Pre-Class Problems 15 for Monday, March 26

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem letter.

1. If $150,000.00 is invested at a rate of 4% per year, then determine the amount in the investment at the end of 5 years for the following compounding options.

   a. compounded annually
   b. compounded quarterly
   c. compounded monthly
   d. compounded weekly
   e. compounded daily
   f. compounded continuously

Discussion of Logarithmic Functions

Sketch of the graph of Logarithmic Functions

2. Graph the following logarithmic functions.

   a. $g(x) = \log_3 x$
   b. $f(x) = \log_{1/2} x$
   c. $h(x) = \log_4(-x)$
   d. $k(x) = -\log_4 x$
   e. $y = \log_{3/5}(-x)$
   f. $f(x) = \ln x$
   g. $g(x) = 3\log x$
   h. $h(x) = -2\log_{1/3}(-x)$
   i. $f(t) = -\dfrac{3}{4}\log_2 t$
   j. $g(t) = \dfrac{1}{2}\log_{3/4}(-t)$

k. $h(t) = 5\log_{1/4} t$

3. Sketch the graph of the following functions. State the domain of the function and use the sketch to state the range of the function.

a. $f(x) = \log_5(x - 3)$

b. $g(x) = 3\log x - 4$

c. $h(x) = \log_{2/3}(x + 5) + 8$

d. $f(x) = \ln(-x) - 2$

e. $g(t) = -2\log_{3/4}(t - 1) + 6$

f. $h(x) = -\ln(x + 4) - 3$

g. $f(x) = \sqrt{3}\,\log_{12/19}(4x + 8)$

h. $g(x) = \log_{\pi}(6 - x) - 1$

i. $h(t) = \dfrac{1}{3}\log_{1/2}(-3t - 5) + 12$

j. $f(x) = -4\log(8 - x) - 15$

4. Find the domain of the following functions.

a. $f(x) = \ln(9 - x^2)$

b. $g(x) = \log \dfrac{3x + 5}{x - 2}$

c. $h(x) = \log_5 \dfrac{7}{\sqrt{x + 4}}$

d. $f(x) = \log_{1/2}(4x - 3)^2$

Problems available in the textbook: Page 423 … 45 – 56 and Examples 4 and 5 starting on page 420. Page 438 … 9 – 50, 55 – 92 and Examples 1 – 9 starting on page 428.

SOLUTIONS:

1a. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ Back to Problem 1.

$P = \$150{,}000.00,\ r = 4\% = 0.04,\ n = 1$, and $t = 5$

$$A = 150000\left(1 + \frac{0.04}{1}\right)^{1(5)} = 150000(1 + 0.04)^5 = 150000(1.04)^5 = 182497.94$$

Answer: \$182,497.94

NOTE: The investment made \$32,497.94 in 5 years.

1b. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ Back to Problem 1.

$P = \$150{,}000.00,\ r = 4\% = 0.04,\ n = 4$, and $t = 5$

$$A = 150000\left(1 + \frac{0.04}{4}\right)^{4(5)} = 150000(1 + 0.01)^{20} = 150000(1.01)^{20} = 183028.51$$

Answer: \$183,028.51

NOTE: The investment made \$33,028.51 in 5 years.

1c. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ Back to Problem 1.

$P = \$150{,}000.00,\ r = 4\% = 0.04,\ n = 12$, and $t = 5$

$$A = 150000\left(1 + \frac{0.04}{12}\right)^{12(5)} = 150000\left(1 + \frac{0.04}{12}\right)^{60} = 183149.49$$

Answer: \$183,149.49

NOTE: The investment made \$33,149.49 in 5 years.

1d. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ Back to Problem 1.

$P = \$150{,}000.00,\ r = 4\% = 0.04,\ n = 52$, and $t = 5$

$$A = 150000\left(1 + \frac{0.04}{52}\right)^{52(5)} = 150000\left(1 + \frac{0.04}{52}\right)^{260} = 183196.33$$

Answer: \$183,196.33

NOTE: The investment made \$33,196.33 in 5 years.

1e. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ Back to Problem 1.

$P = \$150{,}000.00,\ r = 4\% = 0.04,\ n = 365$, and $t = 5$

$$A = 150000\left(1 + \frac{0.04}{365}\right)^{365(5)} = 150000\left(1 + \frac{0.04}{365}\right)^{1825} = 183208.41$$

Answer: $183,208.41

NOTE: The investment made $33,208.41 in 5 years.

1f. You will need the topics in calculus to show that as $n \to \infty$, $\left(1 + \frac{r}{n}\right)^{nt} \to e^{rt}$. Thus, as $n \to \infty$, $A = P\left(1 + \frac{r}{n}\right)^{nt} \to Pe^{rt}$

$A = Pe^{rt}$ Back to Problem 1.

$P = \$150{,}000.00$, $r = 4\% = 0.04$, and $t = 5$

$$A = 150000e^{0.04(5)} = 150000e^{0.2} = 183210.41$$

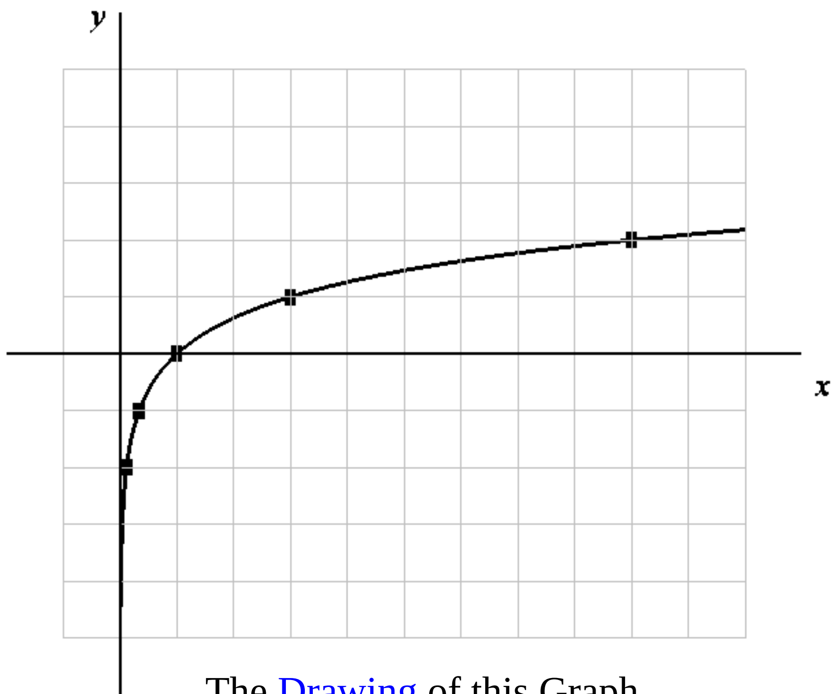
Answer: $183,210.41

NOTE: The investment made $33,210.41 in 5 years.

2a. $g(x) = \log_3 x$ Back to Problem 2.

Note that the domain of the logarithmic function g is $(0, \infty)$. In order to graph the function g given by $g(x) = \log_3 x$, we set $g(x) = y$ and graph the equation $y = \log_3 x$. By the definition of logarithm, $y = \log_3 x$ if and only if $x = 3^y$.

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2



The Drawing of this Graph

The x-intercept of the graph of the function is the point $(1, 0)$.

Note that as $x \to 0$ from the right, $y = \log_3 x \to -\infty$. Thus, the vertical line of $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

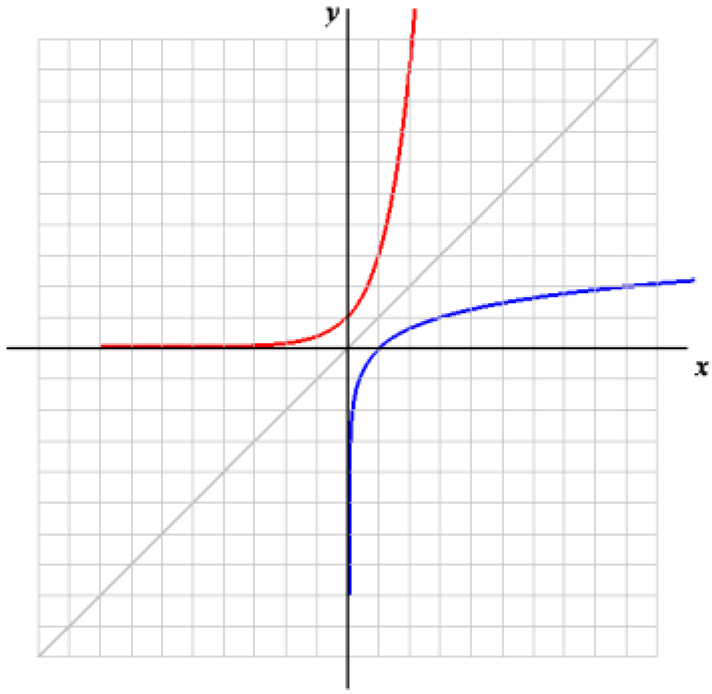
The functions $y = 3^x$ and $y = \log_3 x$ are inverse functions of one another:

$$y = 3^x \Rightarrow x = \log_3 y$$

$$y = \log_3 x \Rightarrow x = 3^y$$

We graphed the function $f(x) = 3^x$ in Pre-Class Problems 19.

The graph of $y = 3^x$ is red and the graph of $y = \log_3 x$ is blue:



The Drawing of these Graphs

Each graph is a reflection of the other through the line $y = x$, which is gray.

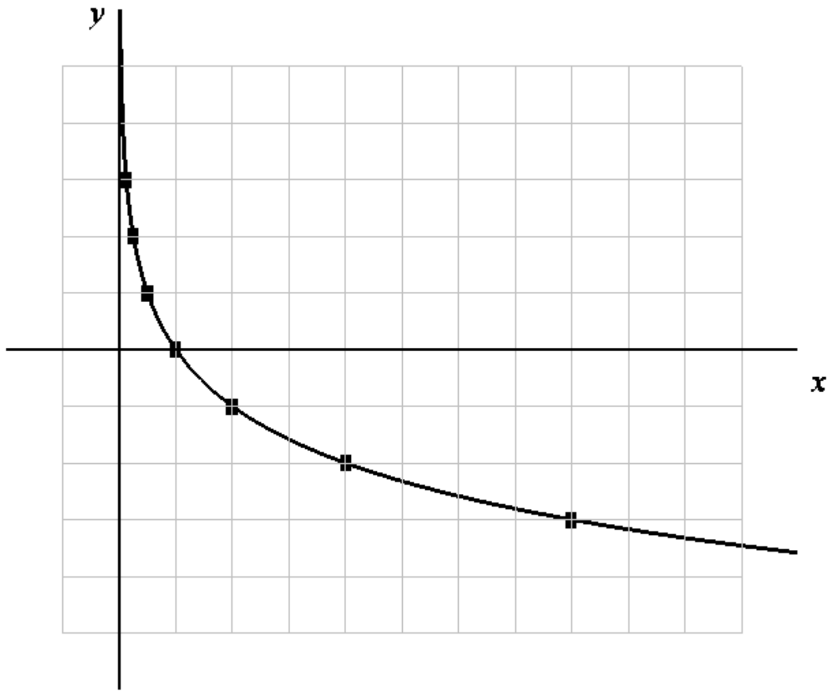
2b. $f(x) = \log_{1/2} x$

Back to Problem 2.

Note that the domain of the logarithmic function f is $(0, \infty)$. In order to graph the function f given by $f(x) = \log_{1/2} x$, we set $f(x) = y$ and graph the equation $y = \log_{1/2} x$. By the definition of logarithm,

$y = \log_{1/2} x$ if and only if $x = \left(\frac{1}{2}\right)^y$.

x	y
8	-3
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2
$\frac{1}{8}$	3



The Drawing of this Graph

The x-intercept of the graph of the function is the point $(1, 0)$.

Note that as $x \to 0$ from the right, $y = \log_{1/2} x \to \infty$. Thus, the vertical line of $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

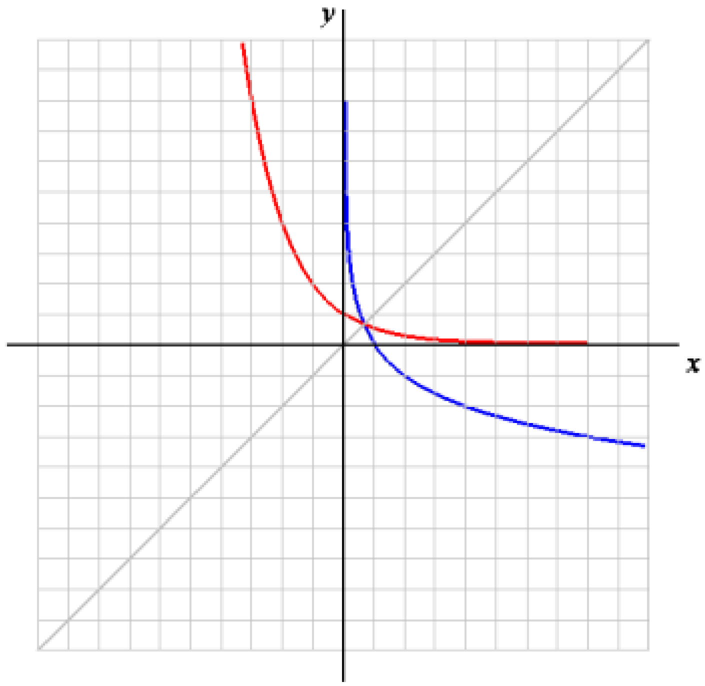
The functions $y = \log_{1/2} x$ and $y = \left(\frac{1}{2}\right)^x$ are inverse functions of one another:

$$y = \left(\frac{1}{2}\right)^x \Rightarrow x = \log_{1/2} y$$

$$y = \log_{1/2} x \Rightarrow x = \left(\frac{1}{2}\right)^y$$

We graphed the function $g(x) = \left(\frac{1}{2}\right)^x$ in Pre-Class Problems 19.

The graph of $y = \left(\frac{1}{2}\right)^x$ is red and the graph of $y = \log_{1/2} x$ is blue:



The Drawing of these Graphs

Each graph is a reflection of the other through the line $y = x$, which is gray.

2c. $h(x) = \log_4(-x)$ Back to Problem 2.

Note that the domain of the logarithmic function h is $(-\infty, 0)$. In order to graph the function h given by $h(x) = \log_4(-x)$, we set $h(x) = y$ and graph the equation $y = \log_4(-x)$. By the definition of logarithm, $y = \log_4(-x)$ if and only if $-x = 4^y \Rightarrow x = -4^y$.

x	y
$-\frac{1}{16}$	-2
$-\frac{1}{4}$	-1
-1	0
-4	1
-16	2

The Drawing of this Graph

The x-intercept of the graph of the function is the point $(-1, 0)$.

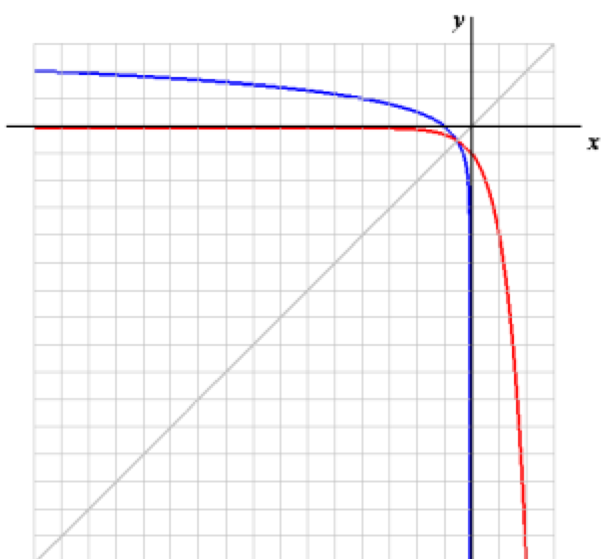
Note that as $x \to 0$ from the left, $y = \log_4(-x) \to -\infty$. Thus, the vertical line of $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

The functions $y = \log_4(-x)$ and $y = -4^x$ are inverse functions of one another:

$$y = \log_4(-x) \Rightarrow -x = 4^y \Rightarrow x = -4^y$$

$$y = -4^x \Rightarrow -y = 4^x \Rightarrow x = \log_4(-y)$$

The graph of $y = -4^x$ is red and the graph of $y = \log_4(-x)$ is blue:



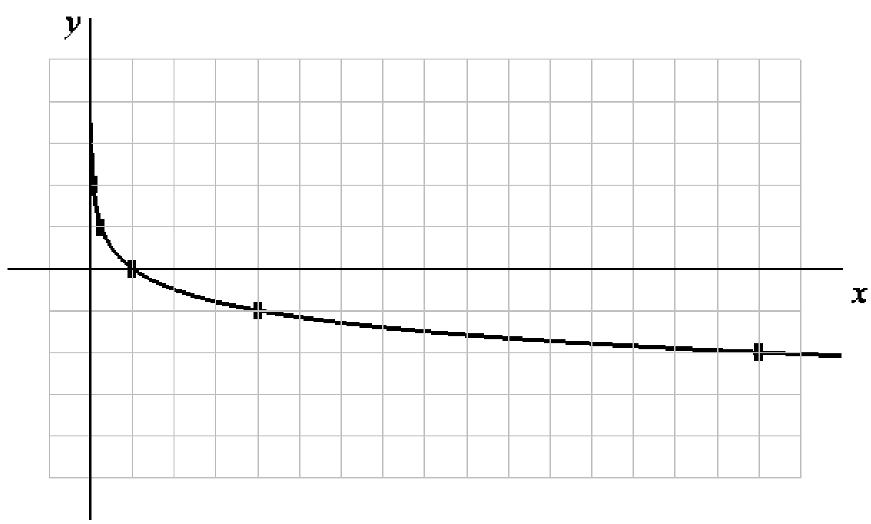
The Drawing of these Graphs

Each graph is a reflection of the other through the line $y = x$, which is gray.

2d. $k(x) = -\log_4 x$ Back to Problem 2.

Note that the domain of the logarithmic function k is $(0, \infty)$. In order to graph the function k given by $k(x) = -\log_4 x$, we set $h(x) = y$ and graph the equation $y = -\log_4 x$. Since $y = -\log_4 x \Rightarrow -y = \log_4 x$, then by the definition of logarithm, $-y = \log_4 x$ if and only if $x = 4^{-y}$.

x	y
16	-2
4	-1
1	0
$\frac{1}{4}$	1
$\frac{1}{16}$	2



The Drawing of this Graph

The x-intercept of the graph of the function is the point $(1, 0)$.

Note that as $x \to 0$ from the right, $y = -\log_4 x \to \infty$. Thus, the vertical line of $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

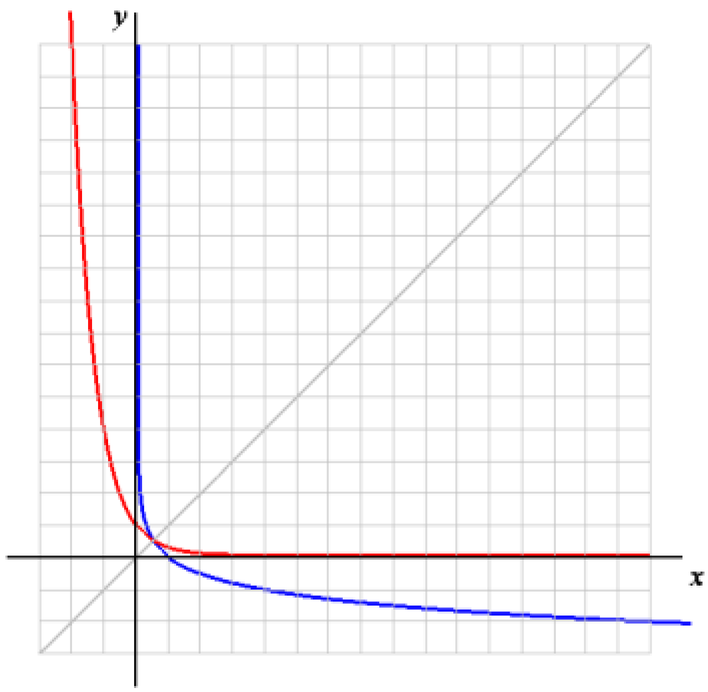
The functions $k(x) = -\log_4 x$ and $h(x) = 4^{-x}$ are inverse functions of one another:

$$y = 4^{-x} \;\Rightarrow\; -x = \log_4 y \;\Rightarrow\; x = -\log_4 y$$

$$y = -\log_4 x \;\Rightarrow\; -y = \log_4 x \;\Rightarrow\; x = 4^{-y}$$

We graphed the function $h(x) = 4^{-x}$ in Pre-Class Problems 19.

The graph of $y = 4^{-x}$ is red and the graph of $y = -\log_4 x$ is blue:



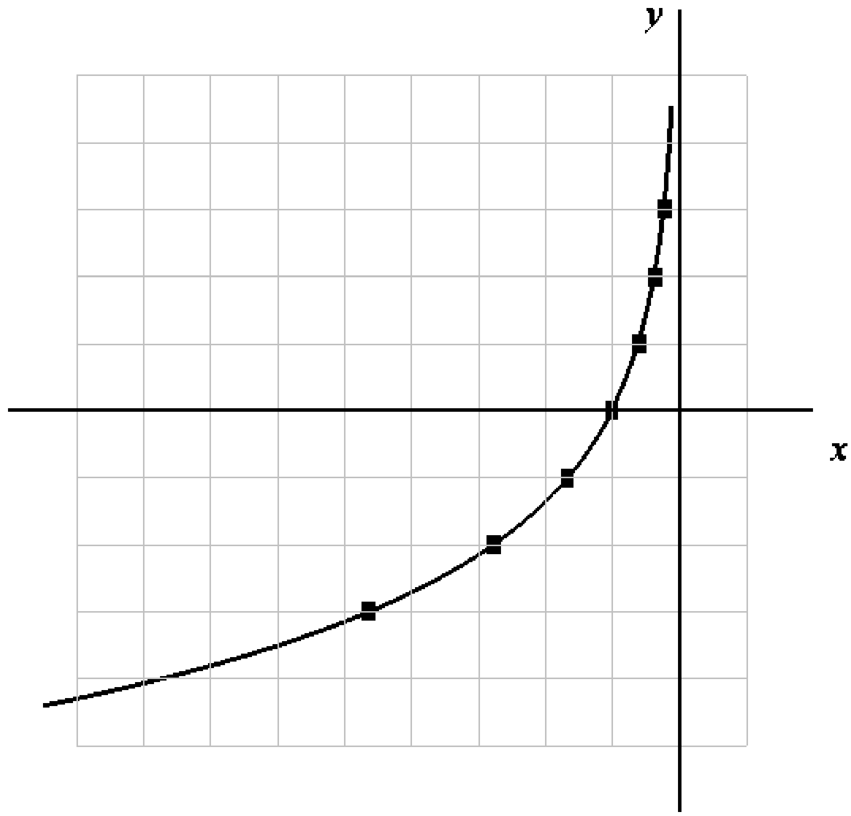
The Drawing of these Graphs

Each graph is a reflection of the other through the line $y = x$, which is gray.

2e. $y = \log_{3/5}(-x)$ Back to Problem 2.

Note that the domain of the logarithmic function is $(-\infty, 0)$. By the definition of logarithm, $y = \log_{3/5}(-x)$ if and only if $-x = \left(\frac{3}{5}\right)^y \Rightarrow$ $x = -\left(\frac{3}{5}\right)^y$.

x	y
$-\frac{125}{27}$	-3
$-\frac{25}{9}$	-2
$-\frac{5}{3}$	-1
-1	0
$-\frac{3}{5}$	1
$-\frac{9}{25}$	2
$-\frac{27}{125}$	3



The Drawing of this Graph

The x-intercept of the graph of the function is the point $(-1, 0)$.

Note that as $x \to 0$ from the left, $y = \log_{3/5}(-x) \to \infty$. Thus, the vertical line of $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

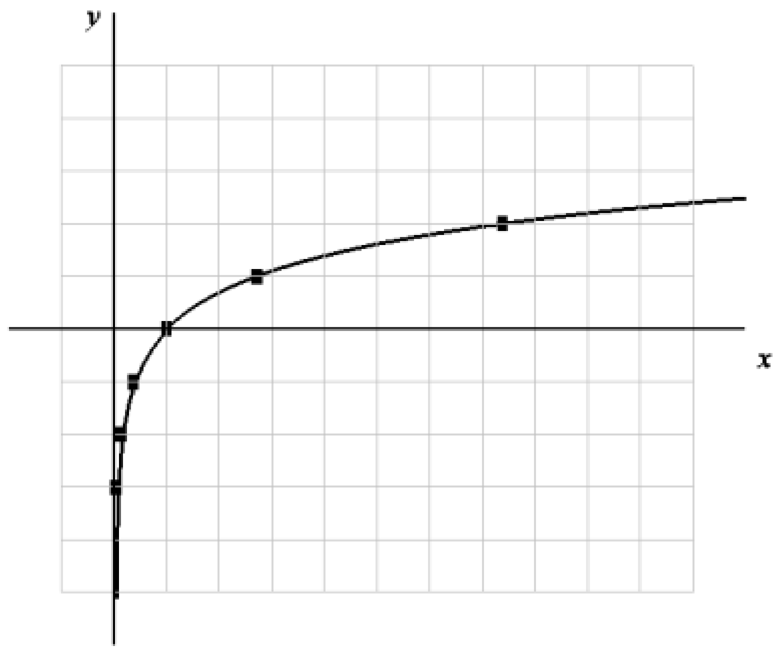
The functions $y = \log_{3/5}(-x)$ and $y = -\left(\frac{3}{5}\right)^x$ are inverse functions of one another.

2f. $f(x) = \ln x$ Back to Problem 2.

Recall: $\ln x = \log_e x$, where $e = 2.718281828....$

Note that the domain of the logarithmic function f is $(0, \infty)$. In order to graph the function f given by $f(x) = \ln x$, we set $f(x) = y$ and graph the equation $y = \ln x$. By the definition of logarithm, $y = \ln x$ if and only if $x = e^y$.

x	y
$e^{-3} \approx 0.04979$	-3
$e^{-2} \approx 0.13534$	-2
$e^{-1} \approx 0.36788$	-1
1	0
$e \approx 2.71828$	1
$e^2 \approx 7.38906$	2
$e^3 \approx 20.08554$	3



The Drawing of this Graph

The x-intercept of the graph of the function is the point $(1, 0)$.

Note that as $x \to 0$ from the right, $y = \ln x \to -\infty$. Thus, the vertical line of $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

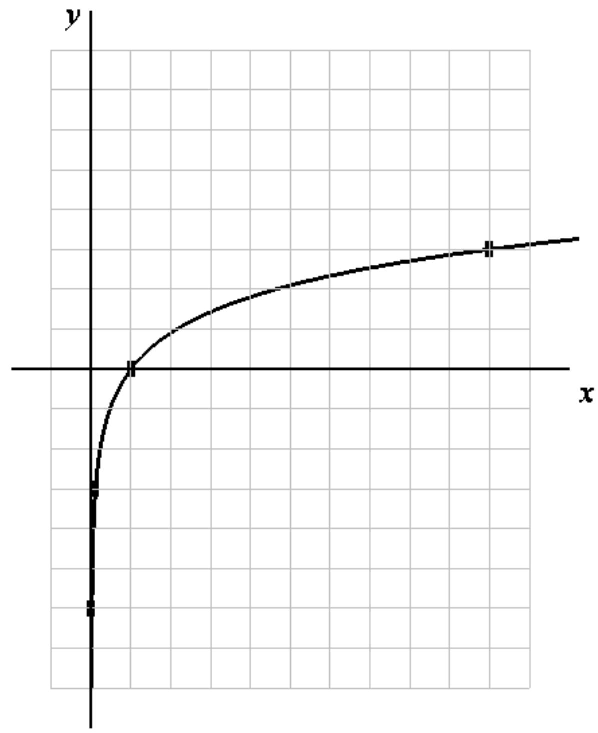
The functions $y = \ln x$ and $y = e^x$ are inverse functions of one another.

2g. $g(x) = 3 \log x$ Back to Problem 2.

Recall: $\log x = \log_{10} x$

Note that the domain of the logarithmic function g is $(-\infty, 0)$. In order to graph the function g given by $g(x) = 3 \log x$, we set $g(x) = y$ and graph the equation $y = 3 \log x$. Since $y = 3 \log x \Rightarrow \dfrac{y}{3} = \log x$, then by the definition of logarithm, $\dfrac{y}{3} = \log x$ if and only if $x = 10^{y/3}$.

x	y
$\frac{1}{100}$	-6
$\frac{1}{10}$	-3
1	0
10	3
100	6



The Drawing of this Graph

The x-intercept of the graph of the function is the point $(1, 0)$.

Note that as $x \to 0$ from the right, $y = \log x \to -\infty$. Thus, the vertical line of $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

The functions $y = 3\log x$ and $y = 10^{x/3}$ are inverse functions of one another.

2h. $h(x) = -2\log_{1/3}(-x)$ Back to Problem 2.

Note that the domain of the logarithmic function h is $(-\infty, 0)$. In order to graph the function h given by $h(x) = -2\log_{1/3}(-x)$, we set $h(x) = y$ and graph the equation $y = -2\log_{1/3}(-x)$. Since $y = -2\log_{1/3}(-x)$ $\Rightarrow -\dfrac{y}{2} = \log_{1/3}(-x)$, then by the definition of logarithm, $-\dfrac{y}{2} = \log_{1/3}(-x)$ if and only if $-x = \left(\dfrac{1}{3}\right)^{-y/2} \Rightarrow -x = 3^{y/2} \Rightarrow x = -3^{y/2}$.

NOTE: $h\left(-\dfrac{1}{9}\right) = -2\log_{1/3}\dfrac{1}{9} = -2(2) = -4$

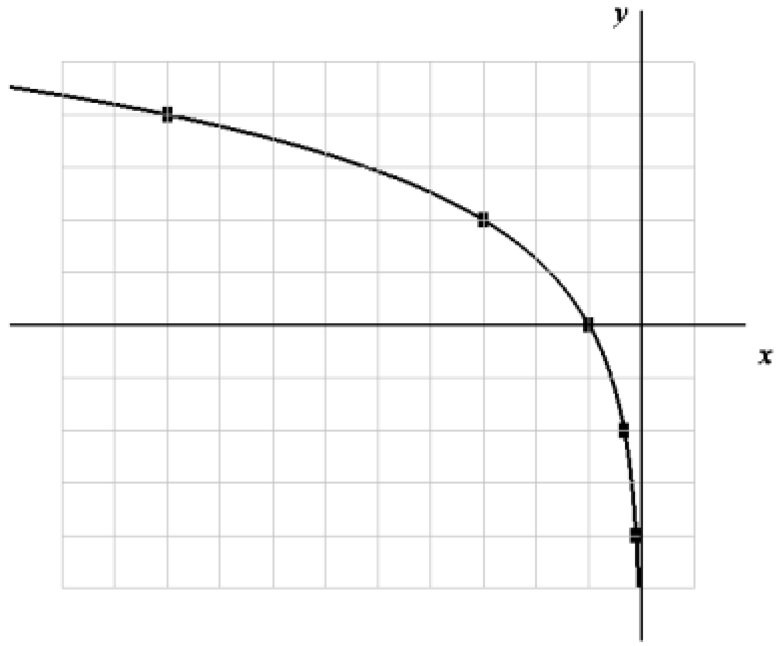
$$h\left(-\frac{1}{3}\right) = -2\log_{1/3}\frac{1}{3} = -2(1) = -2$$

$$h(-1) = -2\log_{1/3}1 = -2(0) = 0$$

$$h(-3) = -2\log_{1/3}3 = -2(-1) = 2$$

$$h(-9) = -2\log_{1/3}9 = -2(-2) = 4$$

x	y
$-\frac{1}{9}$	-4
$-\frac{1}{3}$	-2
-1	0
-3	2
-9	4



The Drawing of this Graph

The x-intercept of the graph of the function is the point $(-1, 0)$.

Note that as $x \to 0$ from the right, $y = -2\log_{1/3}(-x) \to -\infty$. Thus, the vertical line of $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

The functions $y = -2\log_{1/3}(-x)$ and $y = -3^{x/2}$ are inverse functions of one another.

2i. $f(t) = -\dfrac{3}{4}\log_2 t$ Back to Problem 2.

Note that the domain of the logarithmic function f is $(0, \infty)$. In order to graph the function f given by $f(t) = -\dfrac{3}{4}\log_2 t$, we set $f(t) = y$ and graph the equation $y = -\dfrac{3}{4}\log_2 t$. Since $y = -\dfrac{3}{4}\log_2 t \Rightarrow$

$-\frac{4y}{3} = \log_2 t$, then by the definition of logarithm, $-\frac{4y}{3} = \log_2 t$ if and only if $t = 2^{-4y/3}$.

NOTE: $$f\left(\frac{1}{8}\right) = -\frac{3}{4}\log_2\frac{1}{8} = -\frac{3}{4}(-3) = \frac{9}{4}$$

$$f\left(\frac{1}{4}\right) = -\frac{3}{4}\log_2\frac{1}{4} = -\frac{3}{4}(-2) = \frac{3}{2}$$

$$f\left(\frac{1}{2}\right) = -\frac{3}{4}\log_2\frac{1}{2} = -\frac{3}{4}(-1) = \frac{3}{4}$$

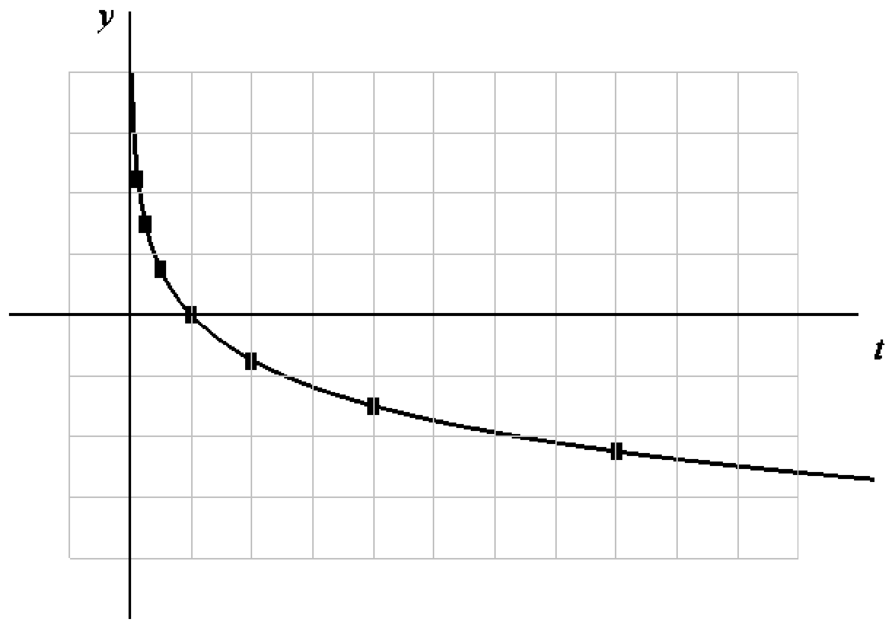
$$f(1) = -\frac{3}{4}\log_2 1 = -\frac{3}{4}(0) = 0$$

$$f(2) = -\frac{3}{4}\log_2 2 = -\frac{3}{4}(1) = -\frac{3}{4}$$

$$f(4) = -\frac{3}{4}\log_2 4 = -\frac{3}{4}(2) = -\frac{3}{2}$$

$$f(8) = -\frac{3}{4}\log_2 8 = -\frac{3}{4}(3) = -\frac{9}{4}$$

t	y
$\frac{1}{8}$	$\frac{9}{4}$
$\frac{1}{4}$	$\frac{3}{2}$
$\frac{1}{2}$	$\frac{3}{4}$
1	0
2	$-\frac{3}{4}$
4	$-\frac{3}{2}$
8	$-\frac{9}{4}$



The Drawing of this Graph

The t-intercept of the graph of the function is the point $(1, 0)$.

Note that as $t \to 0$ from the right, $y = -\frac{3}{4}\log_2 t \to \infty$. Thus, the vertical line of $t = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

The functions $y = -\frac{3}{4}\log_2 t$ and $y = 2^{-4t/3}$ are inverse functions of one another.

2j. $g(t) = \frac{1}{2}\log_{3/4}(-t)$

Back to Problem 2.

Note that the domain of the logarithmic function g is $(-\infty, 0)$. In order to graph the function g given by $g(t) = \frac{1}{2}\log_{3/4}(-t)$, we set $g(t) = y$ and graph the equation $y = \frac{1}{2}\log_{3/4}(-t)$. Since $y = \frac{1}{2}\log_{3/4}(-t) \Rightarrow 2y = \log_{3/4}(-t)$, then by the definition of logarithm, $2y = \log_{3/4}(-t)$ if and only if $-t = \left(\frac{3}{4}\right)^{2y} \Rightarrow t = -\left(\frac{3}{4}\right)^{2y}$.

NOTE: Since $\left(\frac{3}{4}\right)^{2y} = \left[\left(\frac{3}{4}\right)^{2}\right]^{y} = \left(\frac{9}{16}\right)^{y}$, then $t = -\left(\frac{3}{4}\right)^{2y} = -\left(\frac{9}{16}\right)^{y}$

NOTE: $g\left(-\frac{64}{27}\right) = \frac{1}{2}\log_{3/4}\frac{64}{27} = \frac{1}{2}(-3) = -\frac{3}{2}$

$$g\left(-\frac{16}{9}\right) = \frac{1}{2}\log_{3/4}\frac{16}{9} = \frac{1}{2}(-2) = -1$$

$$g\left(-\frac{4}{3}\right) = \frac{1}{2}\log_{3/4}\frac{4}{3} = \frac{1}{2}(-1) = -\frac{1}{2}$$

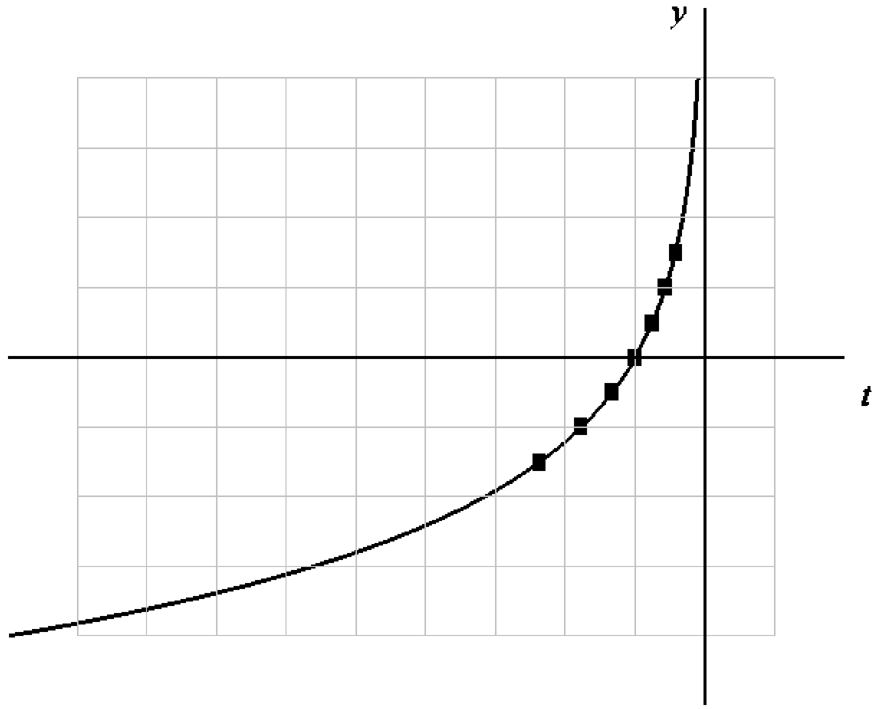
$$g(-1) = \frac{1}{2}\log_{3/4}1 = \frac{1}{2}(0) = 0$$

$$g\left(-\frac{3}{4}\right) = \frac{1}{2}\log_{3/4}\frac{3}{4} = \frac{1}{2}(1) = \frac{1}{2}$$

$$g\left(-\frac{9}{16}\right) = \frac{1}{2}\log_{3/4}\frac{9}{16} = \frac{1}{2}(2) = 1$$

$$g\left(-\frac{27}{64}\right) = \frac{1}{2}\log_{3/4}\frac{27}{64} = \frac{1}{2}(3) = \frac{3}{2}$$

t	y
$-\frac{64}{27}$	$-\frac{3}{2}$
$-\frac{16}{9}$	-1
$-\frac{4}{3}$	$-\frac{1}{2}$
-1	0
$-\frac{3}{4}$	$\frac{1}{2}$
$-\frac{9}{16}$	1
$-\frac{27}{64}$	$\frac{3}{2}$



The Drawing of this Graph

The t-intercept of the graph of the function is the point $(-1, 0)$.

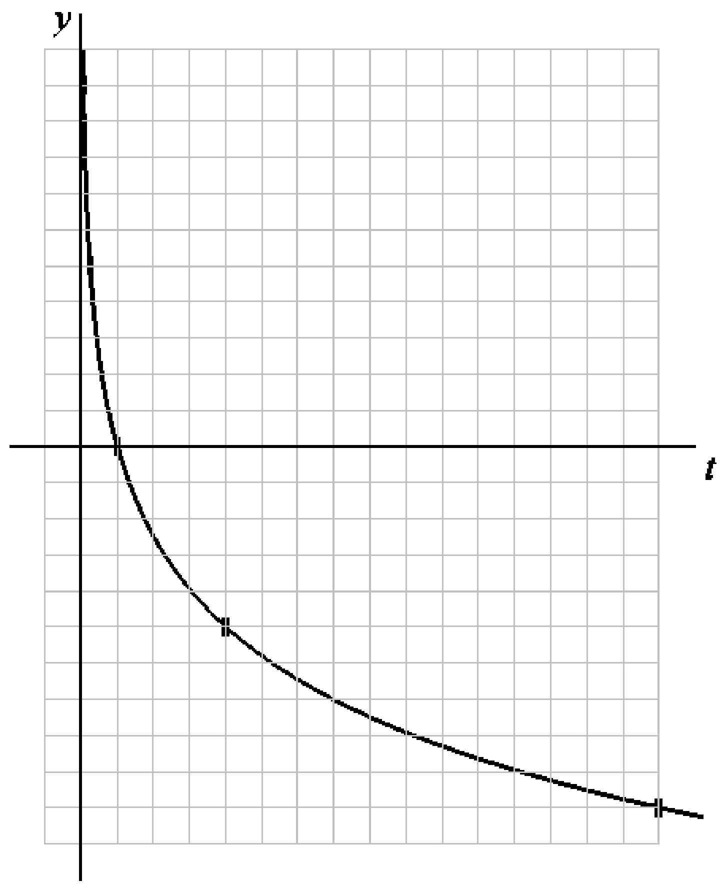
Note that as $t \to 0$ from the left, $y = \frac{1}{2}\log_{3/4}(-t) \to \infty$. Thus, the vertical line of $t = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

The functions $y = \frac{1}{2}\log_{3/4}(-t)$ and $y = -\left(\frac{3}{4}\right)^{2t} = -\left(\frac{9}{16}\right)^{t}$ are inverse functions of one another.

2k. $h(t) = 5\log_{1/4} t$ Back to Problem 2.

Note that the domain of the logarithmic function h is $(0, \infty)$. In order to graph the function h given by $h(t) = 5\log_{1/4} t$, we set $h(t) = y$ and graph the equation $y = 5\log_{1/4} t$. Since $y = 5\log_{1/4} t \Rightarrow \frac{y}{5} = \log_{1/4} t$, then by the definition of logarithm, $\frac{y}{5} = \log_{1/4} t$ if and only if $t = \left(\frac{1}{4}\right)^{y/5} = 4^{-y/5}$.

t	y
$\frac{1}{16}$	10
$\frac{1}{4}$	5
1	0
4	-5
16	-10



The Drawing of this Graph

The t-intercept of the graph of the function is the point $(1, 0)$.

Note that as $t \to 0$ from the right, $y = 5\log_{1/4} t \to \infty$. Thus, the vertical line of $t = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

The functions $y = 5\log_{1/4} t$ and $y = \left(\frac{1}{4}\right)^{t/5} = 4^{-t/5}$ are inverse functions of one another.

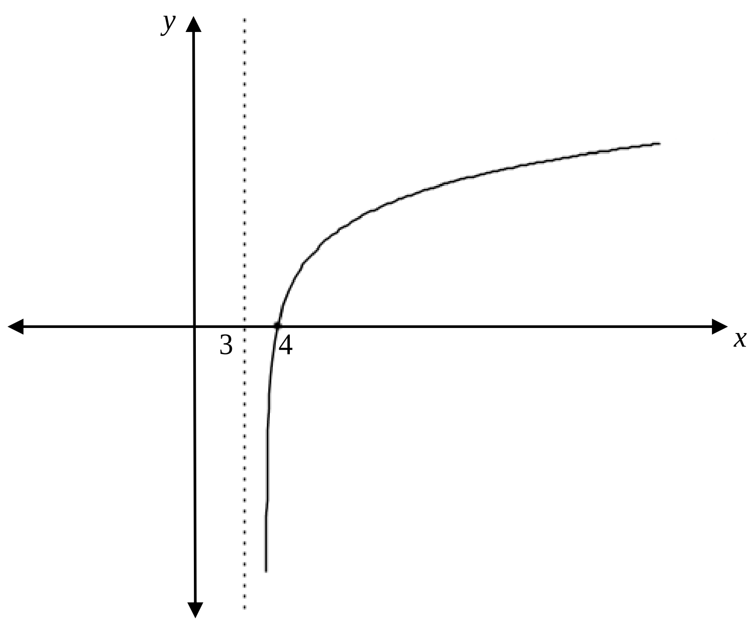
3a. $f(x) = \log_5(x - 3)$ Back to Problem 3.

Since we can only take the logarithm of positive numbers, we need that $x - 3$ be positive. That is, we need that $x - 3 > 0 \Rightarrow x > 3$

Domain: $(3, \infty)$

To graph the function *f*, we set $f(x) = y$ and graph the equation $y = \log_5(x - 3)$.

The graph of $y = \log_5(x - 3)$ is the graph of $y = \log_5 x$ shifted 3 units to the right.



The range of f is $(-\infty, \infty)$. Note that the x-intercept is the point $(4, 0)$.

3b. $g(x) = 3 \log x - 4$ Back to Problem 3.

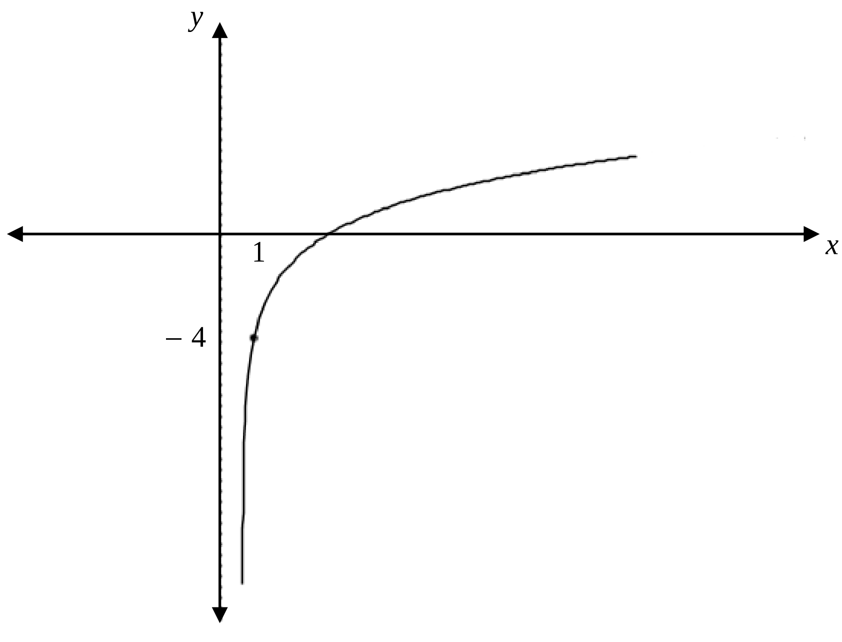
Since we can only take the logarithm of positive numbers, we need that x be positive. That is, we need that $x > 0$

Domain: $(0, \infty)$

To graph the function g, we set $g(x) = y$ and graph the equation $y = 3 \log x - 4$.

$$y = 3 \log x - 4 \Rightarrow y + 4 = 3 \log x$$

The graph of $y = 3 \log x - 4$ is the graph of $y = 3 \log x$ shifted 4 units downward.



The range of g is $(-\infty, \infty)$.

3c. $h(x) = \log_{2/3}(x + 5) + 8$ Back to Problem 3.

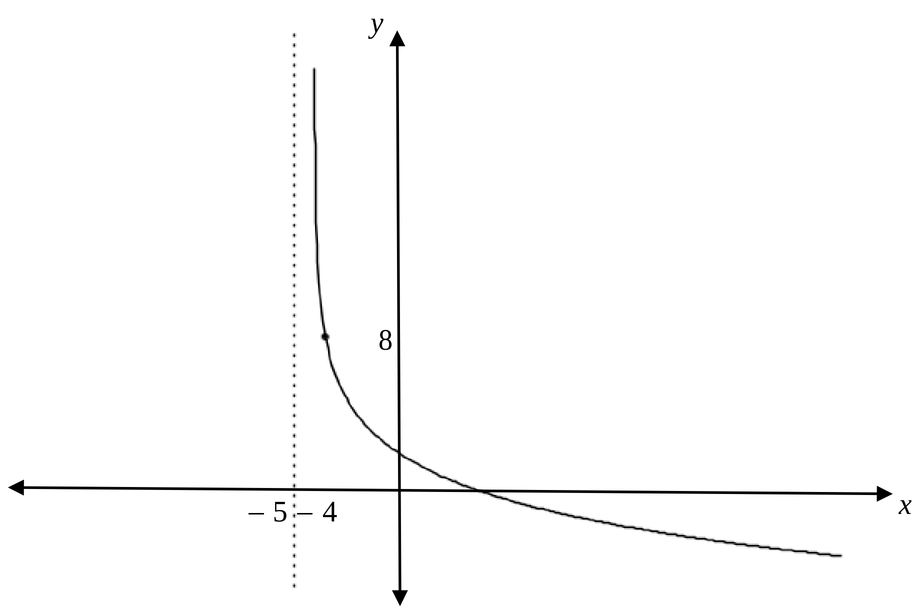
Since we can only take the logarithm of positive numbers, we need that $x + 5$ be positive. That is, we need that $x + 5 > 0 \Rightarrow x > -5$

Domain: $(-5, \infty)$

To graph the function h, we set $h(x) = y$ and graph the equation $y = \log_{2/3}(x + 5) + 8$.

$$y = \log_{2/3}(x + 5) + 8 \Rightarrow y - 8 = \log_{2/3}(x + 5)$$

The graph of $y - 8 = \log_{2/3}(x + 5)$ is the graph of $y = \log_{2/3} x$ shifted 5 units to the left and 8 units upward.



The range of h is $(-\infty, \infty)$.

3d. $f(x) = \ln(-x) - 2$ Back to Problem 3.

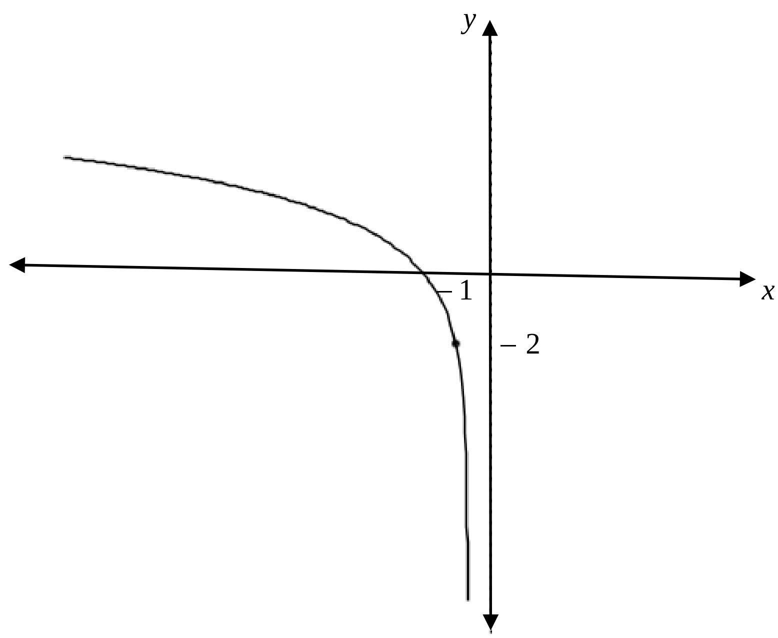
Since we can only take the logarithm of positive numbers, we need that $-x$ be positive. That is, we need that $-x > 0 \Rightarrow x < 0$

Domain: $(-\infty,\ 0)$

To graph the function f, we set $f(x) = y$ and graph the equation $y = \ln(-x) - 2$.

$$y = \ln(-x) - 2 \Rightarrow y + 2 = \ln(-x)$$

The graph of $y + 2 = \ln(-x)$ is the graph of $y = \ln(-x)$ shifted 2 units downward.



The range of f is $(-\infty, \infty)$.

3e. $g(t) = -2\log_{3/4}(t - 1) + 6$ Back to Problem 3.

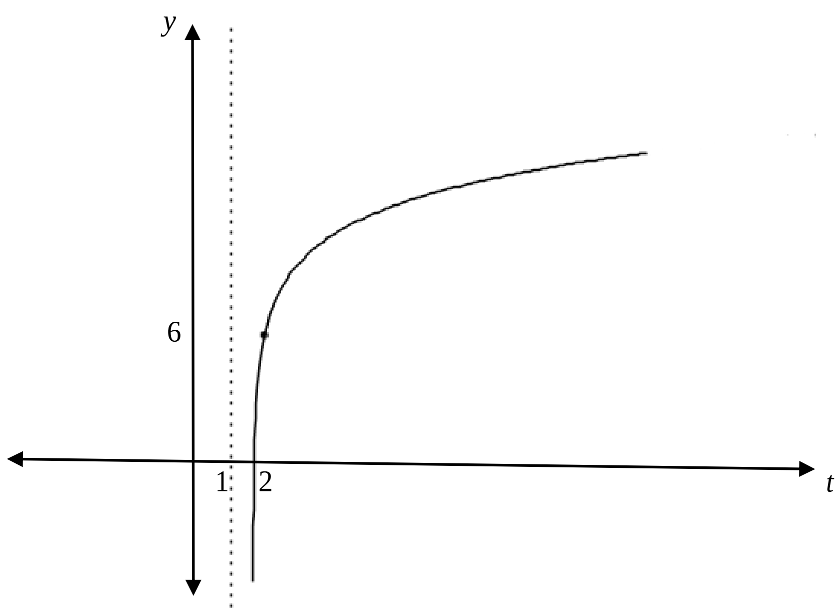
Since we can only take the logarithm of positive numbers, we need that $t - 1$ be positive. That is, we need that $t - 1 > 0 \Rightarrow t > 1$

Domain: $(1, \infty)$

To graph the function g, we set $g(t) = y$ and graph the equation $y = -2\log_{3/4}(t - 1) + 6$.

$$y = -2\log_{3/4}(t - 1) + 6 \Rightarrow y - 6 = -2\log_{3/4}(t - 1)$$

The graph of $y - 6 = -2\log_{3/4}(t - 1)$ is the graph of $y = -2\log_{3/4} t$ shifted 1 units to the right and 6 units upward. The graph of $y = -2\log_{3/4} t$ is the graph of $y = 2\log_{3/4} t$ reflected through the *t*-axis. That is, flipped over with respect to the *t*-axis.



The range of g is $(-\infty, \infty)$.

3f. $h(x) = -\ln(x + 4) - 3$ Back to Problem 3.

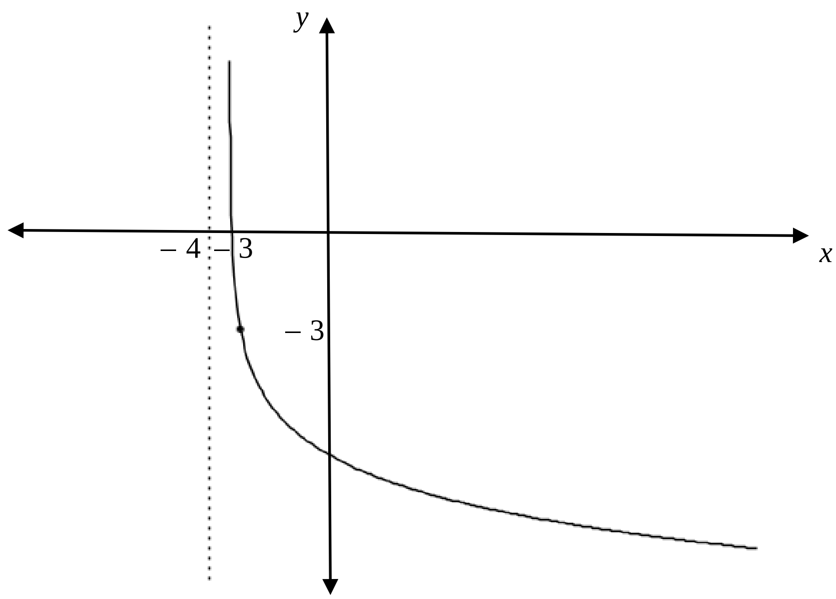
Since we can only take the logarithm of positive numbers, we need that $x + 4$ be positive. That is, we need that $x + 4 > 0 \Rightarrow x > -4$

Domain: $(-4, \infty)$

To graph the function h, we set $h(x) = y$ and graph the equation $y = -\ln(x + 4) - 3$.

$$y = -\ln(x + 4) - 3 \Rightarrow y + 3 = -\ln(x + 4)$$

The graph of $y + 3 = -\ln(x + 4)$ is the graph of $y = -\ln x$ shifted 4 units to the left and 3 units downward. The graph of $y = -\ln x$ is the graph of $y = \ln x$ reflected through the *x*-axis. That is, flipped over with respect to the *x*-axis.



The range of h is $(-\infty, \infty)$.

3g. $f(x) = \sqrt{3}\,\log_{12/19}(4x + 8)$ Back to Problem 3.

Since we can only take the logarithm of positive numbers, we need that $4x + 8$ be positive. That is, we need that $4x + 8 > 0 \Rightarrow 4x > -8 \Rightarrow$ $x > -2$

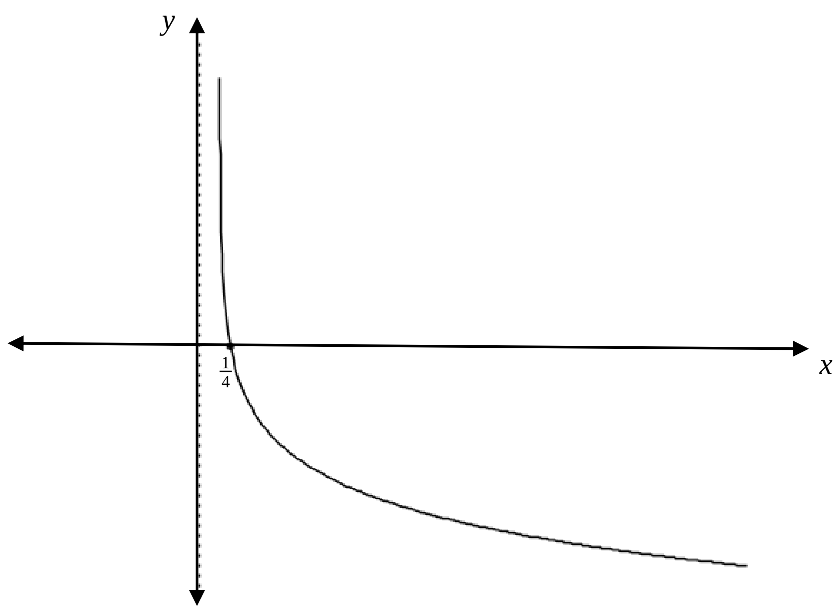
Domain: $(-2, \infty)$

To graph the function f, we set $f(x) = y$ and graph the equation $y = \sqrt{3}\,\log_{12/19}(4x + 8)$.

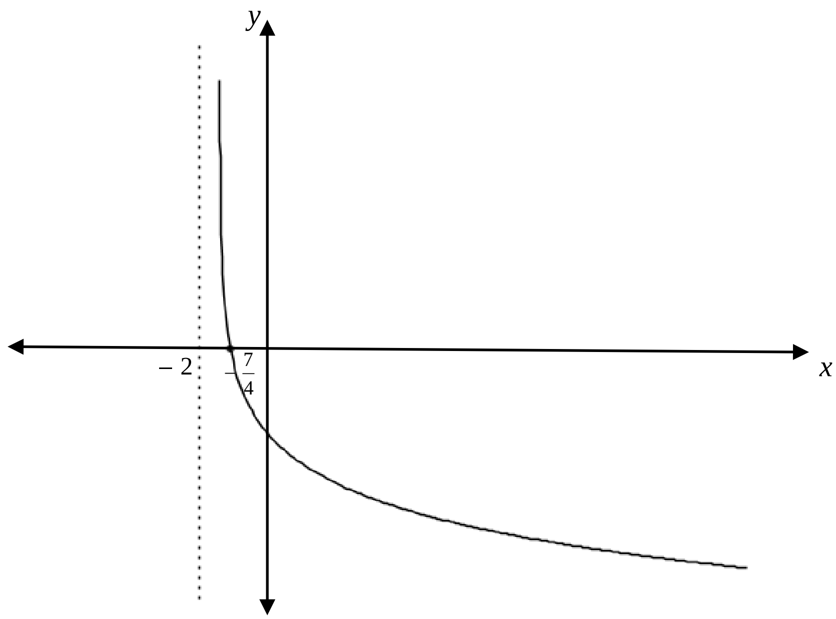
$$y = \sqrt{3}\,\log_{12/19}(4x + 8) \Rightarrow y = \sqrt{3}\,\log_{12/19}[4(x + 2)]$$

The graph of $y = \sqrt{3}\,\log_{12/19}[4(x + 2)]$ is the graph of $y = \sqrt{3}\,\log_{12/19} 4x$ shifted 2 units to the left.

The graph of $y = \sqrt{3}\,\log_{12/19} 4x$:



The graph of $y = \sqrt{3}\,\log_{12/19}[4(x + 2)]$:



The range of f is $(-\infty, \infty)$.

3h. $g(x) = \log_{\pi}(6 - x) - 1$

Back to Problem 3.

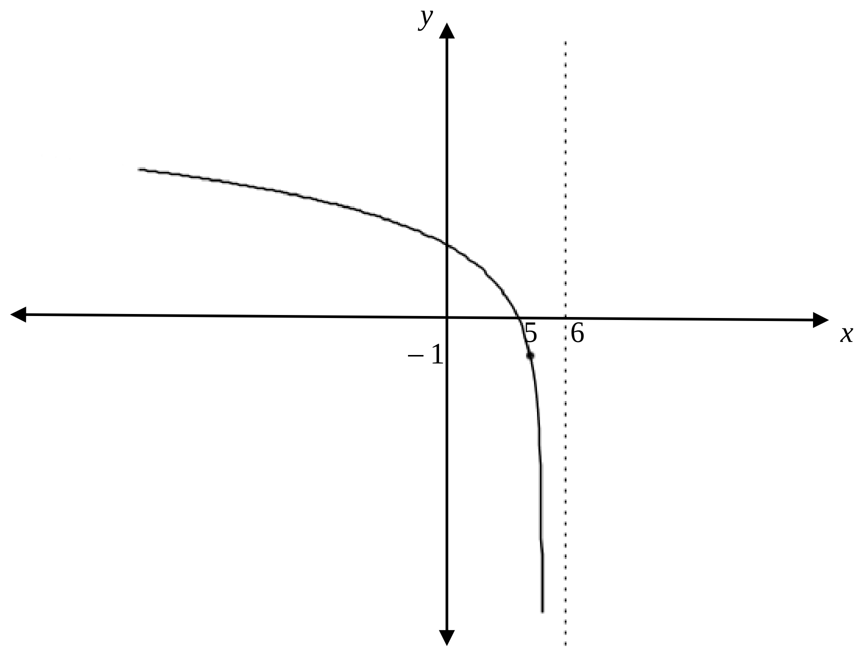
Since we can only take the logarithm of positive numbers, we need that $6 - x$ be positive. That is, we need that $6 - x > 0 \Rightarrow 6 > x \Rightarrow x < 6$.

Domain: $(-\infty, 6)$

To graph the function g, we set $g(x) = y$ and graph the equation $y = \log_{\pi}(6 - x) - 1$.

$$y = \log_{\pi}(6 - x) - 1 \Rightarrow y + 1 = \log_{\pi}[-(x - 6)]$$

The graph of $y + 1 = \log_{\pi}(6 - x)$ is the graph of $y = \log_{\pi}(-x)$ shifted 6 units to the right and 1 unit downward.



The range of g is $(-\infty, \infty)$.

3i. $h(t) = \frac{1}{3}\log_{1/2}(-3t - 5) + 12$

Back to Problem 3.

Since we can only take the logarithm of positive numbers, we need that $-3t - 5$ be positive. That is, we need that $-3t - 5 > 0 \Rightarrow -3t > -5$ $\Rightarrow t < \frac{5}{3}$.

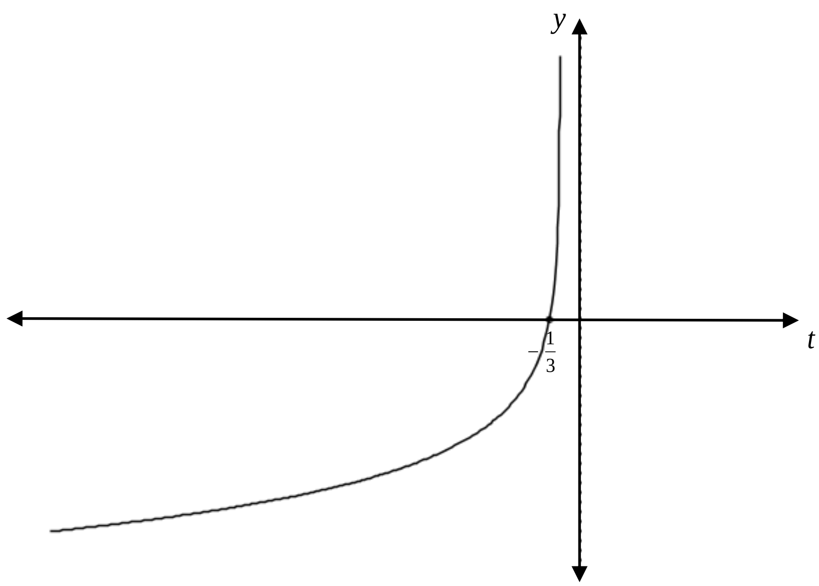
Domain: $\left(-\infty, \frac{5}{3}\right)$

To graph the function h, we set $h(t) = y$ and graph the equation $y = \frac{1}{3}\log_{1/2}(-3t - 5) + 12$.

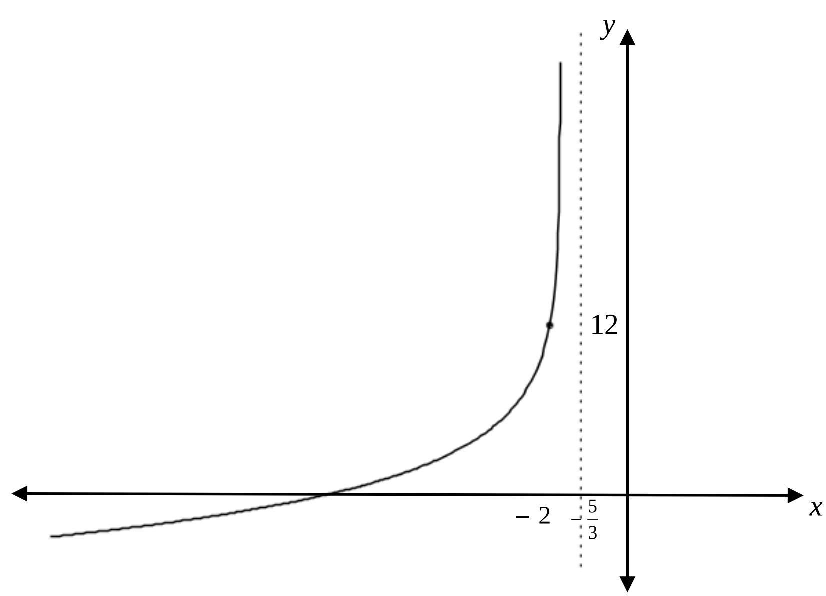
$$y = \frac{1}{3}\log_{1/2}(-3t - 5) + 12 \Rightarrow y - 12 = \frac{1}{3}\log_{1/2}\left[-3\left(t + \frac{5}{3}\right)\right]$$

The graph of $y - 12 = \frac{1}{3}\log_{1/2}\left[-3\left(t + \frac{5}{3}\right)\right]$ is the graph of $y = \frac{1}{3}\log_{1/2}(-3t)$ shifted $\frac{5}{3}$ units to the left and 12 units upward.

The graph of $y = \frac{1}{3}\log_{1/2}(-3t)$:



The graph of $y - 12 = \frac{1}{3}\log_{1/2}\left[-3\left(t + \frac{5}{3}\right)\right]$:



The range of h is $(-\infty, \infty)$.

3j. $f(x) = -4 \log(8 - x) - 15$ Back to Problem 3.

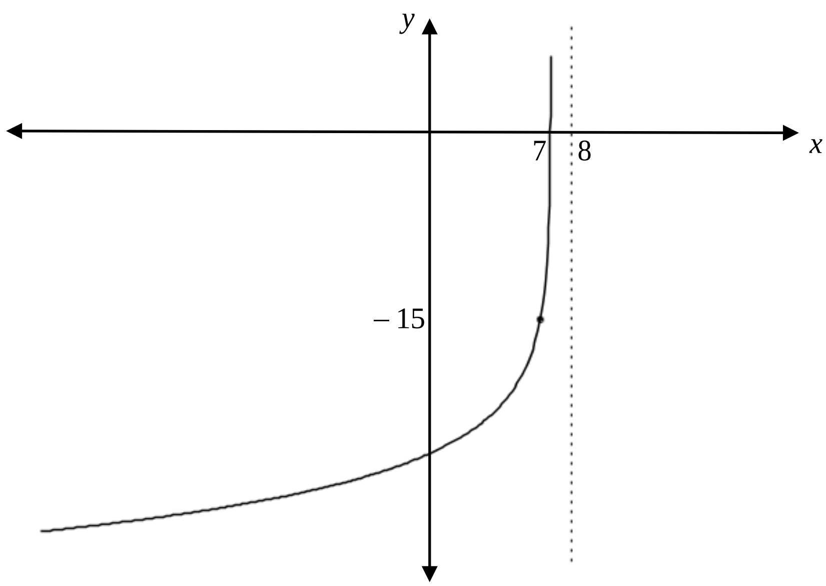
Since we can only take the logarithm of positive numbers, we need that $8 - x$ be positive. That is, we need that $8 - x > 0 \Rightarrow 8 > x \Rightarrow x < 8$.

Domain: $(-\infty, 8)$

To graph the function f, we set $f(x) = y$ and graph the equation $y = -4 \log(8 - x) - 15$.

$$y = -4 \log(8 - x) - 15 \Rightarrow y + 15 = -4 \log[-(x - 8)]$$

The graph of $y + 15 = -4 \log[-(x - 8)]$ is the graph of $y = -4 \log(-x)$ shifted 8 units to the right and 15 units downward. The graph of $y = -4 \log(-x)$ is the graph of $y = 4 \log(-x)$ reflected through the *x*-axis. That is, flipped over with respect to the *x*-axis.



The range of f is $(-\infty, \infty)$.

4a. $f(x) = \ln(9 - x^2)$ Back to Problem 4.

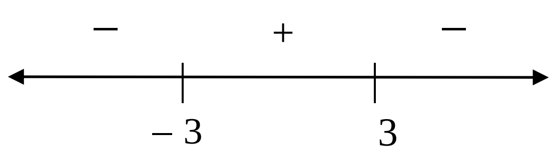
Want (Need): $9 - x^2 > 0$

Step 1:

$9 - x^2 = 0 \Rightarrow 9 = x^2 \Rightarrow x = \pm 3$

$9 - x^2$ is defined for all real numbers *x*.

Step 2: Plot all the numbers found in Step 1 on the real number line.



Sign of $9 - x^2$

Step 3: Use the real number line to identify the open intervals determined by the plotted numbers. Pick a test value for each open interval.

Interval	Test Value	Sign of $9 - x^2 = (3+x)(3-x)$
$(-\infty, -3)$	-4	$(-)(+) = -$
$(-3, 3)$	0	$(+)(+) = +$
$(3, \infty)$	4	$(+)(-) = -$

Answer: $(-3, 3)$

4b. $g(x) = \log \dfrac{3x+5}{x-2}$ Back to Problem 4.

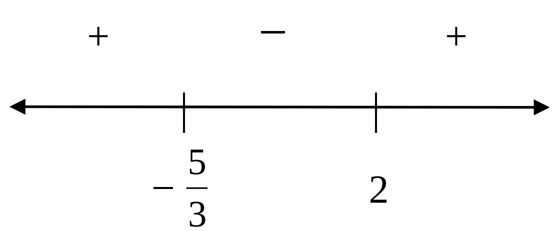
Want (Need): $\dfrac{3x+5}{x-2} > 0$

Step 1:

$$\frac{3x+5}{x-2} = 0 \Rightarrow 3x + 5 = 0 \Rightarrow x = -\frac{5}{3}$$

$$\frac{3x+5}{x-2} \text{ undefined } \Rightarrow x - 2 = 0 \Rightarrow x = 2$$

Step 2: Plot all the numbers found in Step 1 on the real number line.



Sign of $\dfrac{3x+5}{x-2}$

Step 3: Use the real number line to identify the open intervals determined by the plotted numbers. Pick a test value for each open interval.

Interval Test Value Sign of $\dfrac{3x+5}{x-2}$

$\left(-\infty, -\frac{5}{3}\right)$	-2	$\frac{(-)}{(-)} = +$
$\left(-\frac{5}{3}, 2\right)$	0	$\frac{(+)}{(-)} = -$
$(2, \infty)$	3	$\frac{(+)}{(+)} = +$

Answer: $\left(-\infty, -\frac{5}{3}\right) \cup (2, \infty)$

4c. $h(x) = \log_5 \frac{7}{\sqrt{x+4}}$ Back to Problem 4.

NOTE: The nonlinear expression $\frac{7}{\sqrt{x+4}}$ will be defined and will be positive as long as the linear expression $x + 4$ is positive.

Want (Need): $x + 4 > 0$

$x + 4 > 0 \Rightarrow x > -4$

Answer: $(-4, \infty)$

4d. $f(x) = \log_{1/2} (4x - 3)^2$ Back to Problem 4.

NOTE: The nonlinear expression $(4x - 3)^2$ is greater than or equal to zero for all real numbers *x*. Since the logarithm of zero is undefined, we need that $4x - 3 \neq 0 \Rightarrow x \neq \frac{3}{4}$.

Answer: $\left(-\infty, \frac{3}{4} \right) \cup \left(\frac{3}{4}, \infty \right)$

<u>Definition</u> The logarithmic function with base b is the function defined by $f(x) = \log_b x$, where $b > 0$ and $b \neq 1$.

Recall that $y = \log_b x$ if and only if $b^y = x$

Recall the following information about logarithmic functions:

1. The domain of $f(x) = \log_b x$ is the set of positive real numbers. That is, the domain of $f(x) = \log_b x$ is $(0, \infty)$.

2. The range of $f(x) = \log_b x$ is the set of real numbers. That is, the range of $f(x) = \log_b x$ is $(-\infty, \infty)$.

3. The logarithmic function $f(x) = \log_b x$ and the exponential function $g(x) = b^x$ are inverses of one another:

   $(f \circ g)(x) = f(g(x)) = \log_b g(x) = \log_b b^x = x \log_b b = x(1) = x$, for all x in the domain of g, which is the set of all real numbers.

   $(g \circ f)(x) = g(f(x)) = g(\log_b x) = b^{\log_b x} = x$, for all x in the domain of *f*, which is the set of real numbers in the interval $(0, \infty)$.

Definition The natural logarithmic function is the logarithmic function whose base is the irrational number *e*. Thus, the natural logarithmic function is the function defined by $f(x) = \log_e x$, where $e = 2.718281828\ldots.$ Recall that $\log_e x = \ln x$.

Definition The common logarithmic function is the logarithmic function whose base is the number 10. Thus, the common logarithmic function is the function defined by $f(x) = \log_{10} x$. Recall that $\log_{10} x = \log x$.

Theorem (Properties of Logarithms)

1. $\log_b u^r = r \log_b u$

2. $\log_b uv = \log_b u + \log_b v$

3. $\log_b \dfrac{u}{v} = \log_b u - \log_b v$

4. $\log_b b = 1$

5. $\log_b 1 = 0$

6. $b^{\log_b u} = u$

7. $\log_b b^u = u$

8. Change of Bases Formula: $\log_b u = \dfrac{\log_a u}{\log_a b}$

Proof

1. Let $y = \log_b u$. Then by the definition of logarithms, $b^y = u$. Thus, $u^r = (b^y)^r = b^{yr} = b^{ry}$. Writing the exponential equation $u^r = b^{ry}$

in terms of a logarithmic equation, we have that $\log_b u^r = ry$. Since $y = \log_b u$, then we have that $\log_b u^r = r\log_b u$.

2. Let $y = \log_b u$ and $w = \log_b v$. Then by the definition of logarithms, $b^y = u$ and $b^w = v$. Thus, $uv = b^y b^w = b^{y+w}$. Writing the exponential equation $uv = b^{y+w}$ in terms of a logarithmic equation, we have that $\log_b uv = y + w$. Since $y = \log_b u$ and $w = \log_b v$, then $\log_b uv = \log_b u + \log_b v$.

3. Let $y = \log_b u$ and $w = \log_b v$. Then by the definition of logarithms, $b^y = u$ and $b^w = v$. Thus, $\frac{u}{v} = \frac{b^y}{b^w} = b^{y-w}$. Writing the exponential equation $\frac{u}{v} = b^{y-w}$ in terms of a logarithmic equation, we have that $\log_b \frac{u}{v} = y - w$. Since $y = \log_b u$ and $w = \log_b v$, then $\log_b \frac{u}{v} = \log_b u - \log_b v$.

   Alternate proof: Since $\frac{u}{v} = uv^{-1}$, we have that $\log_b \frac{u}{v} = \log_b uv^{-1}$. Now, applying Property 2, we have that $\log_b uv^{-1} = \log_b u + \log_b v^{-1}$. Now, applying Property 1, we have that $\log_b v^{-1} = -\log_b v$. Thus, we have that $\log_b \frac{u}{v} = \log_b uv^{-1} = \log_b u + \log_b v^{-1} = \log_b u - \log_b v$.

6. Let $y = \log_b u$. Then by the definition of logarithms, $b^y = u$. Since $y = \log_b u$, then $b^{\log_b u} = u$.

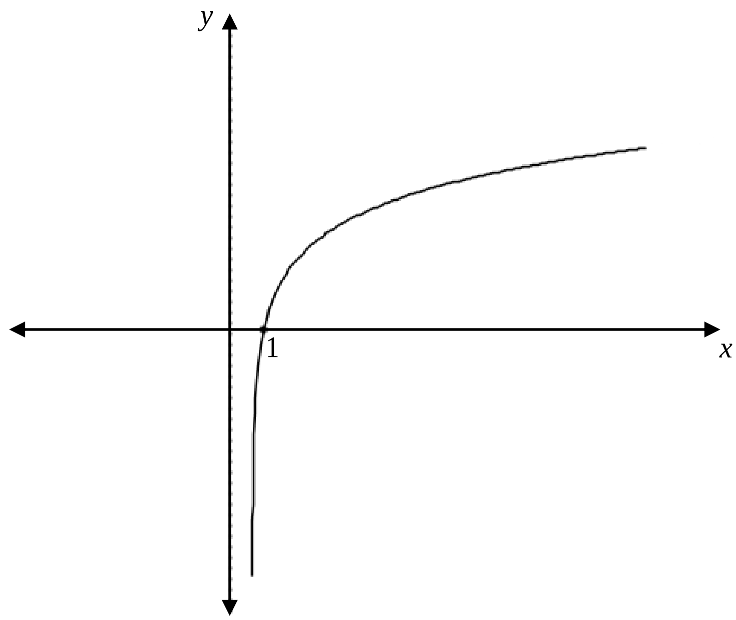
7. Follows from applying Property 1 and then Property 4.

8. Let $y = \log_b u$, $w = \log_a u$, and $z = \log_a b$. Then by the definition of logarithms, we have that $b^y = u$, $a^w = u$, and $a^z = b$. Since $a^z = b$, then $b^y = (a^z)^y = a^{yz}$. Since $b^y = u$ and $b^y = a^{yz}$, then $a^{yz} = u$. Since $a^w = u$, then $a^{yz} = a^w$. Thus, $yz = w$. Since $y = \log_b u$, $z = \log_a b$, and $w = \log_a u$, then $(\log_b u)(\log_a b) = \log_a u$. Since b is the base of a logarithm, then $b \neq 1$. Since $\log_a b = 0$ if and only if $b = 1$, then $\log_a b \neq 0$. So, we can solve for $\log_b u$ by dividing both sides of the equation $(\log_b u)(\log_a b) = \log_a u$ by $\log_a b$. Thus, we obtain that $\log_b u = \dfrac{\log_a u}{\log_a b}$.

Alternate proof: Let $y = \log_b u$. Then by the definition of logarithms, $b^y = u$. Taking the logarithm base a of both sides of this equation, we obtain that $\log_a b^y = \log_a u$. By Property 1, we have that $\log_a b^y = y \log_a b$. Thus, $\log_a b^y = \log_a u \Rightarrow y \log_a b = \log_a u$. Since b is the base of a logarithm, then $b \neq 1$. Since $\log_a b = 0$ if and only if $b = 1$, then $\log_a b \neq 0$. Solving for y, we obtain that $y = \dfrac{\log_a u}{\log_a b}$. Since $y = \log_b u$, then $\log_b u = \dfrac{\log_a u}{\log_a b}$.

Back to Top.

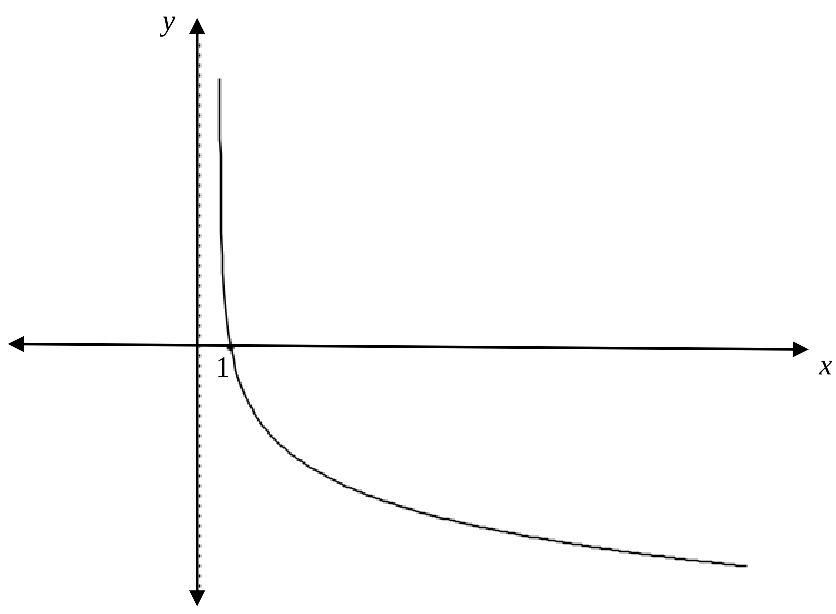
The sketch of the graph of $y = \log_b x$, where $b > 1$:



The x-intercept of the graph of the function is the point $(1, 0)$.

The vertical line $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

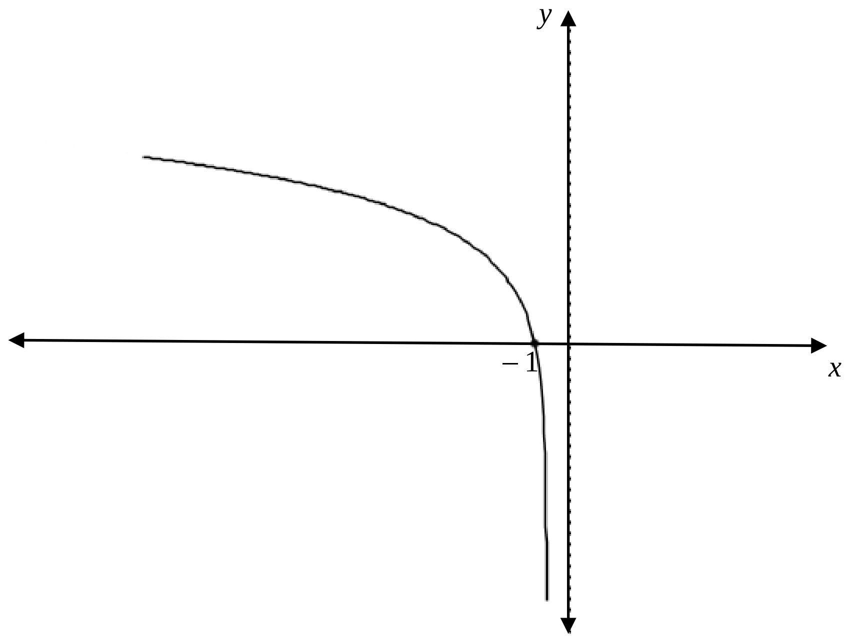
The sketch of the graph of $y = \log_b x$, where $0 < b < 1$:



The x-intercept of the graph of the function is the point $(1, 0)$.

The vertical line $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

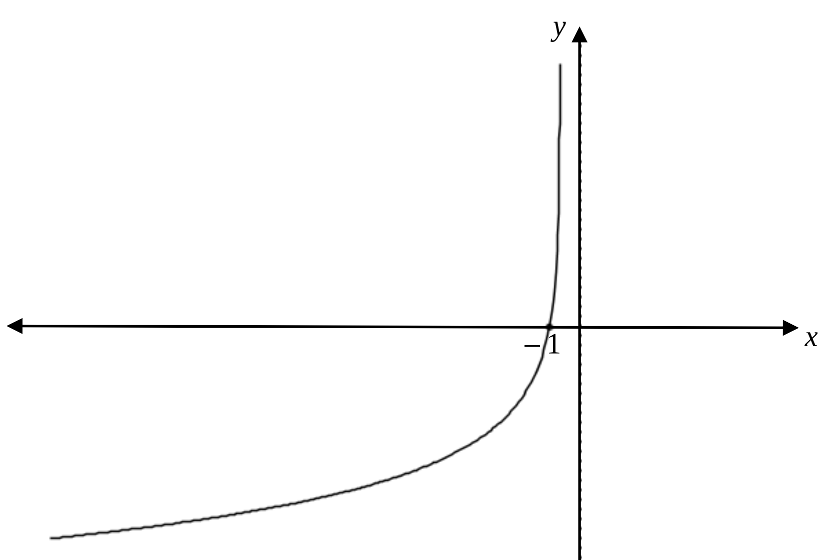
The sketch of the graph of $y = \log_b(-x)$, where $b > 1$:



The x-intercept of the graph of the function is the point $(-1, 0)$.

The vertical line $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

The sketch of the graph of $y = \log_b(-x)$, where $0 < b < 1$:



The x-intercept of the graph of the function is the point $(-1, 0)$.

The vertical line $x = 0$, which is the y-axis, is a vertical asymptote of the graph of the function.

Back to the top.