac{y^2}{36} = 1$$

Answer: $\frac{x^2}{20} + \frac{y^2}{36} = 1$

For the following, $a > b > 0 \Rightarrow a^2 > b^2$ Back to Top

Ellipse:	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Center:	$(0, 0)$	(h, k)	$(0, 0)$	(h, k)
Major Axis:	x-axis $(y = 0)$	horizontal $(y - k = 0 \Rightarrow y = k)$	y-axis $(x = 0)$	vertical $(x - h = 0 \Rightarrow x = h)$
Vertices:	$(\pm a, 0)$	$(\pm a, 0) + (h, k) = (h \pm a, k)$	$(0, \pm a)$	$(0, \pm a) + (h, k) = (h, k \pm a)$
Foci:	$(\pm c, 0)$ $c^2 = a^2 - b^2$	$(\pm c, 0) + (h, k) = (h \pm c, k)$ $c^2 = a^2 - b^2$	$(0, \pm c)$ $c^2 = a^2 - b^2$	$(0, \pm c) + (h, k) = (h, k \pm c)$ $c^2 = a^2 - b^2$
Length of Major Axis:	$2a$	$2a$	$2a$	$2a$
Minor Axis:	y-axis $(x = 0)$	vertical $(x - h = 0 \Rightarrow x = h)$	x-axis $(y = 0)$	horizontal $(y - k = 0 \Rightarrow y = k)$
Endpoints of the Major Axis:	$(0, \pm b)$	$(0, \pm b) + (h, k) = (h, k \pm b)$	$(\pm b, 0)$	$(\pm b, 0) + (h, k) = (h \pm b, k)$
Length of Minor Axis:	$2b$	$2b$	$2b$	$2b$