Pre-Class Problems 19 for Monday, April 9

**These are the type of problems that you will be working on in class.**

**You can go to the solution for each problem by clicking on the problem letter.**

1. Solve the following systems of equations using Gaussian elimination.

a.
$$\begin{aligned} 2x + 3y - 5z &= -9 \\ 6x - 9y + 7z &= 5 \\ 4x - 3y + z &= -2 \end{aligned}$$

b.
$$\begin{aligned} x - 2y + 5z &= 9 \\ -4x + 7y - 23z &= -43 \\ 3x - 5y + 18z &= 34 \end{aligned}$$

c.
$$\begin{aligned} 3x - 5y + 7z &= -11 \\ 9x - 14y + 27z &= -30 \\ -12x + 23y - 10z &= 57 \end{aligned}$$

d.
$$\begin{aligned} 2x + 8y + 10z &= -14 \\ 3x - 11y - 9z &= 2 \\ 5x - 3y + z &= -17 \end{aligned}$$

e.
$$\begin{aligned} -x + 4y - 2z &= 7 \\ 3x - 12y + 6z &= -21 \\ -4x + 16y - 8z &= 28 \end{aligned}$$

2. The ordered triple $\left(\frac{4t - 11}{6}, \frac{11t - 16}{9}, t\right)$, where $t$ is any real number, is the solution to Problem 1a above. Find the solution to this system if

a. $t = 2$ b. $t = -3$ c. $t = -1$

3. The ordered triple $(23 - 11t, 7 - 3t, t)$, where $t$ is any real number, is the solution to Problem 1b above. Find the solution to this system if

a. $t = 0$ b. $t = 4$ c. $t = -2$

Problems available in the textbook: Page 582 … 17 – 44 and Examples 1, 3 – 5 starting on page 575.

**SOLUTIONS:**

1a.
$$\begin{aligned} 2x + 3y - 5z &= -9 \\ 6x - 9y + 7z &= 5 \\ 4x - 3y + z &= -2 \end{aligned}$$

Back to Problem 1.

First, form the augmented matrix for this system of equations:

$$\begin{bmatrix} 2 & 3 & -5 & -9 \\ 6 & -9 & 7 & 5 \\ 4 & -3 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & -5 & -9 \\ 6 & -9 & 7 & 5 \\ 4 & -3 & 1 & -2 \end{bmatrix} \xrightarrow[-2R_1 + R_3]{-3R_1 + R_2} \begin{bmatrix} 2 & 3 & -5 & -9 \\ 0 & -18 & 22 & 32 \\ 0 & -9 & 11 & 16 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_2 + R_3}$$

$$\begin{bmatrix} 2 & 3 & -5 & -9 \\ 0 & -18 & 22 & 32 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-\frac{1}{18}R_2} \begin{bmatrix} 2 & 3 & -5 & -9 \\ 0 & 1 & -\frac{11}{9} & -\frac{16}{9} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 3 reads $0x + 0y + 0z = 0$, which is a true equation.

Row 2 reads $y - \dfrac{11}{9}z = -\dfrac{16}{9}$

Let $z = t$, where $t$ is any real number. Then $y - \frac{11}{9}z = -\frac{16}{9} \Rightarrow$

$$y - \frac{11}{9}t = -\frac{16}{9} \Rightarrow y = \frac{11}{9}t - \frac{16}{9}$$

Row 1 reads $2x + 3y - 5z = -9$

Since $y = \frac{11}{9}t - \frac{16}{9}$ and $z = t$, then $2x + 3\left(\frac{11}{9}t - \frac{16}{9}\right) - 5t = -9 \Rightarrow$

$$2x + \frac{11}{3}t - \frac{16}{3} - \frac{15}{3}t = -9 \Rightarrow 2x - \frac{4}{3}t - \frac{16}{3} = -\frac{27}{3} \Rightarrow$$

$$2x = \frac{4}{3}t - \frac{11}{3} \Rightarrow x = \frac{2}{3}t - \frac{11}{6}$$

**Answer:** $\left(\frac{2}{3}t - \frac{11}{6}, \frac{11}{9}t - \frac{16}{9}, t\right) = \left(\frac{4t - 11}{6}, \frac{11t - 16}{9}, t\right)$, where $t$ is any real number

1b.
$$\begin{aligned} x - 2y + 5z &= 9 \\ -4x + 7y - 23z &= -43 \\ 3x - 5y + 18z &= 34 \end{aligned}$$

Back to Problem 1.

First, form the augmented matrix for this system of equations:

$$\begin{bmatrix} 1 & -2 & 5 & 9 \\ -4 & 7 & -23 & -43 \\ 3 & -5 & 18 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 5 & 9 \\ -4 & 7 & -23 & -43 \\ 3 & -5 & 18 & 34 \end{bmatrix} \xrightarrow[-3R_1 + R_3]{4R_1 + R_2} \begin{bmatrix} 1 & -2 & 5 & 9 \\ 0 & -1 & -3 & -7 \\ 0 & 1 & 3 & 7 \end{bmatrix} \xrightarrow{R_2 + R_3}$$

$$\begin{bmatrix} 1 & -2 & 5 & 9 \\ 0 & -1 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & -2 & 5 & 9 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 3 reads $0x + 0y + 0z = 0$, which is a true equation.

Row 2 reads $y + 3z = 7$

Let $z = t$, where $t$ is any real number. Then $y + 3z = 7 \Rightarrow$

$y + 3t = 7 \Rightarrow y = 7 - 3t$

Row 1 reads $x - 2y + 5z = 9$

Since $y = 7 - 3t$ and $z = t$, then $x - 2(7 - 3t) + 5t = 9 \Rightarrow$

$x - 14 + 6t + 5t = 9 \Rightarrow x - 14 + 11t = 9 \Rightarrow x = 23 - 11t$

**Answer:** $(23 - 11t,\ 7 - 3t,\ t)$, where $t$ is any real number

1c.
$$\begin{aligned} 3x - 5y + 7z &= -11 \\ 9x - 14y + 27z &= -30 \\ -12x + 23y - 10z &= 57 \end{aligned}$$

Back to Problem 1.

First, form the augmented matrix for this system of equations:

$$\begin{bmatrix} 3 & -5 & 7 & -11 \\ 9 & -14 & 27 & -30 \\ -12 & 23 & -10 & 57 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -5 & 7 & -11 \\ 9 & -14 & 27 & -30 \\ -12 & 23 & -10 & 57 \end{bmatrix} \xrightarrow[4R_1 + R_3]{-3R_1 + R_2} \begin{bmatrix} 3 & -5 & 7 & -11 \\ 0 & 1 & 6 & 3 \\ 0 & 3 & 18 & 13 \end{bmatrix} \xrightarrow{-3R_2 + R_3}$$

$$\begin{bmatrix} 3 & -5 & 7 & -11 \\ 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Row 3 reads $0x + 0y + 0z = 4$, which is a false equation.

**Answer:** No solution

1d.
$$\begin{aligned} 2x + 8y + 10z &= -14 \\ 3x - 11y - 9z &= 2 \\ 5x - 3y + z &= -17 \end{aligned}$$

Back to Problem 1.

First, form the augmented matrix for this system of equations:

$$\begin{bmatrix} 2 & 8 & 10 & -14 \\ 3 & -11 & -9 & 2 \\ 5 & -3 & 1 & -17 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 8 & 10 & -14 \\ 3 & -11 & -9 & 2 \\ 5 & -3 & 1 & -17 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & 4 & 5 & -7 \\ 3 & -11 & -9 & 2 \\ 5 & -3 & 1 & -17 \end{bmatrix} \xrightarrow[-5R_1 + R_3]{-3R_1 + R_2}$$

$$\begin{bmatrix} 1 & 4 & 5 & -7 \\ 0 & -23 & -24 & 23 \\ 0 & -23 & -24 & 18 \end{bmatrix} \xrightarrow{-R_2 + R_3} \begin{bmatrix} 1 & 4 & 5 & -7 \\ 0 & -23 & -24 & 23 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

Row 3 reads $0x + 0y + 0z = -5$, which is a false equation.

**Answer:** No solution

1e.
$$\begin{aligned} -x + 4y - 2z &= 7 \\ 3x - 12y + 6z &= -21 \\ -4x + 16y - 8z &= 28 \end{aligned}$$

Back to Problem 1.

First, form the augmented matrix for this system of equations:

$$\begin{bmatrix} -1 & 4 & -2 & 7 \\ 3 & -12 & 6 & -21 \\ -4 & 16 & -8 & 28 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & -2 & 7 \\ 3 & -12 & 6 & -21 \\ -4 & 16 & -8 & 28 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 1 & -4 & 2 & -7 \\ 3 & -12 & 6 & -21 \\ -4 & 16 & -8 & 28 \end{bmatrix} \xrightarrow[4R_1 + R_3]{-3R_1 + R_2}$$

$$\begin{bmatrix} 1 & -4 & 2 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 3 reads $0x + 0y + 0z = 0$, which is a true equation.

Row 2 reads $0x + 0y + 0z = 0$, which is a true equation.

Row 1 reads $x - 4y + 2z = -7$

Let $z = t$, where $t$ is any real number, and let $y = s$, where $s$ is any real number. Then $x - 4y + 2z = -7 \Rightarrow x - 4s + 2t = -7 \Rightarrow$ $x = 4s - 2t - 7$

**Answer:** $(4s - 2t - 7,\ s,\ t)$, where $s$ and $t$ are any real numbers

2a. $\left(\dfrac{4t - 11}{6}, \dfrac{11t - 16}{9}, t\right)$ Back to Problem 2.

$t = 2$: $\left(\dfrac{4t - 11}{6}, \dfrac{11t - 16}{9}, t\right) = \left(\dfrac{8 - 11}{6}, \dfrac{22 - 16}{9}, 2\right) =$

$\left(-\dfrac{3}{6}, \dfrac{6}{9}, 2\right) = \left(-\dfrac{1}{2}, \dfrac{2}{3}, 2\right)$

**Answer:** $\left(-\dfrac{1}{2}, \dfrac{2}{3}, 2\right)$

2b. $\left(\frac{4t-11}{6}, \frac{11t-16}{9}, t\right)$ Back to Problem 2.

$t=-3$: $\left(\frac{4t-11}{6}, \frac{11t-16}{9}, t\right) = \left(\frac{-12-11}{6}, \frac{-33-16}{9}, -3\right) =$

$\left(-\frac{23}{6}, -\frac{49}{9}, -3\right)$

**Answer:** $\left(-\frac{23}{6}, -\frac{49}{9}, -3\right)$

2c. $\left(\frac{4t-11}{6}, \frac{11t-16}{9}, t\right)$ Back to Problem 2.

$t=-1$: $\left(\frac{4t-11}{6}, \frac{11t-16}{9}, t\right) = \left(\frac{-4-11}{6}, \frac{-11-16}{9}, -1\right) =$

$\left(-\frac{15}{6}, -\frac{27}{9}, -1\right) = \left(-\frac{5}{2}, -3, -1\right)$

**Answer:** $\left(-\frac{5}{2}, -3, -1\right)$

3a. $(23-11t, 7-3t, t)$ Back to Problem 3.

$t=0$: $(23-11t, 7-3t, t) = (23-0, 7-0, 0) = (23, 7, 0)$

**Answer:** $(23,\ 7,\ 0)$

3b. $(23 - 11t,\ 7 - 3t,\ t)$ Back to Problem 3.

$t = 4$: $(23 - 11t,\ 7 - 3t,\ t) = (23 - 44,\ 7 - 12,\ 4) =$

$(-21,\ -5,\ 4)$

**Answer:** $(-21,\ -5,\ 4)$

3c. $(23 - 11t,\ 7 - 3t,\ t)$ Back to Problem 3.

$t = -2$: $(23 - 11t,\ 7 - 3t,\ t) = (23 + 22,\ 7 + 6,\ -2) =$

$(45,\ 13,\ -2)$

**Answer:** $(45,\ 13,\ -2)$