Pre-Class Problems 16 for Wednesday, March 28

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem letter.

Properties of Logarithmic Functions

1. Use the properties of logarithms to write the following as a sum and/or difference of logarithms. All variables represent positive numbers.

a. $\log_2 32x$ b. $\ln 5ab$ c. $\log \dfrac{x+y}{100}$

d. $\log_4 \dfrac{64a}{b^5}$ e. $\ln \dfrac{\sqrt{x}}{y^3}$ f. $\log_{3/2} \dfrac{2-u^3}{5u+17}$

g. $\log_6 \sqrt[3]{x^7}\,\sqrt[4]{y}$ h. $\log_{1/4} \dfrac{\sqrt{x^2+4}}{(3x+5)^8}$

i. $\log\left[\sqrt[5]{(9-4w)^3}\,(2w^4-w^2+5)^6\right]$ j. $\ln \dfrac{x(x^2-5)^3}{\sqrt[3]{(4x+7)^4}}$

k. $\log_{1/3} \dfrac{x^3\,\sqrt[4]{3x+5}}{(x+7)^2(4x^3-9)}$ m. $\log_{\pi} \sqrt[3]{\dfrac{t^3-64}{t^3+64}}$

2. Write the following as a single logarithm.

a. b.

c. d.

e. f.

g. h.

i. j.

3. Use the change of base formula and a calculator to approximate the following.

a. b.

c. d.

4. Solve the following logarithmic equations.

a. $\log_x 16 = 2$ b. $\log_x 5 = 3$ c. $\log_3 x = -2$

d. $\log(4t - 9) = 2$ e. $\log_2 x + \log_2(x - 12) = 6$

f. $\ln x = \ln(5 - x)$ g. $\log_8 t = \log_8(7t + 11)$

h. $\log_6(3x + 17) = 2 + \log_6(4 - x)$

Problems available in the textbook: Page 450 … 7 – 68, 79b – 84b and Examples 1 – 6, 8 starting on page 443. Problems available in the textbook: Page 463 … 37 – 60 and Examples 6 – 10 starting on page 457.

SOLUTIONS:

1a. $\log_2 32x$ Back to Problem 1.

$$\log_2 32x = \log_2 32 + \log_2 x = 5 + \log_2 x$$

NOTE: Since $2^5 = 32$, then $\log_2 32 = 5$

Answer: $5 + \log_2 x$

1b. $\ln 5ab$ Back to Problem 1.

$$\ln 5ab = \ln 5 + \ln a + \ln b$$

NOTE: $\ln 5ab = \ln 5 + \ln ab$

$\ln ab = \ln a + \ln b$

Answer: $\ln 5 + \ln a + \ln b$

1c. $\log \dfrac{x + y}{100}$ Back to Problem 1.

$$\log \frac{x + y}{100} = \log(x + y) - \log 100 = \log(x + y) - 2$$

NOTE: Since $10^2 = 100$, then $\log 100 = 2$

Answer: $\log(x + y) - 2$

1d. $\log_4 \dfrac{64a}{b^5}$ Back to Problem 1.

$$\log_4 \frac{64a}{b^5} = \log_4 64 + \log_4 a - \log_4 b^5 = 3 + \log_4 a - \log_4 b^5 =$$

$$3 + \log_4 a - 5\log_4 b$$

NOTE: $\log_4 \dfrac{64a}{b^5} = \log_4 64a - \log_4 b^5$

$\log_4 64a = \log_4 64 + \log_4 a$,

$\log_4 b^5 = 5\log_4 b$.

Answer: $3 + \log_4 a - 5\log_4 b$

1e. $\ln \dfrac{\sqrt{x}}{y^3}$ Back to Problem 1.

$$\ln \frac{\sqrt{x}}{y^3} = \frac{1}{2}\ln x - 3\ln y$$

NOTE: $\ln \frac{\sqrt{x}}{y^3} = \ln \sqrt{x} - \ln y^3$

$$\ln \sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x$$

$$\ln y^3 = 3 \ln y$$

Answer: $\frac{1}{2} \ln x - 3 \ln y$

1f. $\log_{\frac{3}{2}} \frac{2 - u^3}{5u + 17}$ Back to Problem 1.

$$\log_{\frac{3}{2}} \frac{2 - u^3}{5u + 17} = \log_{\frac{3}{2}} (2 - u^3) - \log_{\frac{3}{2}} (5u + 17)$$

Answer: $\log_{\frac{3}{2}} (2 - u^3) - \log_{\frac{3}{2}} (5u + 17)$

1g. $\log_6 \sqrt[3]{x^7} \sqrt[4]{y}$ Back to Problem 1.

$$\log_6 \sqrt[3]{x^7} \sqrt[4]{y} = \frac{7}{3} \log_6 x + \frac{1}{4} \log_6 y$$

NOTE: $\log_6 \sqrt[3]{x^7} \sqrt[4]{y} = \log_6 \sqrt[3]{x^7} + \log_6 \sqrt[4]{y}$

$$\log_6 \sqrt[3]{x^7} = \log_6 x^{7/3} = \frac{7}{3}\log_6 x$$

$$\log_6 \sqrt[4]{y} = \log_6 y^{1/4} = \frac{1}{4}\log_6 y$$

Answer: $\frac{7}{3}\log_6 x + \frac{1}{4}\log_6 y$

1h. $\log_{1/4} \frac{\sqrt{x^2 + 4}}{(3x + 5)^8}$ Back to Problem 1.

$$\log_{1/4} \frac{\sqrt{x^2 + 4}}{(3x + 5)^8} = \frac{1}{2}\log_{1/4}(x^2 + 4) - 8\log_{1/4}(3x + 5)$$

NOTE: $\log_{1/4} \frac{\sqrt{x^2 + 4}}{(3x + 5)^8} = \log_{1/4}\sqrt{x^2 + 4} - \log_{1/4}(3x + 5)^8$

$$\log_{1/4}\sqrt{x^2 + 4} = \log_{1/4}(x^2 + 4)^{1/2} = \frac{1}{2}\log_{1/4}(x^2 + 4)$$

$$\log_{1/4}(3x + 5)^8 = 8\log_{1/4}(3x + 5)$$

Answer: $\frac{1}{2}\log_{1/4}(x^2 + 4) - 8\log_{1/4}(3x + 5)$

1i. $\log\,[\sqrt[5]{(9 - 4w)^3}\,(2w^4 - w^2 + 5)^6]$ Back to Problem 1.

$$\log\left[\sqrt[5]{(9-4w)^3}\,(2w^4 - w^2 + 5)^6\right] =$$

$$\frac{3}{5}\log(9-4w) + 6\log(2w^4 - w^2 + 5)$$

NOTE: $$\log\left[\sqrt[5]{(9-4w)^3}\,(2w^4 - w^2 + 5)^6\right] =$$

$$\log\sqrt[5]{(9-4w)^3} + \log(2w^4 - w^2 + 5)^6$$

$$\log\sqrt[5]{(9-4w)^3} = \log(9-4w)^{3/5} = \frac{3}{5}\log(9-4w)$$

$$\log(2w^4 - w^2 + 5)^6 = 6\log(2w^4 - w^2 + 5)$$

Answer: $\frac{3}{5}\log(9-4w) + 6\log(2w^4 - w^2 + 5)$

1j. $\ln\dfrac{x(x^2-5)^3}{\sqrt[3]{(4x+7)^4}}$

Back to Problem 1.

$$\ln\frac{x(x^2-5)^3}{\sqrt[3]{(4x+7)^4}} = \ln x + 3\ln(x^2-5) - \frac{4}{3}\ln(4x+7)$$

NOTE: $$\ln\frac{x(x^2-5)^3}{\sqrt[3]{(4x+7)^4}} = \ln x(x^2-5)^3 - \ln\sqrt[3]{(4x+7)^4}$$

$$\ln x(x^2-5)^3 = \ln x + \ln(x^2-5)^3 = \ln x + 3\ln(x^2-5)$$

$$\ln \sqrt[3]{(4x+7)^4} = \ln(4x+7)^{4/3} = \frac{4}{3}\ln(4x+7)$$

Answer: $\ln x + 3\ln(x^2-5) - \frac{4}{3}\ln(4x+7)$

1k. $\log_{1/3} \dfrac{x^3\sqrt[4]{3x+5}}{(x+7)^2(4x^3-9)}$ Back to Problem 1.

$$\log_{1/3} \frac{x^3\sqrt[4]{3x+5}}{(x+7)^2(4x^3-9)} =$$

$$3\log_{1/3} x + \frac{1}{4}\log_{1/3}(3x+5) - 2\log_{1/3}(x+7) - \log_{1/3}(4x^3-9)$$

NOTE: $\log_{1/3} \dfrac{x^3\sqrt[4]{3x+5}}{(x+7)^2(4x^3-9)} =$

$$\log_{1/3} x^3\sqrt[4]{3x+5} - \log_{1/3}(x+7)^2(4x^3-9)$$

$$\log_{1/3} x^3\sqrt[4]{3x+5} = \log_{1/3} x^3 + \log_{1/3}(3x+5)^{1/4} =$$

$$3\log_{1/3} x + \frac{1}{4}\log_{1/3}(3x+5)$$

$$\log_{1/3}(x+7)^2(4x^3-9) = \log_{1/3}(x+7)^2 + \log_{1/3}(4x^3-9)$$

$$= 2\log_{1/3}(x+7) + \log_{1/3}(4x^3-9)$$

Answer: $3\log_{1/3} x + \dfrac{1}{4}\log_{1/3}(3x + 5) - 2\log_{1/3}(x + 7) - \log_{1/3}(4x^3 - 9)$

1m. $\log_\pi \sqrt[3]{\dfrac{t^3 - 64}{t^3 + 64}}$ Back to Problem 1.

$$\log_\pi \sqrt[3]{\frac{t^3 - 64}{t^3 + 64}} = \log_\pi \left(\frac{t^3 - 64}{t^3 + 64}\right)^{1/3} = \frac{1}{3}\log_\pi \frac{t^3 - 64}{t^3 + 64} =$$

$$\frac{1}{3}[\log_\pi (t^3 - 64) - \log_\pi (t^3 + 64)]$$

Answer: $\dfrac{1}{3}[\log_\pi (t^3 - 64) - \log_\pi (t^3 + 64)]$

2a. Back to Problem 2.

2b. Back to Problem 2.

2c. Back to Problem 2.

2d. Back to Problem 2.

2e. Back to Problem 2.

2f. Back to Problem 2.

2g. Back to Problem 2.

2h. Back to Problem 2.

2i. Back to Problem 2.

2j. Back to Problem 2.

3a. Back to Problem 3.

3b. Back to Problem 3.

3c. Back to Problem 3.

3d. Back to Problem 3.

4a. $\log_x 16 = 2$ Back to Problem 4.

Using the definition of logarithm ($y = \log_b x$ if and only if $b^y = x$), we will write the logarithmic equation as an exponential equation:

$$\log_x 16 = 2 \Rightarrow x^2 = 16$$

Using square roots to solve the equation $x^2 = 16$, we have that

$$x^2 = 16 \Rightarrow \sqrt{x^2} = \sqrt{16} \Rightarrow |x| = 4 \Rightarrow x = \pm 4$$

Since the base of a logarithm can not be negative, then the solution of $x = -4$ cannot be used. Thus, the only solution of the equation $\log_x 16 = 2$ is $x = 4$.

Answer: $x = 4$

4b. $\log_x 5 = 3$ Back to Problem 4.

Using the definition of logarithm ($y = \log_b x$ if and only if $b^y = x$), we will write the logarithmic equation as an exponential equation:

$$\log_x 5 = 3 \Rightarrow x^3 = 5$$

Using cube roots to solve the equation $x^3 = 5$, we have that

$$x^3 = 5 \Rightarrow \sqrt[3]{x^3} = \sqrt[3]{5} \Rightarrow x = \sqrt[3]{5}$$

Answer: $x = \sqrt[3]{5}$

4c. $\log_3 x = -2$ Back to Problem 4.

Using the definition of logarithm ($y = \log_b x$ if and only if $b^y = x$), we will write the logarithmic equation as an exponential equation:

$$\log_3 x = -2 \Rightarrow x = 3^{-2} \Rightarrow x = \frac{1}{9}$$

We need to check that the number $\frac{1}{9}$ makes the argument of the logarithm positive since the logarithm of a negative number or zero is undefined. The argument of $\log_3 x$ is x. Thus, x will be positive when $x = \frac{1}{9}$. Thus, $\frac{1}{9}$ is a solution of the equation $\log_3 x = -2$. Of course, it is the only solution.

Answer: $x = \frac{1}{9}$

4d. $\log(4t - 9) = 2$ Back to Problem 4.

Recall that log is the notation for the common logarithm, and the base of the common logarithm is 10. Using the definition of logarithm ($y = \log_b x$ if and only if $b^y = x$), we will write the logarithmic equation as an exponential equation:

$$\log(4t - 9) = 2 \Rightarrow 4t - 9 = 10^2 \Rightarrow 4t - 9 = 100$$

Solving the equation $4t - 9 = 100$, we have that $t = \dfrac{109}{4}$.

We need to check that the number $\dfrac{109}{4}$ makes the argument of the logarithm positive since the logarithm of a negative number or zero is undefined. The argument of $\log(4t - 9)$ is $4t - 9$.

When $t = \dfrac{109}{4}$, we have that $4t - 9 = 4\left(\dfrac{109}{4}\right) - 9 = 109 - 9 > 0$.

Thus, $\dfrac{109}{4}$ is a solution of the equation $\log(4t - 9) = 2$. Of course, it is the only solution.

Answer: $t = \dfrac{109}{4}$

4e. $\log_2 x + \log_2(x - 12) = 6$ Back to Problem 4.

First, we'll use the property of logarithms that $\log_b uv = \log_b u + \log_b v$ in order to write $\log_2 x + \log_2(x - 12)$ as $\log_2 x(x - 12)$. Thus,

$$\log_2 x + \log_2(x - 12) = 6 \Rightarrow \log_2 x(x - 12) = 6$$

Now, using the definition of logarithm ($y = \log_b x$ if and only if $b^y = x$), we will write the logarithmic equation $\log_2 x(x - 12) = 6$ as an exponential equation:

$$\log_2 x(x - 12) = 6 \Rightarrow x(x - 12) = 2^6 \Rightarrow x(x - 12) = 64$$

Solving the equation $x(x - 12) = 64$, we have that

$x(x - 12) = 64 \Rightarrow x^2 - 12x = 64 \Rightarrow x^2 - 12x - 64 = 0 \Rightarrow$

$(x + 4)(x - 16) = 0 \Rightarrow x = -4, \ x = 16$

We need to check that the numbers -4 and 16 make the argument of the logarithms positive since the logarithm of a negative number or zero is undefined. The argument of $\log_2 x$ is x and the argument of $\log_2 (x - 12)$ is $x - 12$.

When $x = -4$, we have that $x = -4 < 0$. Thus, when $x = -4$, we have that $\log_2 x = \log_2 (-4)$. However, $\log_2 (-4)$ is undefined. Thus, -4 is a solution of the equation $x(x - 12) = 64$, but it is not a solution of the equation $\log_2 x + \log_2 (x - 12) = 6$.

When $x = 16$, we have that $x = 16 > 0$ and $x - 12 = 16 - 12 > 0$. Thus, 16 is a solution of the equation $\log_2 x + \log_2 (x - 12) = 6$.

Answer: $x = 16$

4f. $\ln x = \ln(5 - x)$ Back to Problem 4.

In order to solve this equation, we will use the fact that any logarithm function is one-to-one. Thus, $\log_b u = \log_b v$ if and only if $u = v$.

Thus, by the one-to-one property, we have that

$$\ln x = \ln(5 - x) \Rightarrow x = 5 - x$$

Solving the equation $x = 5 - x$, we have that $x = \frac{5}{2}$.

We need to check that the number $\frac{5}{2}$ makes the argument of the logarithms positive since the logarithm of a negative number or zero is undefined. The argument of $\ln x$ is x and the argument of $\ln(5 - x)$ is $5 - x$.

When $x = \frac{5}{2}$, we have that $x = \frac{5}{2} > 0$ and $5 - x = 5 - \frac{5}{2} > 0$. Thus, $\frac{5}{2}$ is a solution of the equation $\ln x = \ln(5 - x)$.

Answer: $x = \frac{5}{2}$

4g. $\log_8 t = \log_8(7t + 11)$ Back to Problem 4.

In order to solve this equation, we will use the fact that any logarithm function is one-to-one. Thus, $\log_b u = \log_b v$ if and only if $u = v$.

Thus, by the one-to-one property, we have that

$$\log_8 t = \log_8(7t + 11) \Rightarrow t = 7t + 11$$

Solving the equation $t = 7t + 11$, we have that $t = -\frac{11}{6}$.

We need to check that the number $-\frac{11}{6}$ makes the argument of the logarithms positive since the logarithm of a negative number or zero is undefined. The argument of $\log_8 t$ is t and the argument of $\log_8(7t + 11)$ is $7t + 11$.

When $t = -\dfrac{11}{6}$, we have that $t = -\dfrac{11}{6} < 0$. Thus, when $t = -\dfrac{11}{6}$, we have that $\log_8 t = \log_8 \left(-\dfrac{11}{6} \right)$. However, $\log_8 \left(-\dfrac{11}{6} \right)$ is undefined. Thus, $-\dfrac{11}{6}$ is a solution of the equation $t = 7t + 11$, but it is not a solution of the equation $\log_8 t = \log_8 (7t + 11)$.

Answer: No solution

4h. $\log_6 (3x + 17) = 2 + \log_6 (4 - x)$ Back to Problem 4.

$$\log_6 (3x + 17) = 2 + \log_6 (4 - x) \Rightarrow \log_6 (3x + 17) - \log_6 (4 - x) = 2$$

Now, we'll use the property of logarithms that $\log_b \dfrac{u}{v} = \log_b u - \log_b v$ in order to write $\log_6 (3x + 17) - \log_6 (4 - x)$ as $\log_6 \dfrac{3x + 17}{4 - x}$. Thus,

$$\log_6 (3x + 17) - \log_6 (4 - x) = 2 \Rightarrow \log_6 \frac{3x + 17}{4 - x} = 2$$

Now, using the definition of logarithm ($y = \log_b x$ if and only if $b^y = x$), we will write the logarithmic equation $\log_6 \dfrac{3x + 17}{4 - x} = 2$ as an exponential equation:

$$\log_6 \frac{3x + 17}{4 - x} = 2 \Rightarrow \frac{3x + 17}{4 - x} = 6^2 \Rightarrow \frac{3x + 17}{4 - x} = 36$$

Solving the equation $\frac{3x+17}{4-x} = 36$, we have that

$$\frac{3x+17}{4-x} = 36 \Rightarrow 3x + 17 = 36(4-x) \Rightarrow 3x + 17 = 144 - 36x \Rightarrow$$

$$39x = 127 \Rightarrow x = \frac{127}{39}$$

We need to check that the number $\frac{127}{39}$ makes the argument of the logarithms positive since the logarithm of a negative number or zero is undefined. The argument of $\log_6(3x+17)$ is $3x+17$ and the argument of $\log_6(4-x)$ is $4-x$.

Since $x = \frac{127}{39}$ is a positive number, then $3x$ is a positive number and $3x+17$ is a positive number. Since $x = \frac{127}{39} = 3\frac{10}{39} < 4$, then $4-x$ is a positive number.

Answer: $\frac{127}{39}$

__**Theorem**__ (Properties of Logarithms)

1. $\log_b u^r = r \log_b u$

2. $\log_b uv = \log_b u + \log_b v$

3. $\log_b \dfrac{u}{v} = \log_b u - \log_b v$

4. $\log_b b = 1$

5. $\log_b 1 = 0$

6. $b^{\log_b u} = u$

7. $\log_b b^u = u$

8. Change of Base Formula: $\log_b u = \dfrac{\log_a u}{\log_a b}$

<u>Proof</u>

1. Let $y = \log_b u$. Then by the definition of logarithms, $b^y = u$. Thus, $u^r = (b^y)^r = b^{yr} = b^{ry}$. Writing the exponential equation $u^r = b^{ry}$ in terms of a logarithmic equation, we have that $\log_b u^r = ry$. Since $y = \log_b u$, then we have that $\log_b u^r = r \log_b u$.

2. Let $y = \log_b u$ and $w = \log_b v$. Then by the definition of logarithms, $b^y = u$ and $b^w = v$. Thus, $uv = b^y b^w = b^{y+w}$. Writing the exponential equation $uv = b^{y+w}$ in terms of a logarithmic equation, we have that $\log_b uv = y + w$. Since $y = \log_b u$ and $w = \log_b v$, then $\log_b uv = \log_b u + \log_b v$.

3. Let $y = \log_b u$ and $w = \log_b v$. Then by the definition of logarithms, $b^y = u$ and $b^w = v$. Thus, $\dfrac{u}{v} = \dfrac{b^y}{b^w} = b^{y-w}$. Writing the exponential

equation $\frac{u}{v} = b^{y-w}$ in terms of a logarithmic equation, we have that $\log_b \frac{u}{v} = y - w$. Since $y = \log_b u$ and $w = \log_b v$, then $\log_b \frac{u}{v} = \log_b u - \log_b v$.

Alternate proof: Since $\frac{u}{v} = uv^{-1}$, we have that $\log_b \frac{u}{v} = \log_b uv^{-1}$. Now, applying Property 2, we have that $\log_b uv^{-1} = \log_b u + \log_b v^{-1}$. Now, applying Property 1, we have that $\log_b v^{-1} = -\log_b v$. Thus, we have that $\log_b \frac{u}{v} = \log_b uv^{-1} = \log_b u + \log_b v^{-1} = \log_b u - \log_b v$.

6. Let $y = \log_b u$. Then by the definition of logarithms, $b^y = u$. Since $y = \log_b u$, then $b^{\log_b u} = u$.

7. Follows from applying Property 1 and then Property 4.

8. Let $y = \log_b u$, $w = \log_a u$, and $z = \log_a b$. Then by the definition of logarithms, we have that $b^y = u$, $a^w = u$, and $a^z = b$. Since $a^z = b$, then $b^y = (a^z)^y = a^{yz}$. Since $b^y = u$ and $b^y = a^{yz}$, then $a^{yz} = u$. Since $a^w = u$, then $a^{yz} = a^w$. Thus, $yz = w$. Since $y = \log_b u$, $z = \log_a b$, and $w = \log_a u$, then $(\log_b u)(\log_a b) = \log_a u$. Since b is the base of a logarithm, then $b \neq 1$. Since $\log_a b = 0$ if and only if $b = 1$, then $\log_a b \neq 0$. So, we can solve for $\log_b u$ by dividing both sides of the equation $(\log_b u)(\log_a b) = \log_a u$ by $\log_a b$. Thus, we obtain that $\log_b u = \dfrac{\log_a u}{\log_a b}$.

Alternate proof: Let $y = \log_b u$. Then by the definition of logarithms, $b^y = u$. Taking the logarithm base a of both sides of this equation, we obtain that $\log_a b^y = \log_a u$. By Property 1, we have that $\log_a b^y = y \log_a b$. Thus, $\log_a b^y = \log_a u \Rightarrow y \log_a b = \log_a u$. Since b is the base of a logarithm, then $b \neq 1$. Since $\log_a b = 0$ if and only if $b = 1$, then $\log_a b \neq 0$. Solving for y, we obtain that $y = \dfrac{\log_a u}{\log_a b}$. Since $y = \log_b u$, then $\log_b u = \dfrac{\log_a u}{\log_a b}$.

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