Pre-Class Problems 17 for Monday, April 2

Earn one bonus point because you checked the Pre-Class problems. Send me an email with PC17 in the Subject line.

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem letter or number.

Since any exponential function is one-to-one, then $b^u = b^v$ if and only if $u = v$.

1. Solve the following exponential equations.

a. $3^x = 81$ b. $5^{-t} = 25$ c. $2^{3x - 11} = 32$

d. $4^{x^3} = \frac{1}{16}$ e. $16^t = 64$ f. $9^{x + 4} = \frac{1}{27}$

g. $6^x = 12$ h. $5^t = \frac{2}{3}$ i. $7^{-x} = \frac{3}{4}$

j. $8^{5 - 2x} = 65$ k. $3^{7x + 4} = 49$ l. $2^{3x - 8} = 6^{2x + 9}$

m. $e^{2x} - 2e^x - 24 = 0$ n. $9^x + 16 = 10(3^x)$ o. $e^{-2t} = 5e^{-t}$

2. Determine how long it will take an investment to double in value at an interest rate of 4% if compounded

a. yearly b. quarterly c. monthly d. continuously

3. Determine how long it will take an investment to double in value at an interest rate of 10% compounded continuously.

4. Solve the following system of equations by using the substitution method.

a. $\begin{aligned} 4x + y &= -7 \\ 3x + 5y &= 16 \end{aligned}$

b. $\begin{aligned} 3x + 8y &= 24 \\ x - 16y &= -6 \end{aligned}$

c. $\begin{aligned} x + 2 &= 8y \\ 3(x - 9) + 10y &= 1 \end{aligned}$

d. $\begin{aligned} 5y &= 2x - 30 \\ -6x + y &= 8 \end{aligned}$

e. $\begin{aligned} 2(x + y) &= 11 - 14x \\ y + 8x &= 15 \end{aligned}$

f. $\begin{aligned} x &= 2y - 5 \\ 6y - 3x &= 15 \end{aligned}$

5. Solve the following system of equations by using the addition method.

a. $\begin{aligned} 4x + y &= -7 \\ 3x + 5y &= 16 \end{aligned}$

b. $\begin{aligned} 3x + 8y &= 24 \\ x - 16y &= -6 \end{aligned}$

c. $\begin{aligned} 4x + 3y &= 10 \\ -5x + 6y &= -32 \end{aligned}$

d. $\begin{aligned} 8x + 3y &= -16 \\ 2x - 5y &= 19 \end{aligned}$

e. $\begin{aligned} 8x - 6y &= -3 \\ 5x + 4y &= -9 \end{aligned}$

f. $\begin{aligned} -12x + 2y &= 5 \\ 6x - y &= 9 \end{aligned}$

6. How many liters of a 6% salt solution and how many liter of a 25% salt solution are needed to make 38 liters of a 20% salt solution?

7. Solve the following system of equations.

a. $\begin{aligned} x - 3y - 2z &= -1 \\ 3x + y + 5z &= 32 \\ -4x + 6y - z &= -29 \end{aligned}$

b. $\begin{aligned} 5x + 4y - 3z &= -36 \\ 3x - 2y + 7z &= -15 \\ -2x - 6y + 9z &= 21 \end{aligned}$

c. $$\begin{aligned} 2x \qquad\quad - 7z &= 19 \\ 3y + z &= 9 \\ 4x - 5y \qquad &= -24 \end{aligned}$$

d. $$\begin{aligned} 2x + 3y - 5z &= -9 \\ 6x - 9y + 7z &= 5 \\ 4x - 3y + z &= -2 \end{aligned}$$

e. $$\begin{aligned} 3x - 5y + 7z &= -11 \\ 9x - 14y + 27z &= -30 \\ -12x + 23y - 10z &= 57 \end{aligned}$$

Problems available in the textbook: Page 462 … 5 – 34, 61 – 70 and Examples 1 – 5 starting on page 453. Problems available in the textbook: Page 501 … 7 – 10, 15 – 34, 37 – 66 and Examples 1 – 8 starting on page 492. Page 514 … 5 – 44 and Examples 1 – 7 starting on page 506.

SOLUTIONS:

1a. $3^x = 81$

Back to Problem 1.

Using the one-to-one property: $3^x = 81 \Rightarrow 3^x = 3^4 \Rightarrow x = 4$

Using logarithms base 3: $3^x = 81 \Rightarrow \log_3 3^x = \log_3 81 \Rightarrow$

$x \log_3 3 = \log_3 81 \Rightarrow x = \log_3 81 = 4$

NOTE: $\log_3 3 = 1$

Using natural logarithms: $3^x = 81 \Rightarrow \ln 3^x = \ln 81 \Rightarrow x \ln 3 = \ln 81 \Rightarrow$

$x = \dfrac{\ln 81}{\ln 3} \Rightarrow x = 4$ (using a calculator)

Without a calculator: $\dfrac{\ln 81}{\ln 3} = \dfrac{\ln 3^4}{\ln 3} = \dfrac{4\ln 3}{\ln 3} = 4$

Answer: $x = 4$

1b. $5^{-t} = 25$ Back to Problem 1.

Using the one-to-one property: $5^{-t} = 25 \Rightarrow 5^{-t} = 5^2 \Rightarrow -t = 2 \Rightarrow$

$t = -2$

Using logarithms base 5: $5^{-t} = 25 \Rightarrow \log_5 5^{-t} = \log_5 25 \Rightarrow$

$-t \log_5 5 = \log_5 25 \Rightarrow -t = 2 \Rightarrow t = -2$

NOTE: $\log_5 5 = 1$ and $\log_5 25 = 2$

Using natural logarithms: $5^{-t} = 25 \Rightarrow \ln 5^{-t} = \ln 25 \Rightarrow$

$-t \ln 5 = \ln 25 \Rightarrow t = -\dfrac{\ln 25}{\ln 5} \Rightarrow t = -2$ (using a calculator)

Without a calculator: $\dfrac{\ln 25}{\ln 5} = \dfrac{\ln 5^2}{\ln 5} = \dfrac{2\ln 5}{\ln 5} = 2$

Answer: $t = -2$

1c. $2^{3x - 11} = 32$ Back to Problem 1.

Using the one-to-one property: $2^{3x - 11} = 32 \Rightarrow 2^{3x - 11} = 2^{5} \Rightarrow$

$$3x - 11 = 5 \Rightarrow 3x = 16 \Rightarrow x = \frac{16}{3}$$

Using logarithms base 2: $2^{3x - 11} = 32 \Rightarrow \log_{2} 2^{3x - 11} = \log_{2} 32 \Rightarrow$

$$(3x - 11)\log_{2} 2 = \log_{2} 32 \Rightarrow 3x - 11 = 5 \Rightarrow 3x = 16 \Rightarrow x = \frac{16}{3}$$

NOTE: $\log_{2} 2 = 1$

Using natural logarithms: $2^{3x - 11} = 32 \Rightarrow \ln 2^{3x - 11} = \ln 32 \Rightarrow$

$$(3x - 11)\ln 2 = \ln 32 \Rightarrow 3x \ln 2 - 11 \ln 2 = \ln 32 \Rightarrow$$

$$3x \ln 2 = \ln 32 + 11 \ln 2 \Rightarrow x = \frac{\ln 32 + 11 \ln 2}{3 \ln 2} = \frac{\ln 32(2^{11})}{\ln 8} = \frac{16}{3}$$

(using a calculator)

Without a calculator: $\dfrac{\ln 32(2^{11})}{\ln 8} = \dfrac{\ln 2^{5}(2^{11})}{\ln 2^{3}} = \dfrac{\ln 2^{16}}{\ln 2^{3}} = \dfrac{16 \ln 2}{3 \ln 2} = \dfrac{16}{3}$

Answer: $x = \dfrac{16}{3}$

1d. $4^{x^{3}} = \dfrac{1}{16}$ Back to Problem 1.

Using the one-to-one property: $4^{x^3} = \frac{1}{16} \Rightarrow 4^{x^3} = 4^{-2} \Rightarrow x^3 = -2 \Rightarrow$

$x = \sqrt[3]{-2} = -\sqrt[3]{2}$

Using logarithms base 4: $4^{x^3} = \frac{1}{16} \Rightarrow \log_4 4^{x^3} = \log_4 \frac{1}{16} \Rightarrow$

$x^3 \log_4 4 = \log_4 \frac{1}{16} \Rightarrow x^3 = -2 \Rightarrow x = \sqrt[3]{-2} = -\sqrt[3]{2}$

NOTE: $\log_4 4 = 1$ and $\log_4 \frac{1}{16} = -2$

Using natural logarithms: $4^{x^3} = \frac{1}{16} \Rightarrow \ln 4^{x^3} = \ln \frac{1}{16} \Rightarrow$

$x^3 \ln 4 = \ln \frac{1}{16} \Rightarrow x^3 = \frac{\ln \frac{1}{16}}{\ln 4} \Rightarrow x^3 = -2 \Rightarrow x = \sqrt[3]{-2} = -\sqrt[3]{2}$

NOTE: $\frac{\ln \frac{1}{16}}{\ln 4} = -2$ (using a calculator)

Without a calculator: $\frac{\ln \frac{1}{16}}{\ln 4} = \frac{\ln 4^{-2}}{\ln 4} = \frac{-2 \ln 4}{\ln 4} = -2$

Answer: $x = -\sqrt[3]{2}$

1e. $16^t = 64$ Back to Problem 1.

Using the one-to-one property: $16^t = 64 \Rightarrow (4^2)^t = 4^3 \Rightarrow 4^{2t} = 4^3 \Rightarrow$

$2t = 3 \Rightarrow t = \dfrac{3}{2}$

Using logarithms base 4: $16^t = 64 \Rightarrow \log_4 16^t = \log_4 64 \Rightarrow$

$t \log_4 16 = \log_4 64 \Rightarrow 2t = 3 \Rightarrow t = \dfrac{3}{2}$

NOTE: $\log_4 16 = 2$ and $\log_4 64 = 3$

Using natural logarithms: $16^t = 64 \Rightarrow \ln 16^t = \ln 64 \Rightarrow$

$t \ln 16 = \ln 64 \Rightarrow t = \dfrac{\ln 64}{\ln 16} \Rightarrow t = \dfrac{3}{2}$ (using a calculator)

Without a calculator: $\dfrac{\ln 64}{\ln 16} = \dfrac{\ln 4^3}{\ln 4^2} = \dfrac{3\ln 4}{2\ln 4} = \dfrac{3}{2}$

Answer: $t = \dfrac{3}{2}$

1f. $9^{x+4} = \dfrac{1}{27}$ Back to Problem 1.

Using the one-to-one property: $9^{x+4} = \dfrac{1}{27} \Rightarrow (3^2)^{x+4} = 3^{-3} \Rightarrow$

$$3^{2(x+4)} = 3^{-3} \Rightarrow 2(x+4) = -3 \Rightarrow 2x + 8 = -3 \Rightarrow x = -\frac{11}{2}$$

Using logarithms base 3: $9^{x+4} = \frac{1}{27} \Rightarrow \log_3 9^{x+4} = \log_3 \frac{1}{27} \Rightarrow$

$(x+4)\log_3 9 = \log_3 \frac{1}{27} \Rightarrow 2(x+4) = -3 \Rightarrow 2x + 8 = -3 \Rightarrow$

$x = -\frac{11}{2}$

NOTE: $\log_3 9 = 2$ and $\log_3 \frac{1}{27} = -3$

Using natural logarithms: $9^{x+4} = \frac{1}{27} \Rightarrow \ln 9^{x+4} = \ln \frac{1}{27} \Rightarrow$

$(x+4)\ln 9 = \ln \frac{1}{27} \Rightarrow x\ln 9 + 4\ln 9 = -\ln 27 \Rightarrow$

$x\ln 9 + 4\ln 9 = -\ln 27 \Rightarrow x\ln 9 = -\ln 27 - 4\ln 9 \Rightarrow$

$x = -\frac{\ln 27 + 4\ln 9}{\ln 9} = \frac{\ln 27 + \ln 9^4}{\ln 9} = \frac{\ln 27(9^4)}{\ln 9} = -\frac{11}{2}$ (using a calculator)

NOTE: $\ln \frac{1}{27} = \ln 27^{-1} = -1 \cdot \ln 27 = -\ln 27$

Without a calculator: $\frac{\ln 27(9^4)}{\ln 9} = \frac{\ln 3^3(3^8)}{\ln 3^2} = \frac{\ln 3^{11}}{\ln 3^2} = \frac{11\ln 3}{2\ln 3} = \frac{11}{2}$

Answer: $x = -\dfrac{11}{2}$

1g. $6^x = 12$ Back to Problem 1.

Using natural logarithms: $6^x = 12 \Rightarrow \ln 6^x = \ln 12 \Rightarrow$

$x \ln 6 = \ln 12 \Rightarrow x = \dfrac{\ln 12}{\ln 6}$

NOTE: $x = \dfrac{\ln 12}{\ln 6} \approx 1.38685$ and $6^{1.38685} \approx 11.99994$

Using logarithms base 6: $6^x = 12 \Rightarrow \log_6 6^x = \log_6 12 \Rightarrow$

$x \log_6 6 = \log_6 12 \Rightarrow x = \log_6 12$ NOTE: $\log_6 6 = 1$

Since your calculator does not have logarithm base 6 key, you would have to do a change of bases to obtain an approximation for $\log_6 12$. Since your calculator has a natural logarithm key $\boxed{\text{LN}}$, then we obtain that $\log_6 12 = \dfrac{\ln 12}{\ln 6}$ using the change of base formula that $\log_b u = \dfrac{\log_a u}{\log_a b}$, where $u = 12$, $b = 6$, and $a = e$. Or, Since your calculator has a common logarithm key $\boxed{\text{LOG}}$, then we obtain that $\log_6 12 = \dfrac{\log 12}{\log 6}$ using the change of base formula that $\log_b u = \dfrac{\log_a u}{\log_a b}$, where $u = 12$, $b = 6$, and $a = 10$.

NOTE: $x = \dfrac{\ln 12}{\ln 6} \approx 1.38685$ and $6^{1.38685} \approx 11.99994$

Answer: $x = \dfrac{\ln 12}{\ln 6}$ **or** $x = \log_6 12$

1h. $5^t = \dfrac{2}{3}$

Back to Problem 1.

Using natural logarithms: $5^t = \dfrac{2}{3} \Rightarrow \ln 5^t = \ln \dfrac{2}{3} \Rightarrow t \ln 5 = \ln \dfrac{2}{3} \Rightarrow$

$$t = \frac{\ln \frac{2}{3}}{\ln 5}$$

NOTE: $t = \dfrac{\ln \frac{2}{3}}{\ln 5} \approx -0.25193$ and $5^{-0.25193} \approx 0.666666277$

Using logarithms base 5: $5^t = \dfrac{2}{3} \Rightarrow \log_5 5^t = \log_5 \dfrac{2}{3} \Rightarrow$

$t \log_5 5 = \log_5 \dfrac{2}{3} \Rightarrow t = \log_5 \dfrac{2}{3}$ NOTE: $\log_5 5 = 1$

Since your calculator does not have logarithm base 5 key, you would have to do a change of bases to obtain an approximation for $\log_5 \dfrac{2}{3}$. Since your calculator has a natural logarithm key $\boxed{\text{LN}}$, then we obtain that $\log_5 \dfrac{2}{3} =$

$\dfrac{\ln \frac{2}{3}}{\ln 5}$ using the change of base formula that $\log_b u = \dfrac{\log_a u}{\log_a b}$, where $u = \dfrac{2}{3}$, $b = 5$, and $a = e$. Or, Since your calculator has a common logarithm key $\boxed{\text{LOG}}$, then we obtain that $\log_5 \dfrac{2}{3} = \dfrac{\log \frac{2}{3}}{\log 5}$ using the change of base formula that $\log_b u = \dfrac{\log_a u}{\log_a b}$, where $u = \dfrac{2}{3}$, $b = 5$, and $a = 10$.

Answer: $t = \dfrac{\ln \frac{2}{3}}{\ln 5}$ **or** $t = \log_5 \dfrac{2}{3}$

1i. $7^{-x} = \dfrac{3}{4}$ Back to Problem 1.

Using natural logarithms: $7^{-x} = \dfrac{3}{4} \Rightarrow \ln 7^{-x} = \ln \dfrac{3}{4} \Rightarrow$

$$-x \ln 7 = \ln \frac{3}{4} \Rightarrow x = -\frac{\ln \frac{3}{4}}{\ln 7}$$

NOTE: $x = -\dfrac{\ln \frac{3}{4}}{\ln 7} \approx 0.14784$ and $7^{-0.14784} \approx 0.749999037$

Answer: $x = -\dfrac{\ln \frac{3}{4}}{\ln 7}$ **or** $x = -\log_7 \dfrac{3}{4}$

1j. $8^{5-2x} = 65$ Back to Problem 1.

Using natural logarithms: $8^{5-2x} = 65 \Rightarrow \ln 8^{5-2x} = \ln 65 \Rightarrow$

$(5 - 2x)\ln 8 = \ln 65 \Rightarrow 5\ln 8 - 2x\ln 8 = \ln 65 \Rightarrow$

$$5\ln 8 - \ln 65 = 2x\ln 8 \Rightarrow x = \frac{5\ln 8 - \ln 65}{2\ln 8} = \frac{\ln \frac{8^5}{65}}{\ln 64}$$

NOTE: $x = \dfrac{\ln \frac{8^5}{65}}{\ln 64} \approx 1.49627$, $5 - 2x \approx 2.00746$, and $8^{2.00746} \approx 65.00055$

Answer: $x = \dfrac{5\ln 8 - \ln 65}{2\ln 8}$

1k. $3^{7x+4} = 49$ Back to Problem 1.

Using natural logarithms: $3^{7x+4} = 49 \Rightarrow \ln 3^{7x+4} = \ln 49 \Rightarrow$

$(7x + 4)\ln 3 = \ln 49 \Rightarrow 7x\ln 3 + 4\ln 3 = \ln 49 \Rightarrow$

$$7x\ln 3 = \ln 49 - 4\ln 3 \Rightarrow x = \frac{\ln 49 - 4\ln 3}{7\ln 3} = \frac{\ln \frac{49}{81}}{\ln 2187}$$

NOTE: $4\ln 3 = \ln 3^4 = \ln 81$ and $7\ln 3 = \ln 3^7 = \ln 2187$

NOTE: $x = \dfrac{\ln \frac{49}{81}}{\ln 2187} \approx -0.065359$, $7x + 4 \approx 3.542487$, and

$3^{3.542487} \approx 48.99997$

Answer: $x = \dfrac{\ln 49 - 4\ln 3}{7\ln 3}$

1l. $2^{3x - 8} = 6^{2x + 9}$ Back to Problem 1.

Using natural logarithms: $2^{3x - 8} = 6^{2x + 9} \Rightarrow \ln 2^{3x - 8} = \ln 6^{2x + 9} \Rightarrow$

$(3x - 8)\ln 2 = (2x + 9)\ln 6 \Rightarrow 3x\ln 2 - 8\ln 2 = 2x\ln 6 + 9\ln 6 \Rightarrow$

$3x\ln 2 - 2x\ln 6 = 9\ln 6 + 8\ln 2 \Rightarrow x(3\ln 2 - 2\ln 6) = 9\ln 6 + 8\ln 2 \Rightarrow$

$x = \dfrac{9\ln 6 + 8\ln 2}{3\ln 2 - 2\ln 6}$

Answer: $x = \dfrac{9\ln 6 + 8\ln 2}{3\ln 2 - 2\ln 6}$

1m. $e^{2x} - 2e^{x} - 24 = 0$ Back to Problem 1.

This equation is quadratic in the expression e^{x}. Let $a = e^{x}$. Then $a^2 = (e^{x})^2 = e^{2x}$. Thus,

$e^{2x} - 2e^{x} - 24 = 0 \Rightarrow a^2 - 2a - 24 = 0 \Rightarrow (a + 4)(a - 6) = 0 \Rightarrow$

$(e^{x} + 4)(e^{x} - 6) = 0 \Rightarrow e^{x} + 4 = 0,\ e^{x} - 6 = 0$

$e^x + 4 = 0 \Rightarrow e^x = -4$. Since $e^x > 0$ for all x, then this equation has no solution.

$e^x - 6 = 0 \Rightarrow e^x = 6 \Rightarrow \ln e^x = \ln 6 \Rightarrow x \ln e = \ln 6 \Rightarrow x = \ln 6$

Answer: $x = \ln 6$

1n. $9^x + 16 = 10(3^x)$ Back to Problem 1.

$9^x + 16 = 10(3^x) \Rightarrow (3^2)^x + 16 = 10(3^x) \Rightarrow 3^{2x} + 16 = 10(3^x) \Rightarrow$

$3^{2x} - 10(3^x) + 16 = 0$

This equation is quadratic in the expression 3^x. Let $a = 3^x$. Then $a^2 = (3^x)^2 = 3^{2x}$. Thus,

$3^{2x} - 10(3^x) + 16 = 0 \Rightarrow a^2 - 10a + 16 = 0 \Rightarrow (a - 2)(a - 8) = 0 \Rightarrow$

$(3^x - 2)(3^x - 8) = 0 \Rightarrow 3^x - 2 = 0,\ 3^x - 8 = 0$

$3^x - 2 = 0 \Rightarrow 3^x = 2 \Rightarrow \ln 3^x = \ln 2 \Rightarrow x \ln 3 = \ln 2 \Rightarrow x = \dfrac{\ln 2}{\ln 3}$

$3^x - 8 = 0 \Rightarrow 3^x = 8 \Rightarrow \ln 3^x = \ln 8 \Rightarrow x \ln 3 = \ln 8 \Rightarrow x = \dfrac{\ln 8}{\ln 3}$

Answer: $x = \dfrac{\ln 2}{\ln 3}, \dfrac{\ln 8}{\ln 3}$ **or** $x = \log_3 2,\ \log_3 8$

1o. $e^{-2t} = 5e^{-t}$ Back to Problem 1.

This equation is quadratic in the expression e^{-t}. Let $a = e^{-t}$. Then $a^2 = (e^{-t})^2 = e^{-2t}$. Thus,

$$e^{-2t} = 5e^{-t} \Rightarrow a^2 = 5a \Rightarrow a^2 - 5a = 0 \Rightarrow a(a - 5) = 0 \Rightarrow$$

$$e^{-t}(e^{-t} - 5) = 0 \Rightarrow e^{-t} = 0,\ e^{-t} - 5 = 0$$

Since $e^{-t} > 0$ for all t, then the equation $e^{-t} = 0$ has no solution.

$$e^{-t} - 5 = 0 \Rightarrow e^{-t} = 5 \Rightarrow \ln e^{-t} = \ln 5 \Rightarrow -t\ln e = \ln 5 \Rightarrow$$

$$-t = \ln 5 \Rightarrow t = -\ln 5$$

Answer: $t = -\ln 5$

2a. $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ Back to Problem 2.

$$A = 2P,\ r = 4\% = 0.04,\ n = 1$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \Rightarrow 2P = P\left(1 + \frac{0.04}{1}\right)^{1t} \Rightarrow 2 = (1 + 0.04)^t \Rightarrow$$

$$2 = (1.04)^t \Rightarrow \ln 2 = \ln (1.04)^t \Rightarrow \ln 2 = t\ln (1.04) \Rightarrow$$

$$t = \frac{\ln 2}{\ln 1.04} \approx 17.67299$$

$$0.67299 \text{ year} = 0.67299 \cdot 12 \approx 8.07588 \text{ months}$$

Thus, it will take approximately 17 years and 8 months for the investment to double in value if the interest is compounded yearly at a rate of 4%.

Answer: 17 years and 8 months

2b. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ Back to Problem 2.

$$A = 2P, \quad r = 4\% = 0.04, \quad n = 4$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \Rightarrow 2P = P\left(1 + \frac{0.04}{4}\right)^{4t} \Rightarrow 2 = (1 + 0.01)^{4t} \Rightarrow$$

$$2 = (1.01)^{4t} \Rightarrow \ln 2 = \ln (1.01)^{4t} \Rightarrow \ln 2 = 4t \ln (1.01) \Rightarrow$$

$$t = \frac{\ln 2}{4 \ln 1.01} \approx 17.41518$$

$$0.41518 \text{ year} = 0.41518 \cdot 12 \approx 4.98216 \text{ months}$$

Thus, it will take approximately 17 years and 5 months for the investment to double in value if the interest is compounded quarterly at a rate of 4%.

Answer: 17 years and 5 months

2c. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ Back to Problem 2.

$A = 2P, \; r = 4\% = 0.04, \; n = 12$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \Rightarrow 2P = P\left(1 + \frac{0.04}{12}\right)^{12t} \Rightarrow 2 = \left(1 + \frac{0.04}{12}\right)^{12t} \Rightarrow$$

$$2 = \left(1 + \frac{0.01}{3}\right)^{12t} \Rightarrow 2 = \left(\frac{3.01}{3}\right)^{12t} \Rightarrow \ln 2 = \ln\left(\frac{3.01}{3}\right)^{12t} \Rightarrow$$

$$\ln 2 = 12t \ln\left(\frac{3.01}{3}\right) \Rightarrow t = \frac{\ln 2}{12 \ln\left(\frac{3.01}{3}\right)} \approx 17.35754$$

$0.35754 \text{ year} = 0.35754 \cdot 12 \approx 4.29048 \text{ months}$

Thus, it will take approximately 17 years and 4 months for the investment to double in value if the interest is compounded monthly at a rate of 4%.

Answer: 17 years and 4 months

2d. $A = Pe^{rt}$ Back to Problem 2.

$A = 2P, \; r = 4\% = 0.04$

$$A = Pe^{rt} \Rightarrow 2P = Pe^{0.04t} \Rightarrow 2 = e^{0.04t} \Rightarrow \ln 2 = \ln e^{0.04t} \Rightarrow$$

$$\ln 2 = 0.04t \ln e \Rightarrow \ln 2 = 0.04t \Rightarrow \ln 2 = \frac{4}{100}t \Rightarrow \ln 2 = \frac{1}{25}t \Rightarrow$$

$$t = 25 \ln 2 = \ln 2^{25} \approx 17.32868$$

$$0.32868 \text{ year} = 0.32868 \cdot 12 \approx 3.94416 \text{ months}$$

Thus, it will take approximately 17 years and 4 months for the investment to double in value if the interest is compounded continuously at a rate of 4%.

Answer: 17 years and 4 months

3. $A = Pe^{rt}$

Back to Problem 3.

$$A = 2P, \quad r = 10\% = 0.1$$

$$A = Pe^{rt} \Rightarrow 2P = Pe^{0.1t} \Rightarrow 2 = e^{0.1t} \Rightarrow \ln 2 = \ln e^{0.1t} \Rightarrow$$

$$\ln 2 = 0.1t \ln e \Rightarrow \ln 2 = 0.1t \Rightarrow \ln 2 = \frac{1}{10}t \Rightarrow$$

$$t = 10 \ln 2 = \ln 2^{10} \approx 6.931472$$

$$0.931472 \text{ year} = 0.931472 \cdot 12 \approx 11.17766 \text{ months}$$

Thus, it will take approximately 6 years and 11 months for the investment to double in value if the interest is compounded continuously at a rate of 10%.

Answer: 6 years and 11 months

4a. $\begin{aligned} 4x + y &= -7 \\ 3x + 5y &= 16 \end{aligned}$ Back to Problem 4.

Use the first equation of $4x + y = -7$ to solve for *y* in terms of *x*:

$$4x + y = -7 \Rightarrow y = -4x - 7$$

Now, replace the *y* variable in the second equation of $3x + 5y = 16$ by the expression $-4x - 7$ and then solve for *x*:

$3x + 5y = 16 \Rightarrow 3x + 5(-4x - 7) = 16 \Rightarrow 3x - 20x - 35 = 16 \Rightarrow$

$-17x - 35 = 16 \Rightarrow -17x = 51 \Rightarrow x = -3$

Now, use the equation $y = -4x - 7$ to find the value of *y* when $x = -3$:

$y = -4x - 7,\ x = -3 \Rightarrow y = -4(-3) - 7 = 12 - 7 = 5$

Answer: $(-3,\ 5)$

NOTE: Geometrically, if you graph the two lines in the system of equations, the lines will intersect at the point $(-3,\ 5)$.

4b. $\begin{aligned} 3x + 8y &= 24 \\ x - 16y &= -6 \end{aligned}$ Back to Problem 4.

Use the second equation of $x - 16y = -6$ to solve for x in terms of y:

$$x - 16y = -6 \Rightarrow x = 16y - 6$$

Now, replace the x variable in the first equation of $3x + 8y = 24$ by the expression $16y - 6$ and then solve for y:

$$3x + 8y = 24 \Rightarrow 3(16y - 6) + 8y = 24 \Rightarrow 48y - 18 + 8y = 24 \Rightarrow$$

$$56y - 18 = 24 \Rightarrow 56y = 42 \Rightarrow y = \frac{42}{56} = \frac{21}{28} = \frac{3}{4}$$

Now, use the equation $x = 16y - 6$ to find the value of x when $y = \frac{3}{4}$:

$$x = 16y - 6,\ y = \frac{3}{4} \Rightarrow x = 16\left(\frac{3}{4}\right) - 6 = 12 - 6 = 6$$

Answer: $\left(6, \frac{3}{4}\right)$

NOTE: Geometrically, if you graph the two lines in the system of equations, the lines will intersect at the point $\left(6, \frac{3}{4}\right)$.

4c. $\begin{aligned} x + 2 &= 8y \\ 3(x - 9) + 10y &= 1 \end{aligned}$

Back to Problem 4.

Simplifying the second equation, we obtain the equation $3x + 10y = 28$:

$$3(x - 9) + 10y = 1 \Rightarrow 3x - 27 + 10y = 1 \Rightarrow 3x + 10y = 28$$

Use the first equation of $x + 2 = 8y$ to solve for *x* in terms of *y*:

$$x + 2 = 8y \Rightarrow x = 8y - 2$$

Now, replace the *x* variable in the simplified second equation of $3x + 10y = 28$ by the expression $x = 8y - 2$ and then solve for *y*:

$$3x + 10y = 28 \Rightarrow 3(8y - 2) + 10y = 28 \Rightarrow 24y - 6 + 10y = 28 \Rightarrow$$

$$34y - 6 = 28 \Rightarrow 34y = 34 \Rightarrow y = 1$$

Now, use the equation $x = 8y - 2$ to find the value of *x* when $y = 1$:

$$x = 8y - 2, \; y = 1 \Rightarrow x = 8(1) - 2 = 8 - 2 = 6$$

Answer: $(6, 1)$

NOTE: Geometrically, if you graph the two lines in the system of equations, the lines will intersect at the point $(6, 1)$.

4d. $\begin{array}{l} 5y = 2x - 30 \\ -6x + y = 8 \end{array}$ Back to Problem 4.

Use the second equation of $-6x + y = 8$ to solve for *y* in terms of *x*:

$$-6x + y = 8 \Rightarrow y = 6x + 8$$

Now, replace the *y* variable in the first equation of $5y = 2x - 30$ by the expression $6x + 8$ and then solve for *x*:

$$5y = 2x - 30 \Rightarrow 5(6x + 8) = 2x - 30 \Rightarrow 30x + 40 = 2x - 30 \Rightarrow$$

$$28x = -70 \Rightarrow x = -\frac{70}{28} = -\frac{10}{4} = -\frac{5}{2}$$

Now, use the equation $y = 6x + 8$ to find the value of *y* when $x = -\frac{5}{2}$:

$$y = 6x + 8,\ x = -\frac{5}{2} \Rightarrow y = 6\left(-\frac{5}{2}\right) + 8 = -15 + 8 = -7$$

Answer: $\left(-\frac{5}{2}, -7\right)$

NOTE: Geometrically, if you graph the two lines in the system of equations, the lines will intersect at the point $\left(-\frac{5}{2}, -7\right)$.

4e. $\begin{array}{l} 2(x + y) = 11 - 14x \\ y + 8x = 15 \end{array}$

Back to Problem 4.

Simplifying the first equation, we obtain the equation $16x + 2y = 11$:

$$2(x + y) = 11 - 14x \Rightarrow 2x + 2y = 11 - 14x \Rightarrow 16x + 2y = 11$$

Use the second equation of $y + 8x = 15$ to solve for *y* in terms of *x*:

$$y + 8x = 15 \Rightarrow y = 15 - 8x$$

Now, replace the *y* variable in the simplified first equation of $16x + 2y = 11$ by the expression $15 - 8x$ and then solve for *x*:

$$16x + 2y = 11 \Rightarrow 16x + 2(15 - 8x) = 11 \Rightarrow 16x + 30 - 16x = 11 \Rightarrow$$

$30 = 11$. This is a false equation. Thus, the system of equations does not have a solution.

Answer: No solution

NOTE: Geometrically, the two lines in the system of equations are parallel. Thus, the line will not intersect.

4f. $\begin{aligned} x &= 2y - 5 \\ 6y - 3x &= 15 \end{aligned}$

Back to Problem 4.

NOTE: In the first equation, the *x* variable is already solved in terms of *y*: $x = 2y - 5$

Replace the *x* variable in the second equation of $6y - 3x = 15$ by the expression $2y - 5$ and then solve for *y*:

$6y - 3x = 15 \Rightarrow 6y - 3(2y - 5) = 15 \Rightarrow 6y - 6y + 15 = 15 \Rightarrow$

$15 = 15$. This is a true equation. Thus, solution to the system of equations is every point on the line $x = 2y - 5$ or $y = \dfrac{1}{2}x + \dfrac{5}{2}$.

NOTE: The two equations in the system of equations represent the same line. If you solve both equations for *y*, you will obtain the equation

$$y = \frac{1}{2}x + \frac{5}{2}.$$

Answer: Every point on the line $y = \dfrac{1}{2}x + \dfrac{5}{2}$.

NOTE: Geometrically, the two lines in the system of equations are the same.

NOTE: We may also write the solution to this system of equations in the following way. Since $x = 2y - 5$, then let $y = t$, where *t* represents any real number. Then $x = 2y - 5 = 2t - 5$. Then the solution to this system of equations may be written as the set $\{(2t - 5, t) : t$ is any real number$\}$.

5a. $$\begin{aligned} 4x + y &= -7 \\ 3x + 5y &= 16 \end{aligned}$$ Back to Problem 5.

Multiply both sides of the first equation by -5 and then add both sides of the equations.

$$\begin{aligned} 4x + y &= -7 \\ 3x + 5y &= 16 \end{aligned} \quad \Rightarrow \quad \begin{array}{rcrcr} -20x & - & 5y & = & 35 \\ 3x & + & 5y & = & 16 \\ \hline -17x & & & = & 51 \end{array} \Rightarrow x = -3$$

Now, use the first equation to find the value of y when $x = -3$:

$$4x + y = -7 \Rightarrow -12 + y = -7 \Rightarrow y = 5$$

Answer: $(-3, 5)$

NOTE: This is the same system of equations that we solve in Problem 4a above.

5b. $$\begin{aligned} 3x + 8y &= 24 \\ x - 16y &= -6 \end{aligned}$$ Back to Problem 5.

Multiply both sides of the second equation by -3 and then add both sides of the equations.

$$\begin{aligned} 3x + 8y &= 24 \\ x - 16y &= -6 \end{aligned} \quad \Rightarrow \quad \begin{array}{rcrcr} 3x & + & 8y & = & 24 \\ -3x & + & 48y & = & 18 \\ \hline & & 56y & = & 42 \end{array} \Rightarrow y = \frac{42}{56} = \frac{21}{28} = \frac{3}{4}$$

Now, use the second equation to find the value of x when $y = \frac{3}{4}$:

$$x - 16y = -6 \Rightarrow x - 16\left(\frac{3}{4}\right) = -6 \Rightarrow x - 12 = -6 \Rightarrow x = 6$$

Answer: $\left(6, \frac{3}{4}\right)$

NOTE: This is the same system of equations that we solve in Problem 4b above.

5c. $\begin{aligned} 4x + 3y &= 10 \\ -5x + 6y &= -32 \end{aligned}$

Back to Problem 5.

Multiply both sides of the first equation by -2 and then add both sides of the equations.

$$\begin{aligned} 4x + 3y &= 10 \\ -5x + 6y &= -32 \end{aligned} \Rightarrow \begin{array}{rcl} -8x - 6y &=& -20 \\ \underline{-5x + 6y} &\underline{=}& \underline{-32} \\ -13x &=& -52 \end{array} \Rightarrow x = 4$$

Now, use the first equation to find the value of y when $x = 4$:

$$4x + 3y = 10,\ x = 4 \Rightarrow 16 + 3y = 10 \Rightarrow 3y = -6 \Rightarrow y = -2$$

Answer: $(4, -2)$

NOTE: Geometrically, if you graph the two lines in the system of equations, the lines will intersect at the point $(4, -2)$.

5d. $\begin{aligned} 8x + 3y &= -16 \\ 2x - 5y &= 19 \end{aligned}$

Back to Problem 5.

Multiply both sides of the second equation by -4 and then add both sides of the equations.

$$\begin{aligned} 8x + 3y &= -16 \\ 2x - 5y &= 19 \end{aligned} \;\Rightarrow\; \begin{array}{r} 8x + 3y = -16 \\ \underline{-8x + 20y = -76} \\ 23y = -92 \end{array} \;\Rightarrow\; y = -4$$

Now, use the second equation to find the value of x when $y = -4$:

$$2x - 5y = 19,\; y = -4 \;\Rightarrow\; 2x + 20 = 19 \;\Rightarrow\; 2x = -1 \;\Rightarrow\; x = -\frac{1}{2}$$

Answer: $\left(-\frac{1}{2},\, -4\right)$

NOTE: Geometrically, if you graph the two lines in the system of equations, the lines will intersect at the point $\left(-\frac{1}{2},\, -4\right)$.

5e. $\begin{aligned} 8x - 6y &= -3 \\ 5x + 4y &= -9 \end{aligned}$

Back to Problem 5.

Multiply both sides of the first equation by 2 and multiply the second equation by 3 and then add both sides of the equations.

$$\begin{aligned} 8x - 6y &= -3 \\ 5x + 4y &= -9 \end{aligned} \;\Rightarrow\; \begin{array}{l} 16x - 12y = -6 \\ \underline{15x + 12y = -27} \\ 31x \qquad\quad = -33 \end{array} \;\Rightarrow\; x = -\frac{33}{31}$$

Multiply the first equation by -5 and multiply the second equation by 8 and then add both sides of the equations.

$$\begin{aligned} 8x - 6y &= -3 \\ 5x + 4y &= -9 \end{aligned} \Rightarrow \begin{array}{r} -40x + 30y = 15 \\ 40x + 32y = -72 \\ \hline 62y = -57 \end{array} \Rightarrow y = -\frac{57}{62}$$

Answer: $\left(-\frac{33}{31}, -\frac{57}{62} \right)$

NOTE: Geometrically, if you graph the two lines in the system of equations, the lines will intersect at the point $\left(-\frac{33}{31}, -\frac{57}{62} \right)$.

5f. $\begin{aligned} -12x + 2y &= 5 \\ 6x - y &= 9 \end{aligned}$

Back to Problem 5.

Multiply both sides of the second equation by 2 and then add both sides of the equations.

$$\begin{aligned} -12x + 2y &= 5 \\ 6x - y &= 9 \end{aligned} \Rightarrow \begin{array}{r} -12x + 2y = 5 \\ 12x - 2y = 18 \\ \hline 0 = 23 \end{array}$$

$0 = 23$ is a false equation. Thus, the system of equations doesn't have a solution.

Answer: No solution

NOTE: You can show that the two lines in this system of equations are parallel. Thus, they don't intersect.

6. Back to Problem 6.

Solution	Amount of Solution	Percent of Salt	Amount of Salt
6% Salt	x	$6\% = 0.06$	$0.06\,x$
25% Salt	y	$25\% = 0.25$	$0.25\,y$
20% Salt	38	$20\% = 0.2$	$0.2(38) = 7.6$

We obtain the following equations: $x + y = 38$ and $0.06x + 0.25y = 7.6$.

Thus, we need to solve the following system of equations:

$$x + y = 38$$
$$0.06x + 0.25y = 7.6$$

We can simplify the second equation by multiplying both sides of the equation by 100:

$$0.06x + 0.25y = 7.6 \Rightarrow 6x + 25y = 760$$

Use the first equation of $x + y = 38$ to solve for x in terms of y:

$$x + y = 38 \Rightarrow x = 38 - y$$

Now, replace the x variable in the simplified second equation of $6x + 25y = 760$ by the expression $38 - y$ and then solve for y:

$$6x + 25y = 760 \Rightarrow 6(38 - y) + 25y = 760 \Rightarrow$$

$$228 - 6y + 25y = 760 \Rightarrow 228 + 19y = 760 \Rightarrow 19y = 532 \Rightarrow$$

$$y = 28$$

Now, use the equation $x = 38 - y$ to find the value of x when $y = 28$:

$x = 38 - y,\ y = 28\ \ x = 38 - 28 = 10$

Answer: Amount of 6% salt solution: 10 liters

Amount of 25% salt solution: 28 liters

7a.
$$\begin{aligned} x - 3y - 2z &= -1 \\ 3x + y + 5z &= 32 \\ -4x + 6y - z &= -29 \end{aligned}$$

Back to Problem 7.

Multiply the first equation by -3 and add this new equation to the second equation:

$$\begin{aligned} x - 3y - 2z &= -1 \\ 3x + y + 5z &= 32 \end{aligned} \Rightarrow \begin{array}{r} -3x + 9y + 6z = 3 \\ 3x + y + 5z = 32 \\ \hline 10y + 11z = 35 \end{array}$$

$$\begin{aligned} x - 3y - 2z &= -1 \\ 3x + y + 5z &= 32 \\ -4x + 6y - z &= -29 \end{aligned}$$

Now, multiply the first equation by 4 and add this new equation to the third equation:

$$\begin{aligned} x - 3y - 2z &= -1 \\ -4x + 6y - z &= -29 \end{aligned} \Rightarrow \begin{array}{r} 4x - 12y - 8z = -4 \\ -4x + 6y - z = -29 \\ \hline -6y - 9z = -33 \end{array} \Rightarrow 2y + 3z = 11$$

Now, solve this new system of equations: $\begin{aligned} 10y + 11z &= 35 \\ 2y + 3z &= 11 \end{aligned}$

Multiply the second equation by -5 and then add both sides of the equations:

$$\begin{aligned} 10y + 11z &= 35 \\ 2y + 3z &= 11 \end{aligned} \Rightarrow \begin{array}{r} 10y + 11z = \ \ 35 \\ \underline{-10y - 15z = -55} \\ -4z = -20 \end{array} \Rightarrow z = 5$$

Now, use the second equation in the new system of equations to find the value of y when $z = 5$:

$$2y + 3z = 11,\ z = 5 \Rightarrow 2y + 15 = 11 \Rightarrow 2y = -4 \Rightarrow y = -2$$

Now, use the first equation in the original system of equations to find the value of x when $y = -2$ and $z = 5$:

$$x - 3y - 2z = -1,\ y = -2,\ z = 5 \Rightarrow x + 6 - 10 = -1 \Rightarrow$$

$$x - 4 = -1 \Rightarrow x = 3$$

Answer: $(3,\ -2,\ 5)$

NOTE: Geometrically, if you graph the three planes in the system of equations, the planes will intersect at the point $(3,\ -2,\ 5)$.

7b. $\begin{aligned} 5x + 4y - 3z &= -36 \\ 3x - 2y + 7z &= -15 \\ -2x - 6y + 9z &= 21 \end{aligned}$

Back to Problem 7.

Multiply the second equation by 2 and add this new equation to the first equation:

$$\begin{aligned} 5x + 4y - 3z &= -36 \\ 3x - 2y + 7z &= -15 \end{aligned} \Rightarrow \begin{array}{r} 5x + 4y - 3z = -36 \\ \underline{6x - 4y + 14z = -30} \\ 11x \qquad + 11z = -66 \end{array} \Rightarrow x + z = -6$$

$$\begin{aligned} 5x + 4y - 3z &= -36 \\ 3x - 2y + 7z &= -15 \\ -2x - 6y + 9z &= 21 \end{aligned}$$

Now, multiply the second equation by -3 and add this new equation to the third equation:

$$\begin{aligned} 3x - 2y + 7z &= -15 \\ -2x - 6y + 9z &= 21 \end{aligned} \Rightarrow \begin{array}{r} -9x + 6y - 21z = 45 \\ \underline{-2x - 6y + 9z = 21} \\ -11x \qquad - 12z = 66 \end{array}$$

Now, solve this new system of equations: $\begin{aligned} x + z &= -6 \\ -11x - 12z &= 66 \end{aligned}$

Multiply the first equation by 11 and then add both sides of the equations:

$$\begin{aligned} x + z &= -6 \\ -11x - 12z &= 66 \end{aligned} \Rightarrow \begin{array}{r} 11x + 11z = -66 \\ \underline{-11x - 12z = 66} \\ -z = 0 \end{array} \Rightarrow z = 0$$

Now, use the first equation in the new system of equations to find the value of x when $z = 0$:

$$x + z = -6,\ z = 0 \Rightarrow x + 0 = -6 \Rightarrow x = -6$$

Now, use the second equation in the original system of equations to find the value of y when $x = -6$ and $z = 0$:

$$3x - 2y + 7z = -15,\ x = -6,\ z = 0 \Rightarrow -18 - 2y + 0 = -15 \Rightarrow$$

$$-18 - 2y = -15 \Rightarrow -2y = 3 \Rightarrow y = -\frac{3}{2}$$

Answer: $\left(-6,\ -\frac{3}{2},\ 0\right)$

NOTE: Geometrically, if you graph the three planes in the system of equations, the planes will intersect at the point $\left(-6,\ -\frac{3}{2},\ 0\right)$.

7c.
$$\begin{aligned} 2x \quad\quad - 7z &= 19 \\ 3y + z &= 9 \\ 4x - 5y \quad\quad &= -24 \end{aligned}$$

Back to Problem 7.

Multiply the second equation by 7 and add this new equation to the first equation:

$$\begin{aligned} 2x \quad\quad - 7z &= 19 \\ 3y + z &= 9 \end{aligned} \Rightarrow \begin{array}{r} 2x \quad\quad - 7z = 19 \\ 21y + 7z = 63 \\ \hline 2x + 21y \quad\quad = 82 \end{array}$$

Notice that the third equation in the system of equations is $4x - 5y = -24$.

Now, solve this new system of equations: $\begin{aligned} 2x + 21y &= 82 \\ 4x - 5y &= -24 \end{aligned}$

Multiply the first equation by -2 and then add both sides of the equations:

$$\begin{aligned} 2x + 21y &= 82 \\ 4x - 5y &= -24 \end{aligned} \Rightarrow \begin{array}{r} -4x - 42y = -164 \\ \underline{4x - 5y = -24} \\ -47y = -188 \end{array} \Rightarrow y = 4$$

Now, use the second equation in the new system of equations to find the value of *x* when $y = 4$:

$$4x - 5y = -24,\ y = 4 \Rightarrow 4x - 20 = -24 \Rightarrow 4x = -4 \Rightarrow x = -1$$

Now, use the second equation in the original system of equations to find the value of *z* when $y = 4$:

$$3y + z = 9,\ y = 4 \Rightarrow 12 + z = 9 \Rightarrow z = -3$$

Answer: $(-1,\ 4,\ -3)$

NOTE: Geometrically, if you graph the three planes in the system of equations, the planes will intersect at the point $(-1,\ 4,\ -3)$.

7d. $\begin{aligned} 2x + 3y - 5z &= -9 \\ 6x - 9y + 7z &= 5 \\ 4x - 3y + z &= -2 \end{aligned}$

Back to Problem 7.

Multiply the third equation by 5 and add this new equation to the first equation:

$$\begin{aligned} 2x + 3y - 5z &= -9 \\ 4x - 3y + z &= -2 \end{aligned} \Rightarrow \begin{array}{r} 2x + 3y - 5z = -9 \\ \underline{20x - 15y + 5z = -10} \\ 22x - 12y \quad\quad = -19 \end{array}$$

$$\begin{aligned} 2x + 3y - 5z &= -9 \\ 6x - 9y + 7z &= 5 \\ 4x - 3y + z &= -2 \end{aligned}$$

Now, multiply the third equation by -7 and add this new equation to the second equation:

$$\begin{aligned} 6x - 9y + 7z &= 5 \\ 4x - 3y + z &= -2 \end{aligned} \Rightarrow \begin{array}{r} 6x - 9y + 7z = 5 \\ \underline{-28x + 21y - 7z = 14} \\ -22x + 12y \quad\quad = 19 \end{array}$$

Now, solve this new system of equations: $\begin{aligned} 22x - 12y &= -19 \\ -22x + 12y &= 19 \end{aligned}$

Notice that if you multiply the first equation by -1, you will obtain the second equation in this new system. These two equations are the same equation.

Let $y = t$, where t is any real number. Now, use the first equation in the new system of equations to find the value of x when $y = t$:

$$22x - 12y = -19, \; y = t \Rightarrow 22x - 12t = -19 \Rightarrow 22x = 12t - 19 \Rightarrow$$

$$x = \frac{12t - 19}{22} = \frac{6}{11}t - \frac{19}{22}$$

Now, use the third equation in the original system of equations to find the value of *z* when $x = \frac{6}{11}t - \frac{19}{22}$ and $y = t$:

$$4x - 3y + z = -2, \; x = \frac{6}{11}t - \frac{19}{22}, \; y = t \Rightarrow$$

$$\frac{24}{11}t - \frac{38}{11} - 3t + z = -2 \Rightarrow \frac{24}{11}t - \frac{38}{11} - \frac{33}{11}t + z = -2 \Rightarrow$$

$$-\frac{9}{11}t - \frac{38}{11} + z = -\frac{22}{11} \Rightarrow z = \frac{9}{11}t + \frac{16}{11}$$

Answer: $\left(\frac{6}{11}t - \frac{19}{22}, \; t, \; \frac{9}{11}t + \frac{16}{11}\right) = \left(\frac{12t - 19}{22}, \; t, \; \frac{9t + 16}{11}\right)$, where *t* is any real number

7e.

$$\begin{aligned} 3x - 5y + 7z &= -11 \\ 9x - 14y + 27z &= -30 \\ -12x + 23y - 10z &= 57 \end{aligned}$$

Back to Problem 7.

Multiply the first equation by -3 and add this new equation to the second equation:

$$\begin{aligned} 3x - 5y + 7z &= -11 \\ 9x - 14y + 27z &= -30 \end{aligned} \Rightarrow \begin{array}{r} -9x + 15y - 21z = 33 \\ 9x - 14y + 27z = -30 \\ \hline y + 6z = 3 \end{array}$$

$$\begin{aligned} 3x - 5y + 7z &= -11 \\ 9x - 14y + 27z &= -30 \\ -12x + 23y - 10z &= 57 \end{aligned}$$

Now, multiply the first equation by 4 and add this new equation to the third equation:

$$\begin{aligned} 3x - 5y + 7z &= -11 \\ -12x + 23y - 10z &= 57 \end{aligned} \Rightarrow \begin{array}{r} 12x - 20y + 28z = -44 \\ \underline{-12x + 23y - 10z = 57} \\ 3y + 18z = 13 \end{array}$$

Now, solve this new system of equations: $\begin{aligned} y + 6z &= 3 \\ 3y + 18z &= 13 \end{aligned}$

Multiply the first equation by -3 and then add both sides of the equations:

$$\begin{aligned} y + 6z &= 3 \\ 3y + 18z &= 13 \end{aligned} \Rightarrow \begin{array}{r} -3y - 18z = -9 \\ \underline{3y + 18z = 13} \\ 0 = 4 \end{array}$$

The equation $0 = 4$ is a false equation. This means that this new system of equation in variables of *y* and *z* does not have a solution. This then means that the original system of equations also does not have a solution.

Answer: No Solution