Pre-Class Problems 1 for Monday, January 22

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem letter or problem number.

1. Solve the following equations.

a. $\frac{3}{4}x + 7 = \frac{1}{2}x - \frac{11}{6}$

b. $0.15y - 6 = 0.75 - 0.5y$

2. Solve the following equations.

a. $\frac{1}{4} - \frac{7}{2t} = \frac{3}{t}$

b. $\frac{3}{5w} - \frac{2}{3} = \frac{1}{3w} - \frac{11}{15}$

c. $\frac{4x - 7}{6x} = \frac{x + 8}{3x} - \frac{19}{12}$

d. $\frac{3}{y - 3} - \frac{5}{8} = \frac{y}{y - 3}$

e. $\frac{15}{t - 4} = \frac{64}{t^2 - 16}$

f. $\frac{4}{w^2 - 36} - \frac{3}{w + 6} = \frac{11}{w - 6}$

g. $\frac{5}{x + 2} - \frac{8}{x - 4} = \frac{3x - 6}{x^2 - 2x - 8}$

h. $\frac{42}{3y^2 + 4y - 15} + \frac{2}{y + 3} = \frac{9}{3y - 5}$

i. $\frac{42}{3y^2 + 4y - 15} + \frac{2}{y + 3} = \frac{9}{5 - 3y}$

j. $\frac{3}{2x - 5} = \frac{2}{3 - 7x}$

k. $\frac{3}{3x + 4} = \frac{8}{8x - 5}$

3. Solve $V = \dfrac{h(b_1 + b_2)}{P}$ for b_2.

4. Write each expression in terms of i and simplify.

a. $\sqrt{-36}$ b. $\sqrt{-48}$ c. $-\sqrt{-23}$ d. $\sqrt{-3}\sqrt{-15}$

e. $\sqrt{-6}\sqrt{-24}$ f. $\sqrt{-16}\sqrt{-49}$ g. $\dfrac{\sqrt{-75}}{\sqrt{-3}}$ h. $\dfrac{\sqrt{-140}}{\sqrt{7}}$

5. Determine the real part and the imaginary part of the following complex numbers.

a. $5 - 8i$ b. $4i - 7$ c. $-\dfrac{2}{3} + \dfrac{1}{2}i$ d. 9 e. $-11i$

Additional problems available in the textbook: Page 92 … 43 – 102 and Examples 1, 2, 5 - 9 starting on page 83. Page 111 … 5 – 28 and Examples 1 and 2 starting on page 104.

Solutions:

1a. $\dfrac{3}{4}x + 7 = \dfrac{1}{2}x - \dfrac{11}{6}$ Back to Problem 1.

LCD(4, 2, 6) = 12

$$\frac{3}{4}x + 7 = \frac{1}{2}x - \frac{11}{6} \Rightarrow 12\left(\frac{3}{4}x + 7\right) = \left(\frac{1}{2}x - \frac{11}{6}\right)12 \Rightarrow$$

$$9x + 84 = 6x - 22 \Rightarrow 3x = -106 \Rightarrow x = -\frac{106}{3}$$

Answer: $x = -\frac{106}{3}$ or the set of solutions is $\left\{-\frac{106}{3}\right\}$

1b. $0.15y - 6 = 0.75 - 0.5y$ Back to Problem 1.

NOTE: $0.15 = \frac{15}{100}$, $0.75 = \frac{75}{100}$, $0.5 = \frac{5}{10}$

LCD(100, 10) = 100

$$0.15y - 6 = 0.75 - 0.5y \Rightarrow 100(0.15y - 6) = (0.75 - 0.5y)100 \Rightarrow$$

$$15y - 600 = 75 - 50y \Rightarrow 65y = 675 \Rightarrow y = \frac{675}{65} = \frac{135}{13}$$

Answer: $y = \frac{135}{13}$ or the set of solutions is $\left\{\frac{135}{13}\right\}$

2a. $\frac{1}{4} - \frac{7}{2t} = \frac{3}{t}$ Back to Problem 2.

NOTE: $t \neq 0$ LCD(4, 2t, t) = 4t

$$\frac{1}{4} - \frac{7}{2t} = \frac{3}{t} \Rightarrow 4t\left(\frac{1}{4} - \frac{7}{2t}\right) = \left(\frac{3}{t}\right)4t \Rightarrow t - 14 = 12 \Rightarrow t = 26$$

Answer: 26 or the set of solutions is {26}

2b. $\frac{3}{5w} - \frac{2}{3} = \frac{1}{3w} - \frac{11}{15}$ Back to Problem 2.

NOTE: $w \neq 0$ LCD($5w$, 3, $3w$, 15) $= 15w$

$$\frac{3}{5w} - \frac{2}{3} = \frac{1}{3w} - \frac{11}{15} \Rightarrow 15w\left(\frac{3}{5w} - \frac{2}{3}\right) = \left(\frac{1}{3w} - \frac{11}{15}\right)15w \Rightarrow$$

$$9 - 10w = 5 - 11w \Rightarrow w = -4$$

Answer: $w = -4$ or the set of solutions is $\{-4\}$

2c. $$\frac{4x - 7}{6x} = \frac{x + 8}{3x} - \frac{19}{12}$$ Back to Problem 2.

NOTE: $x \neq 0$ LCD($6x$, $3x$, 12) $= 12x$

$$\frac{4x - 7}{6x} = \frac{x + 8}{3x} - \frac{19}{12} \Rightarrow 12x\left(\frac{4x - 7}{6x}\right) = \left(\frac{x + 8}{3x} - \frac{19}{12}\right)12x \Rightarrow$$

$$2(4x - 7) = 4(x + 8) - 19x \Rightarrow 8x - 14 = 4x + 32 - 19x \Rightarrow$$

$$8x - 14 = -15x + 32 \Rightarrow 23x = 46 \Rightarrow x = 2$$

Answer: $x = 2$ or the set of solutions is $\{2\}$

2d. $$\frac{3}{y - 3} - \frac{5}{8} = \frac{y}{y - 3}$$ Back to Problem 2.

NOTE: $y \neq 3$ LCD($y - 3$, 8) $= 8(y - 3)$

$$\frac{3}{y - 3} - \frac{5}{8} = \frac{y}{y - 3} \Rightarrow 8(y - 3)\left(\frac{3}{y - 3} - \frac{5}{8}\right) = \left(\frac{y}{y - 3}\right)8(y - 3) \Rightarrow$$

$$24 - 5(y - 3) = 8y \Rightarrow 24 - 5y + 15 = 8y \Rightarrow 39 = 13y \Rightarrow y = 3$$

This equation does not have a solution because $y \neq 3$. If $y = 3$, then two of the fractions in the equation are undefined because you would have division by zero.

Answer: No solution, or the set of solutions is the empty set.

2e. $\dfrac{15}{t - 4} = \dfrac{64}{t^2 - 16}$ Back to Problem 2.

Since $t^2 - 16 = (t + 4)(t - 4)$, then $t \neq -4$, $t \neq 4$.

$$\frac{15}{t - 4} = \frac{64}{t^2 - 16} \Rightarrow \frac{15}{t - 4} = \frac{64}{(t + 4)(t - 4)}$$

$$\text{LCD}[t - 4, (t + 4)(t - 4)] = (t + 4)(t - 4)$$

$$\frac{15}{t - 4} = \frac{64}{(t + 4)(t - 4)} \Rightarrow$$

$$(t + 4)(t - 4)\left(\frac{15}{t - 4}\right) = \left[\frac{64}{(t + 4)(t - 4)}\right](t + 4)(t - 4) \Rightarrow$$

$$15(t + 4) = 64 \Rightarrow 15t + 60 = 64 \Rightarrow 15t = 4 \Rightarrow t = \frac{4}{15}$$

Answer: $t = \dfrac{4}{15}$ or the set of solutions is $\left\{\dfrac{4}{15}\right\}$

2f. $\dfrac{4}{w^2 - 36} - \dfrac{3}{w + 6} = \dfrac{11}{w - 6}$ Back to Problem 2.

Since $w^2 - 36 = (w + 6)(w - 6)$, then $w \neq -6$, $w \neq 6$.

$$\frac{4}{w^2 - 36} - \frac{3}{w + 6} = \frac{11}{w - 6} \Rightarrow \frac{4}{(w + 6)(w - 6)} - \frac{3}{w + 6} = \frac{11}{w - 6}$$

$$\text{LCD}[(w + 6)(w - 6), w + 6, w - 6)] = (w + 6)(w - 6)$$

$$\frac{4}{(w + 6)(w - 6)} - \frac{3}{w + 6} = \frac{11}{w - 6} \Rightarrow$$

$$(w + 6)(w - 6)\left[\frac{4}{(w + 6)(w - 6)} - \frac{3}{w + 6}\right] = \left(\frac{11}{w - 6}\right)(w + 6)(w - 6) \Rightarrow$$

$$4 - 3(w - 6) = 11(w + 6) \Rightarrow 4 - 3w + 18 = 11w + 66 \Rightarrow$$

$$\Rightarrow 22 - 3w = 11w + 66 \Rightarrow -44 = 14w \Rightarrow w = -\frac{44}{14} = -\frac{22}{7}$$

Answer: $w = -\dfrac{22}{7}$ or the set of solutions is $\left\{-\dfrac{22}{7}\right\}$

2g. $\dfrac{5}{x + 2} - \dfrac{8}{x - 4} = \dfrac{3x - 6}{x^2 - 2x - 8}$ Back to Problem 2.

Since $x^2 - 2x - 8 = (x + 2)(x - 4)$, then $x \neq -2$, $x \neq 4$.

$$\frac{5}{x + 2} - \frac{8}{x - 4} = \frac{3x - 6}{x^2 - 2x - 8} \Rightarrow \frac{5}{x + 2} - \frac{8}{x - 4} = \frac{3x - 6}{(x + 2)(x - 4)}$$

$$\text{LCD}[x+2,\ x-4,\ (x+2)(x-4)] = (x+2)(x-4)$$

$$\frac{5}{x+2} - \frac{8}{x-4} = \frac{3x-6}{(x+2)(x-4)} \Rightarrow$$

$$(x+2)(x-4)\left(\frac{5}{x+2} - \frac{8}{x-4}\right) = \left[\frac{3x-6}{(x+2)(x-4)}\right](x+2)(x-4) \Rightarrow$$

$$5(x-4) - 8(x+2) = 3x - 6 \Rightarrow 5x - 20 - 8x - 16 = 3x - 6 \Rightarrow$$

$$-3x - 36 = 3x - 6 \Rightarrow -30 = 6x \Rightarrow x = -5$$

Answer: $x = -5$ or the set of solutions is $\{-5\}$

2h. $\dfrac{42}{3y^2 + 4y - 15} + \dfrac{2}{y+3} = \dfrac{9}{3y-5}$ Back to Problem 2.

Since $3y^2 + 4y - 15 = (y+3)(3y-5)$, then $y \neq -3$, $x \neq \dfrac{5}{3}$.

$$\frac{42}{3y^2 + 4y - 15} + \frac{2}{y+3} = \frac{9}{3y-5} \Rightarrow \frac{42}{(y+3)(3y-5)} + \frac{2}{y+3} = \frac{9}{3y-5}$$

$$\text{LCD}[(y+3)(3y-5),\ y+3,\ 3y-5] = (y+3)(3y-5)$$

$$\frac{42}{(y+3)(3y-5)} + \frac{2}{y+3} = \frac{9}{3y-5} \Rightarrow$$

$$(y+3)(3y-5)\left[\frac{42}{(y+3)(3y-5)} + \frac{2}{y+3}\right] = \left(\frac{9}{3y-5}\right)(y+3)(3y-5) \Rightarrow$$

$$42 + 2(3y-5) = 9(y+3) \Rightarrow 42 + 6y - 10 = 9y + 27 \Rightarrow$$

$$6y + 32 = 9y + 27 \Rightarrow 5 = 3y \Rightarrow y = \frac{5}{3}$$

This equation does not have a solution because $y \neq \frac{5}{3}$. If $y = \frac{5}{3}$, then two of the fractions in the equation are undefined because you would have division by zero.

Answer: No solution, or the set of solutions is the empty set.

2i. $$\frac{42}{3y^2 + 4y - 15} + \frac{2}{y + 3} = \frac{9}{5 - 3y}$$

Back to Problem 2.

Since $3y^2 + 4y - 15 = (y + 3)(3y - 5)$, then $y \neq -3$, $y \neq \frac{5}{3}$.

NOTE: $5 - 3y = -(3y - 5)$

$$\frac{42}{3y^2 + 4y - 15} + \frac{2}{y + 3} = \frac{9}{5 - 3y} \Rightarrow \frac{42}{(y + 3)(3y - 5)} + \frac{2}{y + 3} = \frac{9}{-(3y - 5)}$$

$$\frac{42}{(y + 3)(3y - 5)} + \frac{2}{y + 3} = \frac{-9}{3y - 5}$$

$$\text{LCD}[(y + 3)(3y - 5),\ y + 3,\ 3y - 5] = (y + 3)(3y - 5)$$

$$\frac{42}{(y + 3)(3y - 5)} + \frac{2}{y + 3} = \frac{-9}{3y - 5} \Rightarrow$$

$$(y + 3)(3y - 5)\left[\frac{42}{(y + 3)(3y - 5)} + \frac{2}{y + 3}\right] = \left(\frac{-9}{3y - 5}\right)(y + 3)(3y - 5) \Rightarrow$$

$$42 + 2(3y - 5) = -9(y + 3) \Rightarrow 42 + 6y - 10 = -9y - 27 \Rightarrow$$

$$6y + 32 = -9y - 27 \Rightarrow 15y = -59 \Rightarrow y = -\frac{59}{15}$$

Answer: $y = -\dfrac{59}{15}$ or the set of solutions is $\left\{-\dfrac{59}{15}\right\}$

2j. $\dfrac{3}{2x - 5} = \dfrac{2}{3 - 7x}$ Back to Problem 2.

NOTE: $x \neq \dfrac{5}{2}$, $x \neq \dfrac{3}{7}$

LCD $(2x - 5, 3 - 7x) = (2x - 5)(3 - 7x)$

$$\frac{3}{2x - 5} = \frac{2}{3 - 7x} \Rightarrow$$

$$(2x - 5)(3 - 7x)\left(\frac{3}{2x - 5}\right) = \left(\frac{2}{3 - 7x}\right)(2x - 5)(3 - 7x) \Rightarrow$$

$$3(3 - 7x) = 2(2x - 5) \Rightarrow 9 - 21x = 4x - 10 \Rightarrow 19 = 25x \Rightarrow x = \frac{19}{25}$$

NOTE: We could have obtained the equation $3(3 - 7x) = 2(2x - 5)$ by cross multiplying. That is,

$$\frac{3}{2x - 5} = \frac{2}{3 - 7x}$$

Answer: $x = \dfrac{19}{25}$ or the set of solutions is $\left\{\dfrac{19}{25}\right\}$

2k. $\dfrac{3}{3t + 4} = \dfrac{8}{8t - 5}$ Back to Problem 2.

NOTE: $t \neq -\dfrac{4}{3}$, $t \neq \dfrac{5}{8}$

Cross multiplying, we obtain $3(8t - 5) = 8(3t + 4)$.

$3(8t - 5) = 8(3t + 4) \Rightarrow 24t - 15 = 24t + 32 \Rightarrow -15 = 32$

This is a false equation. Thus, the given equation does not have a solution.

Answer: No solution, or the set of solutions is the empty set.

3. $V = \dfrac{h(b_1 + b_2)}{P} \Rightarrow PV = h(b_1 + b_2) \Rightarrow PV = hb_1 + hb_2 \Rightarrow$

$PV - hb_1 = hb_2 \Rightarrow b_2 = \dfrac{PV - hb_1}{h}$

Answer: $b_2 = \dfrac{PV - hb_1}{h}$ Back to Problem 3.

4a. $\sqrt{-36} = \sqrt{-1}\sqrt{36} = i6 = 6i$

Answer: $6i$ Back to Problem 4.

4b. $\sqrt{-48} = \sqrt{-1}\sqrt{48} = i\sqrt{16 \cdot 3} = i\sqrt{16}\sqrt{3} = i4\sqrt{3} = 4i\sqrt{3}$

Answer: $4i\sqrt{3}$ **or** $4\sqrt{3}\ i$ Back to Problem 4.

4c. $-\sqrt{-23} = -\sqrt{-1}\ \sqrt{23} = -i\sqrt{23}$

Answer: $-i\sqrt{23}$ **or** $-\sqrt{23}\ i$ Back to Problem 4.

4d. $\sqrt{-3}\ \sqrt{-15} = i\sqrt{3}\ i\sqrt{15} = i^2\sqrt{45} = -1\sqrt{9}\ \sqrt{5} = -1\cdot 3\sqrt{5} = -3\sqrt{5}$

Answer: $-3\sqrt{5}$ Back to Problem 4.

4e. $\sqrt{-6}\ \sqrt{-24} = i\sqrt{6}\ i\sqrt{24} = i^2\sqrt{144} = -1\cdot 12 = -12$

Answer: -12 Back to Problem 4.

4f. $\sqrt{-16}\ \sqrt{-49} = i\sqrt{16}\ i\sqrt{49} = i^2 4\cdot 7 = -1\cdot 28 = -28$

Answer: -28 Back to Problem 4.

4g. $\dfrac{\sqrt{-75}}{\sqrt{-3}} = \dfrac{i\sqrt{75}}{i\sqrt{3}} = \dfrac{\sqrt{75}}{\sqrt{3}} = \sqrt{\dfrac{75}{3}} = \sqrt{25} = 5$

Answer: 5 Back to Problem 4.

4h. $\dfrac{\sqrt{-140}}{\sqrt{7}} = \dfrac{i\sqrt{140}}{\sqrt{7}} = i\,\dfrac{\sqrt{140}}{\sqrt{7}} = i\sqrt{\dfrac{140}{7}} = i\sqrt{20} = 2i\sqrt{5}$

Answer: $2i\sqrt{5}$ **or** $2\sqrt{5}\,i$ Back to Problem 4.

5a. $5 - 8i$ Back to Problem 5.

Answer: Real part: 5 Imaginary part: -8

5b. $4i - 7$ Back to Problem 5.

Answer: Real part: -7 Imaginary part: 4

5c. $-\dfrac{2}{3} + \dfrac{1}{2}i$ Back to Problem 5.

Answer: Real part: $-\dfrac{2}{3}$ Imaginary part: $\dfrac{1}{2}$

5d. 9 Back to Problem 5.

Answer: Real part: 9 Imaginary part: 0

5e. $-11i$ Back to Problem 5.

Answer: Real part: 0 Imaginary part: -11