Pre-Class Problems 14 for Wednesday, March 21

**These are the type of problems that you will be working on in class.**

1. Determine the horizontal asymptotes for the graph of the following rational functions (if any). If the function has a horizontal asymptote, determine if the graph crosses the asymptote.

a. $h(x) = \dfrac{x^2 - 4x}{6x^2 - 8x + 15}$

b. $f(x) = \dfrac{4x^2 - 5x + 16}{x^2 + 12x + 36}$

c. $g(x) = \dfrac{3x - 4}{2x^2 - 7x - 9}$

d. $h(x) = \dfrac{x^2 - 4x + 16}{5x^3 + 2x^2 - 7}$

e. $f(x) = \dfrac{x^3 + 27}{x^2 + 9x - 18}$

f. $g(x) = \dfrac{2x^2 - x - 28}{x - 4}$

2. Determine the vertical and horizontal asymptotes for the graph of the following rational functions (if any). If the function has a horizontal asymptote, determine if the graph crosses the asymptote. Then sketch the graph of the rational function.

a. $f(x) = \dfrac{6x - 5}{3x + 7}$

b. $g(x) = \dfrac{9 - x^2}{3x^2 + 16x - 12}$

c. $h(x) = \dfrac{2x + 3}{x^2 + 4x - 12}$

Discussion of Exponential Functions

Sketch of the graph of Exponential Functions

3. Graph the following exponential functions.

a. $f(x) = 3^{x}$

b. $g(x) = \left(\frac{1}{2}\right)^{x}$

c. $h(x) = 4^{-x}$

d. $y = \left(\frac{3}{5}\right)^{-x}$

e. $f(x) = e^{x}$

f. $g(x) = 3(2^{x})$

g. $h(x) = -4\left(\frac{2}{3}\right)^{x}$

h. $f(t) = -\frac{3}{7}\left(\frac{5}{4}\right)^{t}$

i. $g(t) = 2e^{-t}$

j. $h(t) = -3^{t}$

4. Sketch the graph of the following functions. State the domain of the function and use the sketch to state the range of the function.

a. $f(x) = 5^{x} + 3$

b. $g(x) = e^{-x} - 4$

c. $h(t) = 7\left(\frac{3}{4}\right)^{t-5}$

d. $y = 8^{x+2} + 6$

e. $f(x) = -3e^{x-4} - 1$

Problems available in the textbook: Page 362 … 13adj, 14adj, 15adj, 25 – 36, 39 – 62, 67 – 86, 91 – 94 and Examples 1ad, 3 – 11 starting on page 346. Page 423 … 15 – 36 and Examples 1 – 3 starting on page 415.

**SOLUTIONS:**

1a. $h(x) = \dfrac{x^2 - 4x}{6x^2 - 8x + 15}$ Back to Problem 1.

$$h(x) = \frac{x^2\left(1 - \dfrac{4}{x}\right)}{x^2\left(6 - \dfrac{8}{x} + \dfrac{15}{x^2}\right)} = \frac{1 - \dfrac{4}{x}}{6 - \dfrac{8}{x} + \dfrac{15}{x^2}}$$

As $x \to -\infty$ and as $x \to \infty$, $\dfrac{4}{x} \to 0$, $\dfrac{8}{x} \to 0$, and $\dfrac{15}{x^2} \to 0$. Thus, as $x \to -\infty$ and as $x \to \infty$, $h(x) \to \dfrac{1 - 0}{6 - 0 + 0} = \dfrac{1}{6}$.

Thus, $y = \dfrac{1}{6}$ is a horizontal asymptotes for the graph of the rational function *h*.

The graph of the rational function *h* will cross the horizontal asymptote $y = \dfrac{1}{6}$ if the equation $h(x) = \dfrac{1}{6}$ has a real number solution.

$$h(x) = \frac{1}{6} \Rightarrow \frac{x^2 - 4x}{6x^2 - 8x + 15} = \frac{1}{6} \Rightarrow 6x^2 - 8x + 15 =$$

$$6(x^2 - 4x) \Rightarrow 6x^2 - 8x + 15 = 6x^2 - 24x \Rightarrow$$

$$-8x + 15 = -24x \Rightarrow 15 = -16x \Rightarrow x = -\frac{15}{16}.$$

The graph of the rational function $h$ will cross the horizontal asymptote $y = \frac{1}{6}$ at the point $\left( -\frac{15}{16}, \frac{1}{6} \right)$.

**Answer:** Horizontal Asymptote: $y = \frac{1}{6}$

Graph crosses horizontal asymptote at $\left( -\frac{15}{16}, \frac{1}{6} \right)$

1b. $f(x) = \dfrac{4x^2 - 5x + 16}{x^2 + 12x + 36}$

Back to Problem 1.

$$f(x) = \frac{x^2\left(4 - \dfrac{5}{x} + \dfrac{16}{x^2}\right)}{x^2\left(1 + \dfrac{12}{x} + \dfrac{36}{x^2}\right)} = \frac{4 - \dfrac{5}{x} + \dfrac{16}{x^2}}{1 + \dfrac{12}{x} + \dfrac{36}{x^2}}$$

As $x \to -\infty$ and as $x \to \infty$, $f(x) \to \dfrac{4 - 0 + 0}{1 + 0 + 0} = \dfrac{4}{1} = 4$.

Thus, $y = 4$ is a horizontal asymptotes for the graph of the rational function $f$.

The graph of the rational function $f$ will cross the horizontal asymptote $y = 4$ if the equation $f(x) = 4$ has a real number solution.

$$f(x) = 4 \Rightarrow \frac{4x^2 - 5x + 16}{x^2 + 12x + 36} = 4 \Rightarrow 4x^2 - 5x + 16 =$$

$4(x^2 + 12x + 36) \Rightarrow 4x^2 - 5x + 16 = 4x^2 + 48x + 144 \Rightarrow$

$-5x + 16 = 48x + 144 \Rightarrow -53x = 128 \Rightarrow x = -\dfrac{128}{53}$

**Answer:** Horizontal Asymptote: $y = 4$

Graph crosses horizontal asymptote at $\left(-\dfrac{128}{53}, 4\right)$

1c. $g(x) = \dfrac{3x - 4}{2x^2 - 7x - 9}$ Back to Problem 1.

$$g(x) = \frac{x^2\left(\dfrac{3}{x} - \dfrac{4}{x^2}\right)}{x^2\left(2 - \dfrac{7}{x} - \dfrac{9}{x^2}\right)} = \frac{\dfrac{3}{x} - \dfrac{4}{x^2}}{2 - \dfrac{7}{x} - \dfrac{9}{x^2}}$$

As $x \to -\infty$ and as $x \to \infty$, $g(x) \to \dfrac{0 - 0}{2 - 0 - 0} = \dfrac{0}{2} = 0$.

Thus, $y = 0$ is a horizontal asymptotes for the graph of the rational function $g$.

The graph of the rational function $g$ will cross the horizontal asymptote $y = 0$ if the equation $g(x) = 0$ has a real number solution.

$$g(x) = 0 \Rightarrow \frac{3x - 4}{2x^2 - 7x - 9} = 0 \Rightarrow 3x - 4 = 0 \Rightarrow x = \frac{4}{3}$$

**Answer:** Horizontal Asymptote: $y = 0$

Graph crosses horizontal asymptote at $\left( \dfrac{4}{3} , 0 \right)$

1d. $h(x) = \dfrac{x^2 - 4x + 16}{5x^3 + 2x^2 - 7}$ Back to Problem 1.

$$h(x) = \frac{x^3 \left( \dfrac{1}{x} - \dfrac{4}{x^2} + \dfrac{16}{x^3} \right)}{x^3 \left( 5 + \dfrac{2}{x} - \dfrac{7}{x^3} \right)} = \frac{\dfrac{1}{x} - \dfrac{4}{x^2} + \dfrac{16}{x^3}}{5 + \dfrac{2}{x} - \dfrac{7}{x^3}}$$

As $x \to -\infty$ and as $x \to \infty$, $h(x) \to \dfrac{0 - 0 + 0}{5 + 0 - 0} = \dfrac{0}{5} = 0$.

Thus, $y = 0$ is a horizontal asymptotes for the graph of the rational function $h$.

The graph of the rational function $h$ will cross the horizontal asymptote $y = 0$ if the equation $h(x) = 0$ has a real number solution.

$$h(x) = 0 \Rightarrow \frac{x^2 - 4x + 16}{5x^3 + 2x^2 - 7} = 0 \Rightarrow x^2 - 4x + 16 = 0 \Rightarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4(1)(16)}}{2} = \frac{4 \pm \sqrt{16 - 64}}{2} =$$

$\dfrac{4 \pm \sqrt{-48}}{2}$. The solutions to the equation $h(x) = 0$ are complex numbers. Thus, the graph of the rational function $h$ will **not** cross the horizontal asymptote $y = 0$

**Answer:** Horizontal Asymptote: $y = 0$

Graph does not cross the horizontal asymptote.

1e. $f(x) = \dfrac{x^3 + 27}{x^2 + 9x - 18}$ Back to Problem 1.

$$f(x) = \frac{x^3\left(1 + \dfrac{27}{x^3}\right)}{x^3\left(\dfrac{1}{x} + \dfrac{9}{x^2} - \dfrac{18}{x^3}\right)} = \frac{1 + \dfrac{27}{x^3}}{\dfrac{1}{x} + \dfrac{9}{x^2} - \dfrac{18}{x^3}}$$

As $x \to -\infty$ and as $x \to \infty$, $f(x) \to \dfrac{1 + 0}{0 + 0 - 0} = \dfrac{1}{0} \to \pm\infty$.

This rational function does not have any horizontal asymptotes.

**Answer:** None

1f. $g(x) = \dfrac{2x^2 - x - 28}{x - 4}$ Back to Problem 1.

$$g(x) = \frac{x^2\left(2 - \frac{1}{x} - \frac{28}{x^2}\right)}{x^2\left(\frac{1}{x} - \frac{4}{x^2}\right)} = \frac{2 - \frac{1}{x} - \frac{28}{x^2}}{\frac{1}{x} - \frac{4}{x^2}}$$

As $x \to -\infty$ and as $x \to \infty$, $g(x) \to \frac{2 - 0 - 0}{0 - 0} = \frac{2}{0} \to \pm\infty$.

This rational function does not have any horizontal asymptotes.

**Answer:** None

2a. $f(x) = \frac{6x - 5}{3x + 7}$ Back to Problem 2.

$$3x + 7 = 0 \Rightarrow x = -\frac{7}{3}$$

**Vertical Asymptote:** $x = -\frac{7}{3}$

$$f(x) = \frac{x\left(6 - \frac{5}{x}\right)}{x\left(3 + \frac{7}{x}\right)} = \frac{6 - \frac{5}{x}}{3 + \frac{7}{x}}$$

As $x \to -\infty$ and as $x \to \infty$, $f(x) \to \frac{6 - 0}{3 + 0} = \frac{6}{3} = 2$.

**Horizontal Asymptote:** $y = 2$

The graph of the rational function $f$ will cross the horizontal asymptote $y = 2$ if the equation $f(x) = 2$ has a real number solution.

$$f(x) = 2 \Rightarrow \frac{6x - 5}{3x + 7} = 2 \Rightarrow 6x - 5 = 2(3x + 7) \Rightarrow$$

$6x - 5 = 6x + 14 \Rightarrow -5 = 14$ False Equation

Thus, the equation $f(x) = 2$ does not have a solution. Thus, the graph does not cross the horizontal asymptote.

$$y = f(x) = \frac{6x - 5}{3x + 7}$$

***x*-intercept(s):** Set $y = 0$. $0 = \dfrac{6x - 5}{3x + 7} \Rightarrow 6x - 5 = 0 \Rightarrow x = \dfrac{5}{6}$

$\left(\dfrac{5}{6}, 0\right)$ is the only *x*-intercept of the graph

***y*-intercept:** Set $x = 0$. $y = \dfrac{0 - 5}{0 + 7} = -\dfrac{5}{7}$

$\left(0, -\dfrac{5}{7}\right)$ is the *y*-intercept of the graph

**Sketch of Graph:** Given in class.

2b. $g(x) = \dfrac{9 - x^2}{3x^2 + 16x - 12}$ Back to Problem 2.

$$g(x) = \frac{(3 + x)(3 - x)}{(x + 6)(3x - 2)}$$

$$(x + 6)(3x - 2) = 0 \Rightarrow x = -6,\ x = \frac{2}{3}$$

**Vertical Asymptotes:** $x = -6\,,\ x = \dfrac{2}{3}$

$$g(x) = \frac{x^2\left(\dfrac{9}{x^2} - 1\right)}{x^2\left(3 + \dfrac{16}{x} - \dfrac{12}{x^2}\right)} = \frac{\dfrac{9}{x^2} - 1}{3 + \dfrac{16}{x} - \dfrac{12}{x^2}}$$

As $x \to -\infty$ and as $x \to \infty$, $g(x) \to \dfrac{0 - 1}{3 + 0 - 0} = \dfrac{-1}{3} = -\dfrac{1}{3}$.

**Horizontal Asymptote:** $y = -\dfrac{1}{3}$

The graph of the rational function $g$ will cross the horizontal asymptote $y = -\dfrac{1}{3}$ if the equation $g(x) = -\dfrac{1}{3}$ has a real number solution.

$$g(x) = -\frac{1}{3} \Rightarrow \frac{9 - x^2}{3x^2 + 16x - 12} = -\frac{1}{3} \Rightarrow \frac{9 - x^2}{3x^2 + 16x - 12} = \frac{1}{-3} \Rightarrow$$

$$3x^2 + 16x - 12 = -3(9 - x^2) \Rightarrow 3x^2 + 16x - 12 = -27 + 3x^2 \Rightarrow$$

$$16x - 12 = -27 \Rightarrow 16x = -15 \Rightarrow x = -\frac{15}{16}$$

Graph crosses horizontal asymptote at $\left(-\frac{15}{16}, -\frac{1}{3}\right)$

$$y = g(x) = \frac{9 - x^2}{3x^2 + 16x - 12} = \frac{(3 + x)(3 - x)}{(x + 6)(3x - 2)}$$

***x*-intercept(s):** Set $y = 0$. $0 = \frac{(3 + x)(3 - x)}{(x + 6)(3x - 2)} \Rightarrow$

$(3 + x)(3 - x) = 0 \Rightarrow x = -3,\ x = 3$

$(-3, 0)$ and $(3, 0)$ are *x*-intercepts of the graph

***y*-intercept:** Set $x = 0$. $y = \frac{9 - 0}{0 + 0 - 12} = \frac{9}{-12} = -\frac{3}{4}$

$\left(0, -\frac{3}{4}\right)$ is the *y*-intercept of the graph

**Sketch of Graph:** Given in class.

2c. $h(x) = \frac{2x + 3}{x^2 + 4x - 12}$

Back to Problem 2.

$$h(x) = \frac{2x + 3}{x^2 + 4x - 12} = \frac{2x + 3}{(x + 6)(x - 2)}$$

$$(x + 6)(x - 2) = 0 \Rightarrow x = -6,\ x = 2$$

**Vertical Asymptotes:** $x = -6,\ x = 2$

$$h(x) = \frac{2x + 3}{x^2 + 4x - 12} = \frac{x^2\left(\frac{2}{x} + \frac{3}{x^2}\right)}{x^2\left(1 + \frac{4}{x} - \frac{12}{x^2}\right)} = \frac{\frac{2}{x} + \frac{3}{x^2}}{1 + \frac{4}{x} - \frac{12}{x^2}}$$

As $x \to -\infty$ and as $x \to \infty$, $h(x) \to \frac{0 + 0}{1 + 0 - 0} = \frac{0}{1} = 0$

**Horizontal Asymptote:** $y = 0$

The graph of the rational function $h$ will cross the horizontal asymptote $y = 0$ if the equation $h(x) = 0$ has a real number solution.

$$h(x) = 0 \Rightarrow 2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$$

Graph crosses horizontal asymptote at $\left(-\frac{3}{2},\ 0\right)$

$$y = h(x) = \frac{2x+3}{x^2+4x-12} = \frac{2x+3}{(x+6)(x-2)}$$

***x*-intercept(s):** Set $y = 0$. $0 = \dfrac{2x+3}{x^2+4x-12} \Rightarrow 2x+3=0 \Rightarrow$

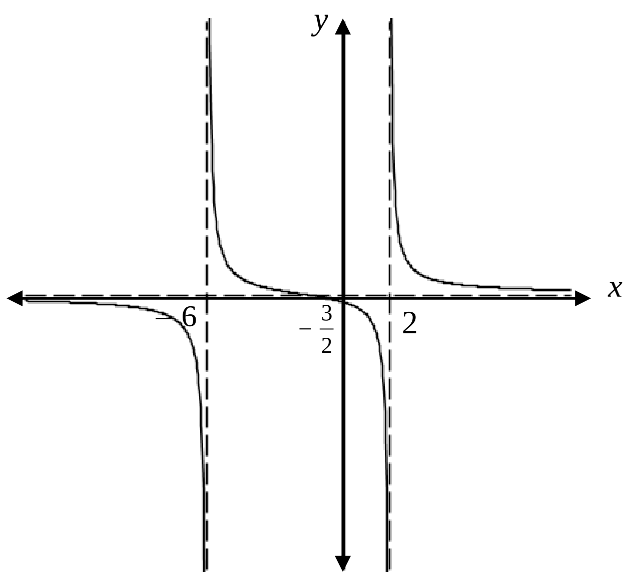
$$x = -\frac{3}{2}$$

$\left(-\frac{3}{2}, 0\right)$ is the only *x*-intercept of the graph

***y*-intercept:** Set $x = 0$. $y = \dfrac{0+3}{0+0-12} = \dfrac{3}{-12} = -\dfrac{1}{4}$

$\left(0, -\frac{1}{4}\right)$ is the *y*-intercept of the graph

**Sketch of Graph:**

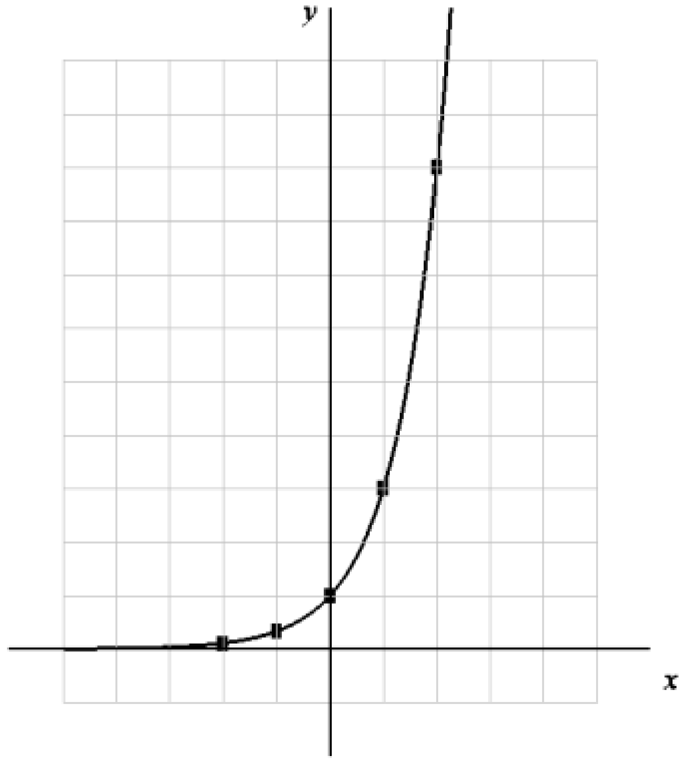



Back to Problem 2.

3a. $f(x) = 3^x$ Back to Problem 3.

In order to graph the function $f$ given by $f(x) = 3^x$, we set $f(x) = y$ and graph the equation $y = 3^x$.

| $x$ | $y$ |
|---|---|
| $-2$ | $\frac{1}{9}$ |
| $-1$ | $\frac{1}{3}$ |
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |




The Drawing of this Graph

The $y$-intercept of the graph of the function is the point $(0, 1)$.

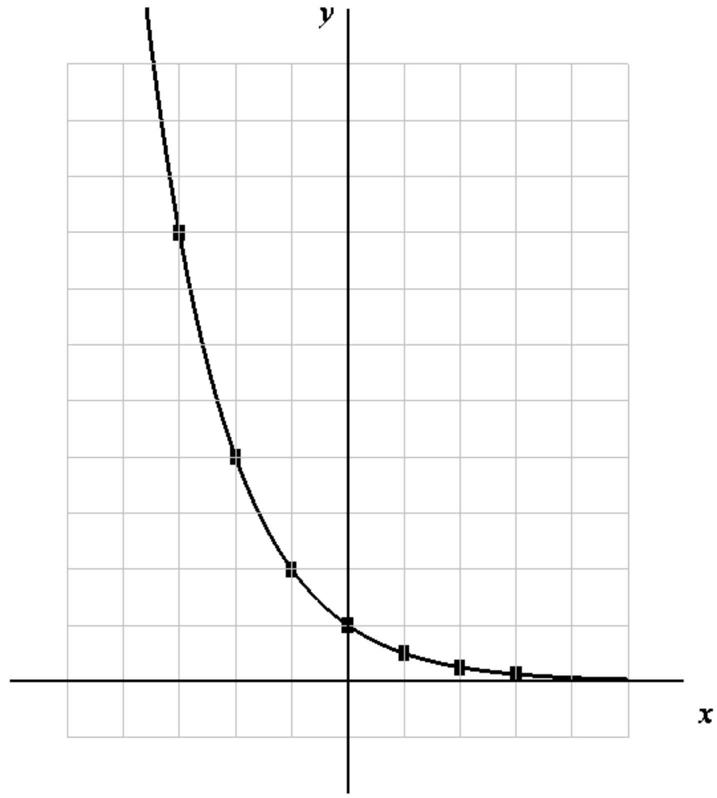
Note that as $x \to \infty$, $y = 3^x \to \infty$ and as $x \to -\infty$, $y = 3^x \to 0$.

Since as $x \to -\infty$, $y = 3^x \to 0$, then the horizontal line of $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

3b. $g(x) = \left(\frac{1}{2}\right)^x$ Back to Problem 3.

In order to graph the function $g$ given by $g(x) = \left(\frac{1}{2}\right)^x$, we set $g(x) = y$ and graph the equation $y = \left(\frac{1}{2}\right)^x$.

| $x$ | $y$ |
|---|---|
| $-3$ | 8 |
| $-2$ | 4 |
| $-1$ | 2 |
| 0 | 1 |
| 1 | $\frac{1}{2}$ |
| 2 | $\frac{1}{4}$ |
| 3 | $\frac{1}{8}$ |




The Drawing of this Graph

$$\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$$

The $y$-intercept of the graph of the function is the point $(0, 1)$.

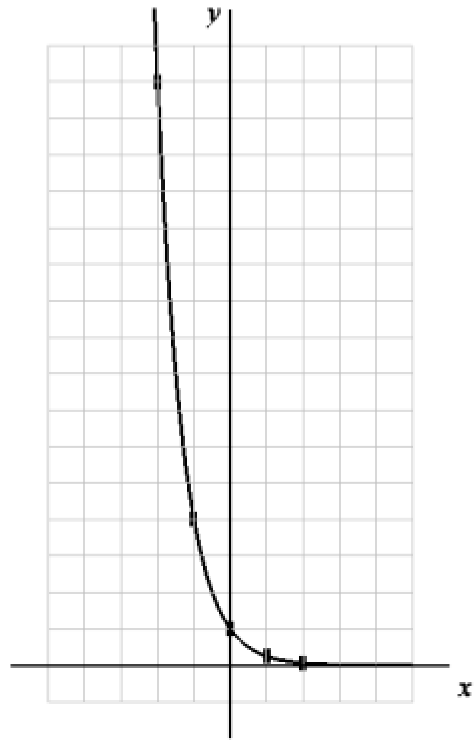
Note that as $x \to \infty$, $y = \left(\frac{1}{2}\right)^x \to 0$ and as $x \to -\infty$, $y = \left(\frac{1}{2}\right)^x \to \infty$.

Since as $x \to \infty$, $y = \left(\frac{1}{2}\right)^x \to 0$, then the horizontal line of $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

3c. $h(x) = 4^{-x}$ Back to Problem 3.

In order to graph the function $h$ given by $h(x) = 4^{-x}$, we set $h(x) = y$ and graph the equation $y = 4^{-x}$.

| $x$ | $y$ |
|---|---|
| $-2$ | 16 |
| $-1$ | 4 |
| 0 | 1 |
| 1 | $\frac{1}{4}$ |
| 2 | $\frac{1}{16}$ |




The Drawing of this Graph

NOTE: $4^{-x} = (4^{-1})^x = \left(\dfrac{1}{4}\right)^x$

The $y$-intercept of the graph of the function is the point $(0, 1)$.

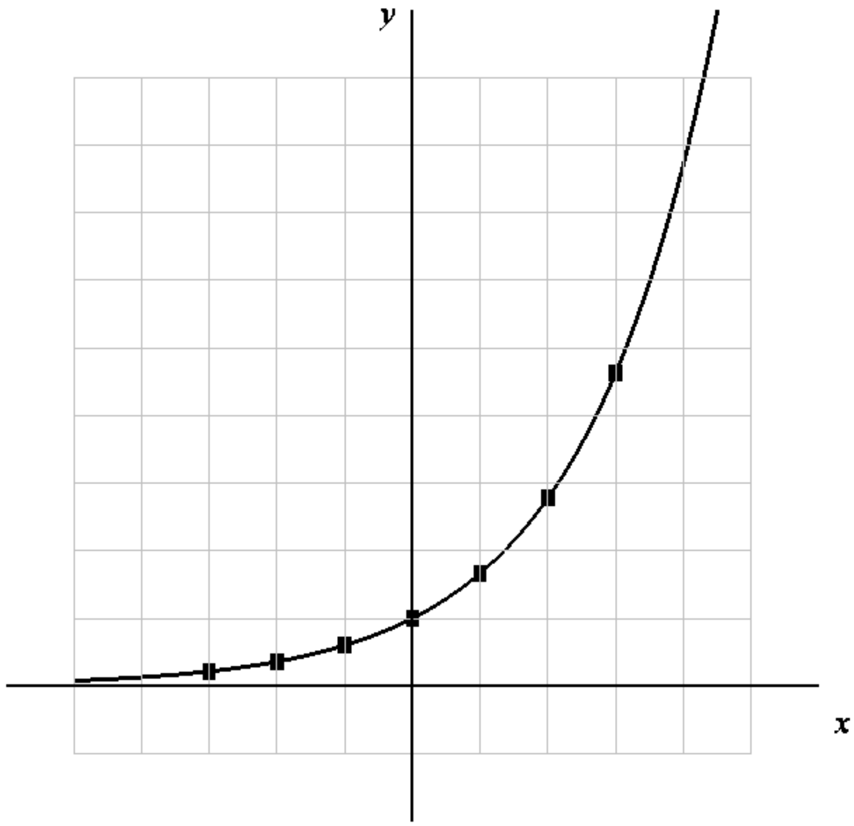
Note that as $x \to \infty$, $y = 4^{-x} \to 0$ and as $x \to -\infty$, $y = 4^{-x} \to \infty$.

Since as $x \to \infty$, $y = 4^{-x} \to 0$, then the horizontal line of $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

3d. $y = \left(\dfrac{3}{5}\right)^{-x}$

Back to Problem 3.

| $x$ | $y$ |
|---|---|
| $-3$ | $\dfrac{27}{125}$ |
| $-2$ | $\dfrac{9}{25}$ |
| $-1$ | $\dfrac{3}{5}$ |
| $0$ | $1$ |
| $1$ | $\dfrac{5}{3}$ |
| $2$ | $\dfrac{25}{9}$ |
| $3$ | $\dfrac{125}{27}$ |




The Drawing of this Graph

NOTE: $\left(\dfrac{3}{5}\right)^{-x} = \left(\dfrac{5}{3}\right)^{x}$

The $y$-intercept of the graph of the function is the point $(0, 1)$.

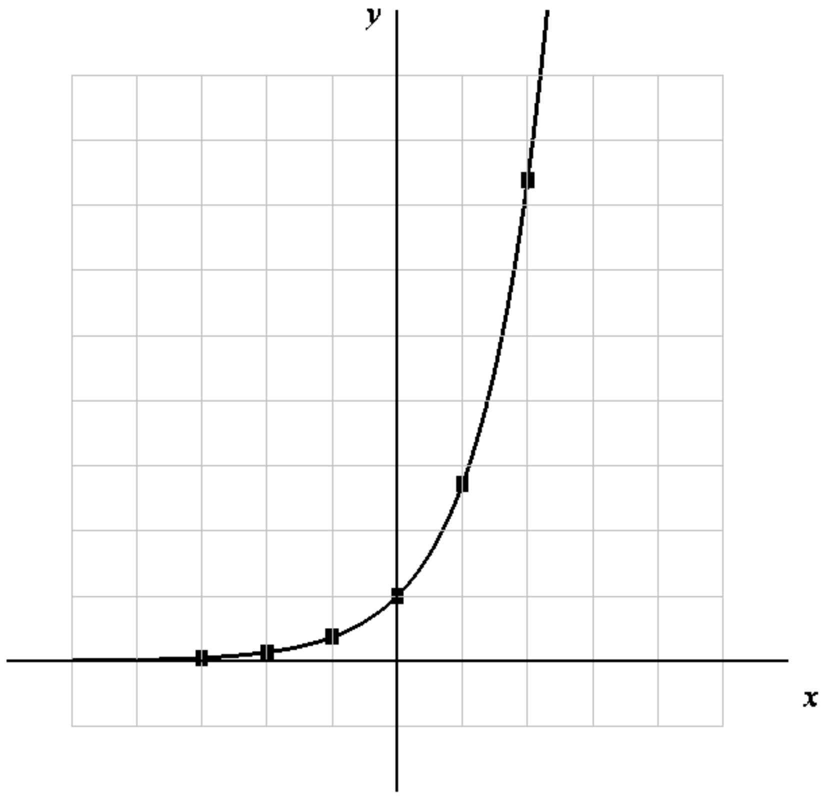
Note that as $x \to \infty$, $y = \left(\dfrac{3}{5}\right)^{-x} \to \infty$ and as $x \to -\infty$, $y = \left(\dfrac{3}{5}\right)^{-x} \to 0$.

Since as $x \to -\infty$, $y = \left(\dfrac{3}{5}\right)^{-x} \to 0$, then the horizontal line of $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

3e. $f(x) = e^{x}$ Back to Problem 3.

In order to graph the function $f$ given by $f(x) = e^{x}$, we set $f(x) = y$ and graph the equation $y = e^{x}$.

| $x$ | $y$ |
|---|---|
| $-3$ | $e^{-3} \approx 0.04979$ |
| $-2$ | $e^{-2} \approx 0.13534$ |
| $-1$ | $e^{-1} \approx 0.36788$ |
| $0$ | $1$ |
| $1$ | $e \approx 2.71828$ |
| $2$ | $e^{2} \approx 7.38906$ |
| $3$ | $e^{3} \approx 20.08554$ |




The Drawing of this Graph

The $y$-intercept of the graph of the function is the point $(0, 1)$.

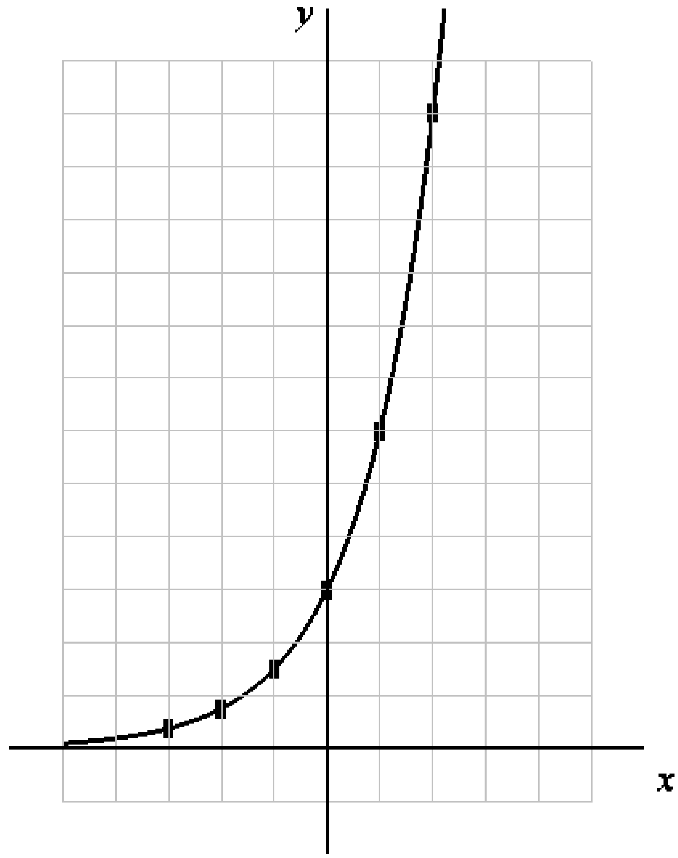
Note that as $x \to \infty$, $y = e^{x} \to \infty$ and as $x \to -\infty$, $y = e^{x} \to 0$.

Since as $x \to -\infty$, $y = e^{x} \to 0$, then the horizontal line of $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

3f. $g(x) = 3(2^{x})$ Back to Problem 3.

In order to graph the function $g$ given by $g(x) = 3(2^x)$, we set $g(x) = y$ and graph the equation $y = 3(2^x)$.

| $x$ | $y$ |
|---|---|
| $-3$ | $\frac{3}{8}$ |
| $-2$ | $\frac{3}{4}$ |
| $-1$ | $\frac{3}{2}$ |
| 0 | 3 |
| 1 | 6 |
| 2 | 12 |
| 3 | 24 |




The Drawing of this Graph

The $y$-intercept of the graph of the function is the point $(0, 3)$.

Note that as $x \to \infty$, $y = 3(2^x) \to \infty$ and as $x \to -\infty$, $y = 3(2^x) \to 0$.

Since as $x \to -\infty$, $y = 3(2^x) \to 0$, then the horizontal line of $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

3g. $h(x) = -4\left(\dfrac{2}{3}\right)^x$

Back to Problem 3.

In order to graph the function $h$ given by $h(x) = -4\left(\frac{2}{3}\right)^{x}$, we set $h(x) = y$ and graph the equation $y = -4\left(\frac{2}{3}\right)^{x}$.

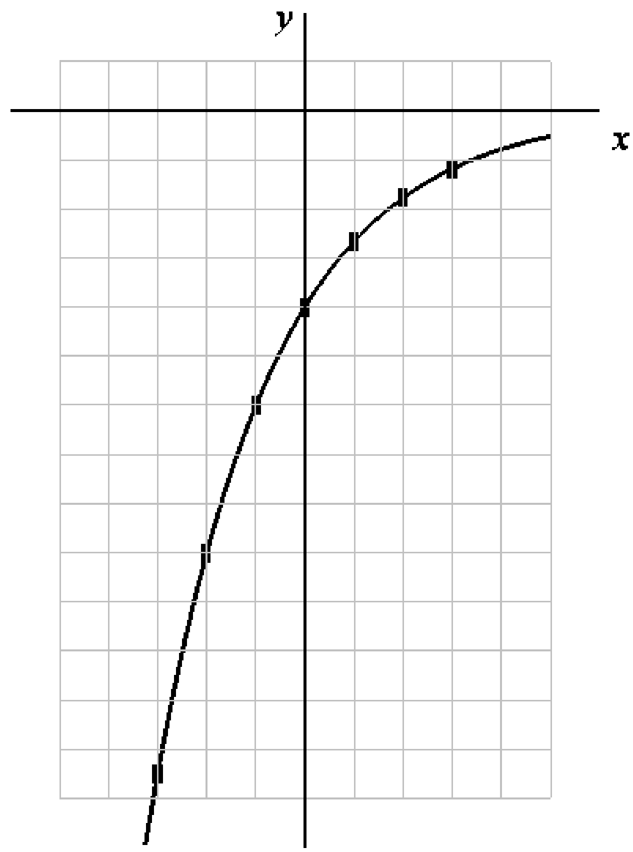
$$h(-3) = -4\left(\frac{2}{3}\right)^{-3} = -4\left(\frac{3}{2}\right)^{3} = -4\left(\frac{27}{8}\right) = -\frac{27}{2}$$

$$h(-2) = -4\left(\frac{2}{3}\right)^{-2} = -4\left(\frac{3}{2}\right)^{2} = -4\left(\frac{9}{4}\right) = -9$$

$$h(-1) = -4\left(\frac{2}{3}\right)^{-1} = -4\left(\frac{3}{2}\right) = -6 \qquad h(1) = -4\left(\frac{2}{3}\right) = -\frac{8}{3}$$

$$h(2) = -4\left(\frac{2}{3}\right)^{2} = -4\left(\frac{4}{9}\right) = -\frac{16}{9} \qquad h(3) = -4\left(\frac{2}{3}\right)^{3} = -4\left(\frac{8}{27}\right) = -\frac{32}{27}$$

| $x$ | $y$ |
|---|---|
| $-3$ | $-\frac{27}{2}$ |
| $-2$ | $-9$ |
| $-1$ | $-6$ |
| $0$ | $-4$ |
| $1$ | $-\frac{8}{3}$ |
| $2$ | $-\frac{16}{9}$ |
| $3$ | $-\frac{32}{27}$ |




The Drawing of this Graph

The $y$-intercept of the graph of the function is the point $(0, -4)$.

Note that as $x \to \infty$, $y = -4\left(\frac{2}{3}\right)^x \to 0$ and as $x \to -\infty$, $y = -4\left(\frac{2}{3}\right)^x \to -\infty$.

Since $x \to \infty$, $y = -4\left(\frac{2}{3}\right)^x \to 0$, then the horizontal line of $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

3h. $f(t) = -\frac{3}{7}\left(\frac{5}{4}\right)^t$ Back to Problem 3.

In order to graph the function $f$ given by $f(t) = -\frac{3}{7}\left(\frac{5}{4}\right)^t$, we set $f(t) = y$ and graph the equation $y = -\frac{3}{7}\left(\frac{5}{4}\right)^t$.

NOTE: $f(-3) = -\frac{3}{7}\left(\frac{5}{4}\right)^{-3} = -\frac{3}{7}\left(\frac{4}{5}\right)^3 = -\frac{3}{7}\left(\frac{64}{125}\right) = -\frac{192}{875}$

$$f(-2) = -\frac{3}{7}\left(\frac{5}{4}\right)^{-2} = -\frac{3}{7}\left(\frac{4}{5}\right)^2 = -\frac{3}{7}\left(\frac{16}{25}\right) = -\frac{48}{175}$$

$$f(-1) = -\frac{3}{7}\left(\frac{5}{4}\right)^{-1} = -\frac{3}{7}\left(\frac{4}{5}\right) = -\frac{12}{35}$$

$$f(1) = -\frac{3}{7}\left(\frac{5}{4}\right) = -\frac{15}{28}$$

$$f(2) = -\frac{3}{7}\left(\frac{5}{4}\right)^2 = -\frac{3}{7}\left(\frac{25}{16}\right) = -\frac{75}{112}$$

$$f(3) = -\frac{3}{7}\left(\frac{5}{4}\right)^3 = -\frac{3}{7}\left(\frac{125}{64}\right) = -\frac{375}{448}$$

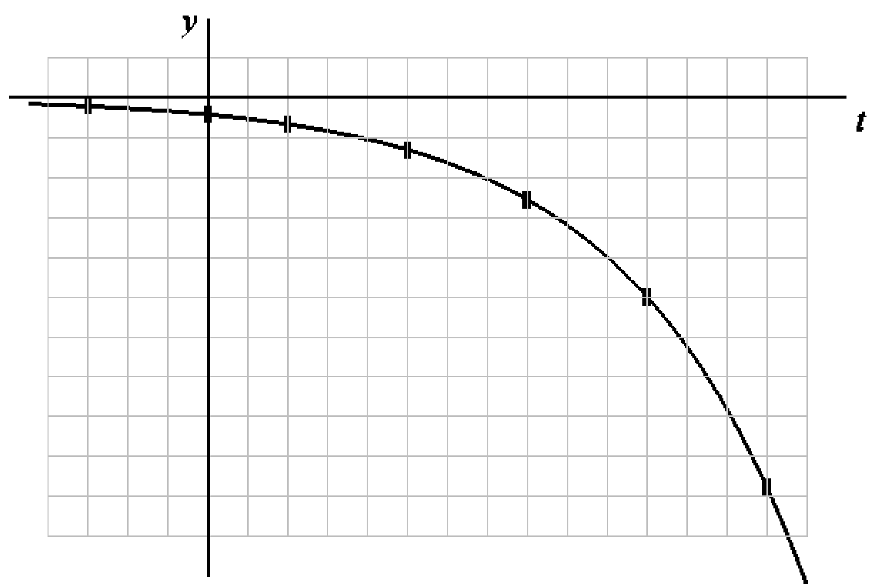
$$f(5) = -\frac{3}{7}\left(\frac{5}{4}\right)^5 = -\frac{3}{7}\left(\frac{3125}{1024}\right) = -\frac{9375}{7168}$$

$$f(8) = -\frac{3}{7}\left(\frac{5}{4}\right)^8 = -\frac{3}{7}\left(\frac{390625}{65536}\right) = -\frac{1171875}{458752} \approx -2.55$$

$$f(11) = -\frac{3}{7}\left(\frac{5}{4}\right)^{11} = -\frac{3}{7}\left(\frac{48828125}{4194304}\right) = -\frac{146484375}{29360128} \approx -4.99$$

$$f(14) = -\frac{3}{7}\left(\frac{5}{4}\right)^{14} = -\frac{18310546875}{1879048192} \approx -9.74$$

| $t$ | $y$ |
|---|---|
| $-3$ | $-\frac{192}{875}$ |
| $0$ | $-\frac{3}{7}$ |
| $2$ | $-\frac{75}{112}$ |
| $5$ | $-\frac{9375}{7168}$ |
| $8$ | $-\frac{1171875}{458752}$ |
| $11$ | $-\frac{146484375}{29360128}$ |
| $14$ | $-\frac{18310546875}{1879048192}$ |




The Drawing of this Graph

The $y$-intercept of the graph of the function is the point $\left(0, -\frac{3}{7}\right)$.

Note that as $t \to \infty$, $y = -\frac{3}{7}\left(\frac{5}{4}\right)^t \to \infty$ and as $t \to -\infty$,
$y = -\frac{3}{7}\left(\frac{5}{4}\right)^t \to 0$.

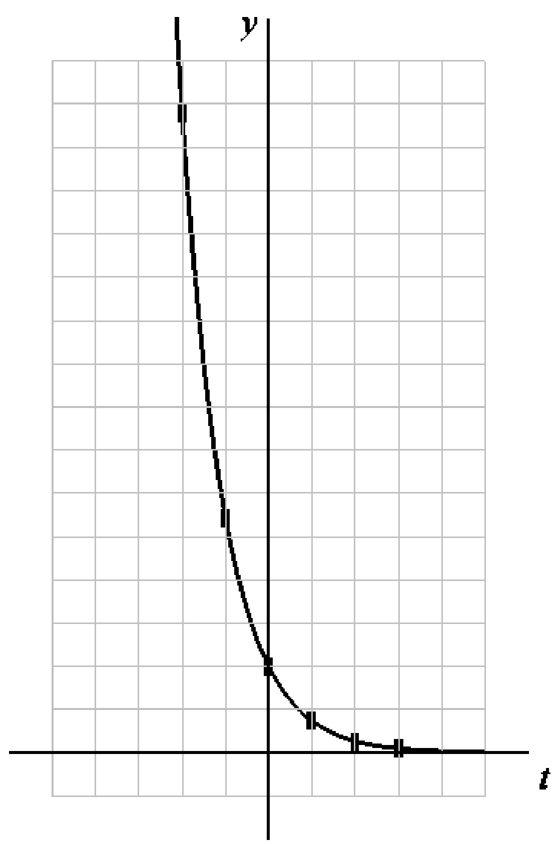
Since as $t \to -\infty$, $y = -\frac{3}{7}\left(\frac{5}{4}\right)^t \to 0$, then the horizontal line of $y = 0$, which is the $t$-axis, is a horizontal asymptote of the graph of the function.

3i. $g(t) = 2e^{-t}$

Back to Problem 3.

In order to graph the function $g$ given by $g(t) = 2e^{-t}$, we set $g(t) = y$ and graph the equation $y = 2e^{-t}$.

| $t$ | $y$ |
|---|---|
| $-3$ | $2e^{3} \approx 40.1711$ |
| $-2$ | $2e^{2} \approx 14.7781$ |
| $-1$ | $2e \approx 5.4366$ |
| $0$ | $2$ |
| $1$ | $2e^{-1} \approx 0.7258$ |
| $2$ | $2e^{-2} \approx 0.2707$ |
| $3$ | $2e^{-3} \approx 0.0996$ |




The Drawing of this Graph

The $y$-intercept of the graph of the function is the point $(0, 2)$.

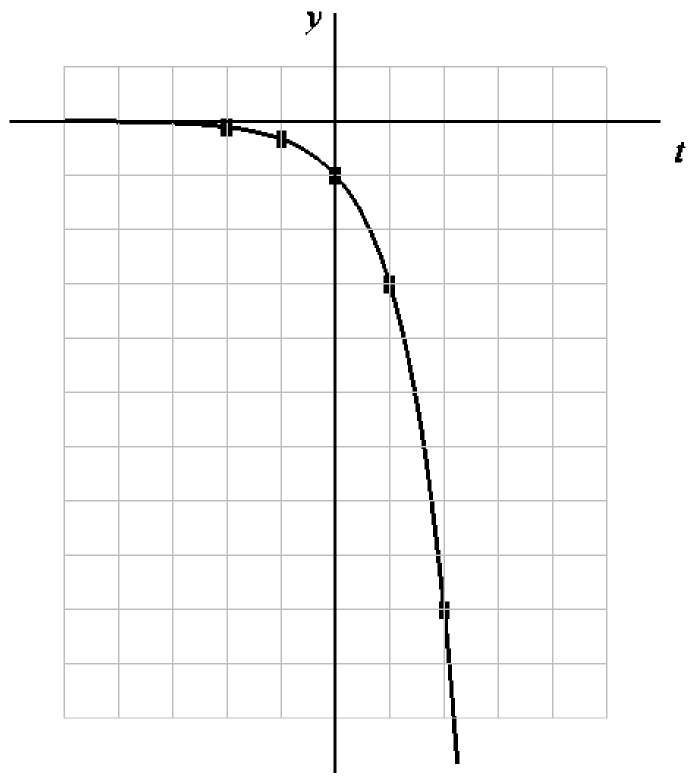
Note that as $t \to \infty$, $y = 2e^{-t} \to 0$ and as $t \to -\infty$, $y = 2e^{-t} \to \infty$.

Since as $t \to \infty$, $y = 2e^{-t} \to 0$, then the horizontal line of $y = 0$, which is the $t$-axis, is a horizontal asymptote of the graph of the function.

3j. $h(t) = -3^{t}$ Back to Problem 3.

In order to graph the function $h$ given by $h(t) = -3^{t}$, we set $h(t) = y$ and graph the equation $y = -3^{t}$.

| $t$ | $y$ |
|---|---|
| $-2$ | $-\frac{1}{9}$ |
| $-1$ | $-\frac{1}{3}$ |
| $0$ | $-1$ |
| $1$ | $-3$ |
| $2$ | $-9$ |




The Drawing of this Graph

The $y$-intercept of the graph of the function is the point $(0, -1)$.

Note that as $t \to \infty$, $y = -3^{t} \to -\infty$ and as $t \to -\infty$, $y = -3^{t} \to 0$.

Since as $t \to -\infty$, $y = -3^t \to 0$, then the horizontal line of $y = 0$, which is the *t*-axis, is a horizontal asymptote of the graph of the function.

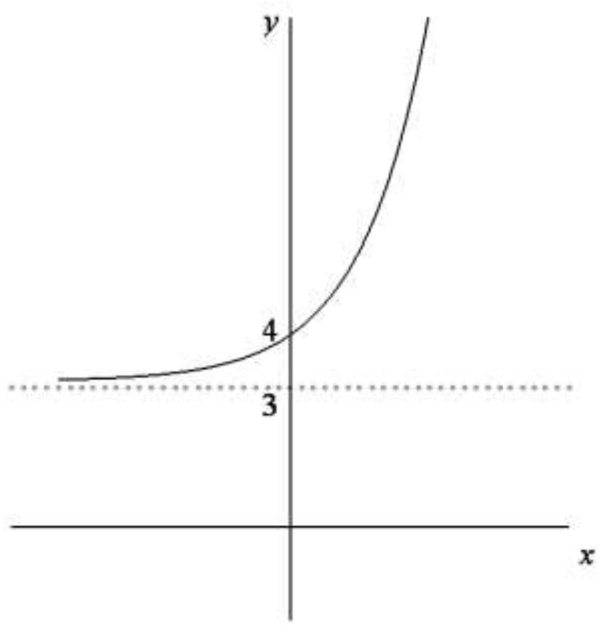
4a. $f(x) = 5^x + 3$ Back to Problem 4.

The domain of $f$ is the set of all real numbers.

To graph the function *f*, we set $f(x) = y$ and graph the equation $y = 5^x + 3$.

$$y = 5^x + 3 \Rightarrow y - 3 = 5^x$$

The graph of $y - 3 = 5^x$ is the graph of $y = 5^x$ shifted 3 units upward.




The Drawing of this Sketch

The range of $f$ is $(3, \infty)$. Note that the *y*-intercept is the point $(0, 4)$.

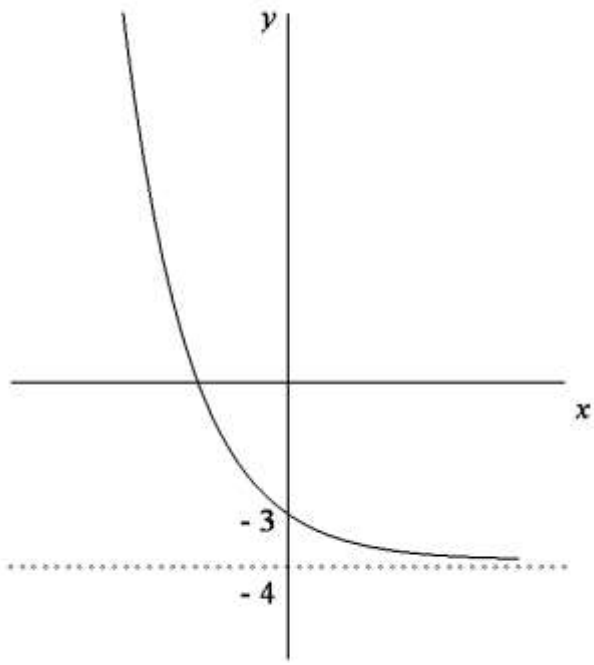
NOTE: The vertical shift of 3 units upward is determined from the expression $y - 3$ in the equation $y - 3 = 5^x$.

4b. $g(x) = e^{-x} - 4$ Back to Problem 4.

The domain of $g$ is the set of all real numbers.
To graph the function $g$, we set $g(x) = y$ and graph the equation $y = e^{-x} - 4$.

$$y = e^{-x} - 4 \Rightarrow y + 4 = e^{-x}$$

The graph of $y + 4 = e^{-x}$ is the graph of $y = e^{-x}$ shifted 4 units downward.




The Drawing of this Sketch

The range of $g$ is $(-4, \infty)$. Note that the *y*-intercept is the point $(0, -3)$.

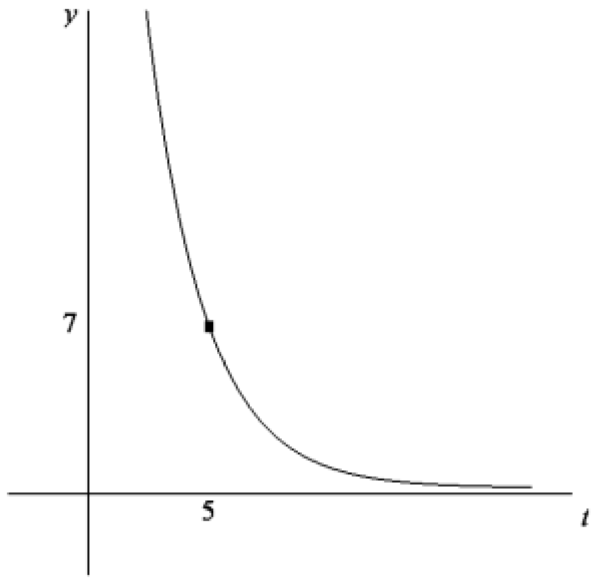
NOTE: The vertical shift of 4 units downward is determined from the expression $y + 4$ in the equation $y + 4 = e^{-x}$.

4c. $h(t) = 7\left(\frac{3}{4}\right)^{t-5}$ Back to Problem 4.

The domain of $h$ is the set of all real numbers.

To graph the function $h$, we set $h(t) = y$ and graph the equation $y = 7\left(\frac{3}{4}\right)^{t-5}$.

The graph of $y = 7\left(\frac{3}{4}\right)^{t-5}$ is the graph of $y = 7\left(\frac{3}{4}\right)^{t}$ shifted 5 units to the right.




The Drawing of this Sketch

The range of $h$ is $(0, \infty)$.

The $y$-coordinate of the $y$-intercept is obtained by setting $t = 0$ in the equation $y = 7\left(\frac{3}{4}\right)^{t-5}$. Thus, we have that $y = 7\left(\frac{3}{4}\right)^{-5} = 7\left(\frac{4}{3}\right)^{5} = 7\left(\frac{1024}{243}\right) = \frac{7168}{243}$. Thus, the $y$-intercept is the point $\left(0, \frac{7168}{243}\right)$.

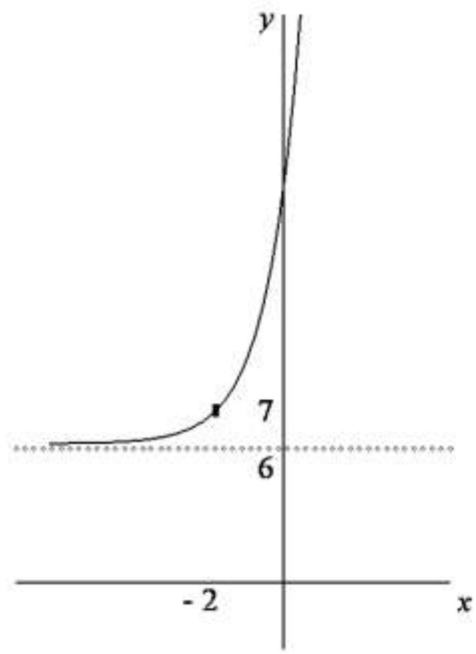
NOTE: The horizontal shift of 5 units to the right is determined from the expression $t - 5$ in the equation $y = 7\left(\frac{3}{4}\right)^{t-5}$.

4d. $y = 8^{x+2} + 6$ Back to Problem 4.

The domain of the function is the set of all real numbers.

$$y = 8^{x+2} + 6 \Rightarrow y - 6 = 8^{x+2}$$

The graph of $y - 6 = 8^{x+2}$ is the graph of $y = 8^{x}$ shifted 2 units to the left and 6 units upward.




The Drawing of this Sketch

The range of the function is $(6, \infty)$.

The $y$-coordinate of the $y$-intercept is obtained by setting $x = 0$ in the equation $y = 8^{x+2} + 6$. Thus, we have that $y = 8^{2} + 6 = 64 + 6 = 70$. Thus, the $y$-intercept is the point $(0, 70)$.

NOTE: The horizontal shift of 2 units to the left is determined from the expression $x + 2$ in the equation $y - 6 = 8^{x + 2}$ and the vertical shift of 6 units upward is determined from the expression $y - 6$ in the equation.

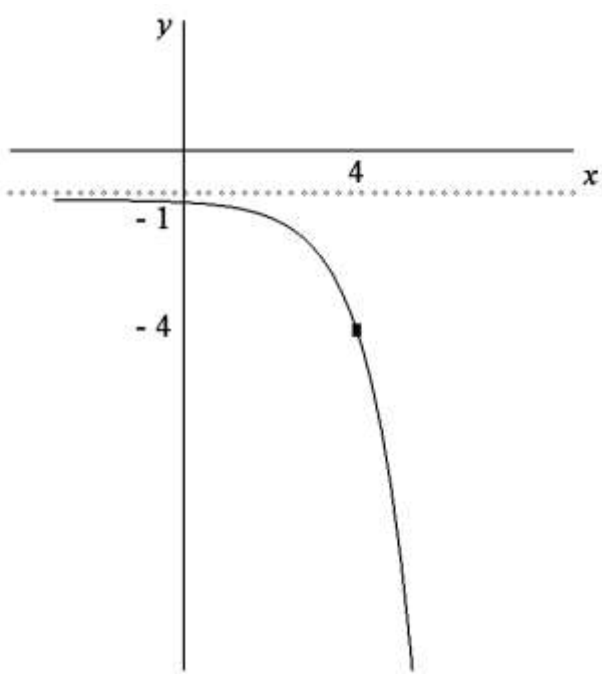
4e. $f(x) = -3e^{x - 4} - 1$ Back to Problem 4.

The domain of $f$ is the set of all real numbers.

To graph the function $f$, we set $f(x) = y$ and graph the equation $y = -3e^{x - 4} - 1$.

$$y = -3e^{x - 4} - 1 \Rightarrow y + 1 = -3e^{x - 4}$$

The graph of $y + 1 = -3e^{x - 4}$ is the graph of $y = -3e^{x}$ shifted 4 units to the right and 1 unit downward.




The Drawing of this Sketch

The range of the function is $(-\infty, -1)$.

The $y$-coordinate of the $y$-intercept is obtained by setting $x = 0$ in the equation $y = -3e^{x-4} - 1$. Thus, we have that $y = -3e^{-4} - 1$. Thus, the $y$-intercept is the point $(0, -3e^{-4} - 1)$.

NOTE: The horizontal shift of 4 units to the right is determined from the expression $x - 4$ in the equation $y + 1 = -3e^{x-4}$ and the vertical shift of 1 unit downward is determined from the expression $y + 1$ in the equation.

**Definition** The exponential function $f$ with base $b$ is the function defined by $f(x) = b^x$, where $b > 0$ and $b \neq 1$.

The domain of $f$ is the set of real numbers. That is, the domain of $f$ is $(-\infty, \infty)$.

The range of $f$ is the set of positive real numbers. That is, the range of $f$ is $(0, \infty)$.

**Definition** The natural exponential function is the exponential function whose base is the irrational number $e$. Thus, the natural exponential function is the function defined by $f(x) = e^x$, where $e = 2.718281828\ldots$.

Notation: Sometimes, $e^x$ is written $\exp(x)$.

Recall the following properties of exponents, where $m$ and $n$ be integers:

1. $(a^m)^n = a^{mn}$

2. $a^m a^n = a^{m+n}$

3. If $a \neq 0$, then $\dfrac{a^m}{a^n} = a^{m-n}$ if $m > n$ OR $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$ if $n > m$.

4. If $a \neq 0$, then $a^0 = 1$.

5. If $a \neq 0$ and $n$ is positive, then $a^{-n} = \dfrac{1}{a^n}$.

6. $(ab)^n = a^n b^n$

7. If $b \neq 0$, then $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$.

8. If $a \neq 0$ and $b \neq 0$ and $n$ is positive, then $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$.

9. $a^{m/n} = \sqrt[n]{a^m}$

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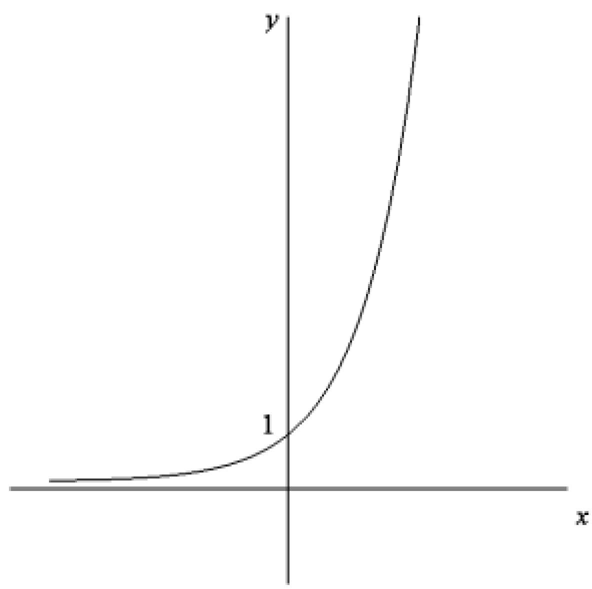
The sketch of the graph of $y = b^x$, where $b > 1$ **OR** the sketch of the graph of $y = b^{-x}$, where $0 < b < 1$:

NOTE: $b^{-x} = (b^{-1})^x = \left(\dfrac{1}{b}\right)^x$

If $0 < b < 1$, then $\dfrac{1}{b} > 1$.



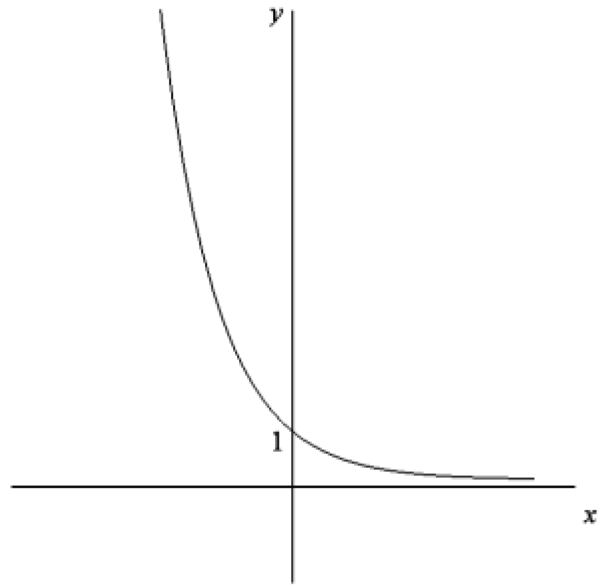

The $y$-intercept of the graph of the function is the point $(0, 1)$.

The horizontal line $y = 0$, which is the *x*-axis, is a horizontal asymptote of the graph of the function.

The sketch of the graph of $y = b^x$, where $0 < b < 1$ **OR** the sketch of the graph of $y = b^{-x}$, where $b > 1$:

NOTE: $b^{-x} = (b^{-1})^x = \left(\dfrac{1}{b}\right)^x$

If $b > 1$, then $0 < \dfrac{1}{b} < 1$.



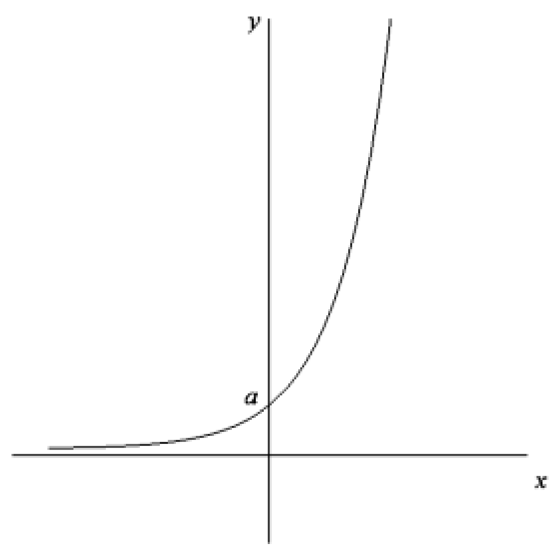

The *y*-intercept of the graph of the function is the point $(0, 1)$.

The horizontal line $y = 0$, which is the *x*-axis, is a horizontal asymptote of the graph of the function.

Let $a > 0$. The sketch of the graph of $y = a(b^x)$, where $b > 1$ **OR** the sketch of the graph of $y = a(b^{-x})$, where $0 < b < 1$:

NOTE: $b^{-x} = (b^{-1})^x = \left(\dfrac{1}{b}\right)^x$

If $0 < b < 1$, then $\dfrac{1}{b} > 1$.



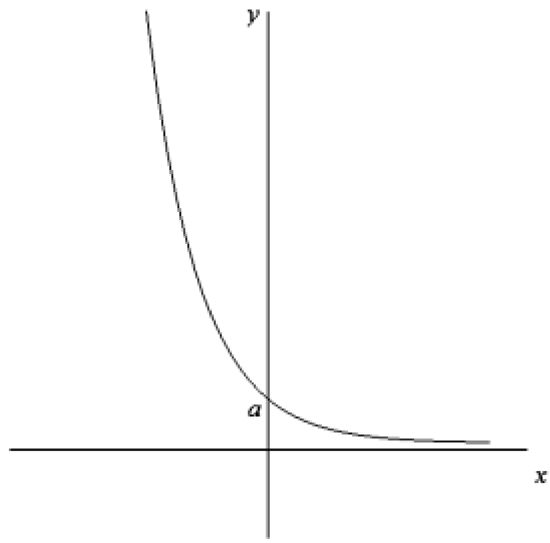
The $y$-intercept of the graph of the function is the point $(0, a)$.

The horizontal line $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

Let $a > 0$. The sketch of the graph of $y = a(b^x)$, where $0 < b < 1$ **OR** the sketch of the graph of $y = a(b^{-x})$, where $b > 1$:

NOTE: $b^{-x} = (b^{-1})^x = \left(\dfrac{1}{b}\right)^x$

If $b > 1$, then $0 < \dfrac{1}{b} < 1$.



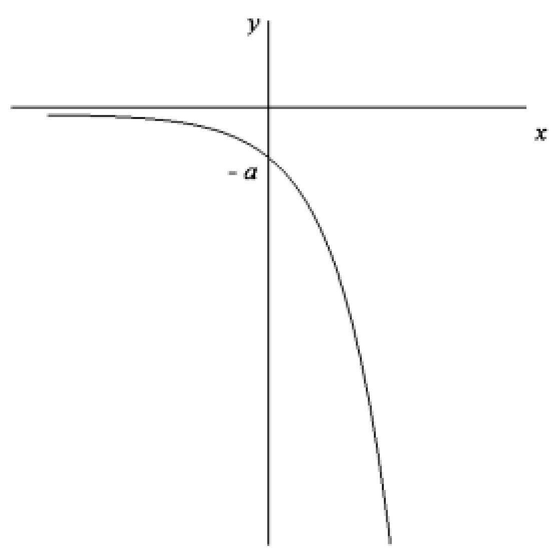

The $y$-intercept of the graph of the function is the point $(0, a)$.

The horizontal line $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

Let $a > 0$. The sketch of the graph of $y = -a(b^x)$, where $b > 1$ **OR** the sketch of the graph of $y = -a(b^{-x})$, where $0 < b < 1$:

NOTE: $b^{-x} = (b^{-1})^x = \left(\dfrac{1}{b}\right)^x$

If $0 < b < 1$, then $\dfrac{1}{b} > 1$.



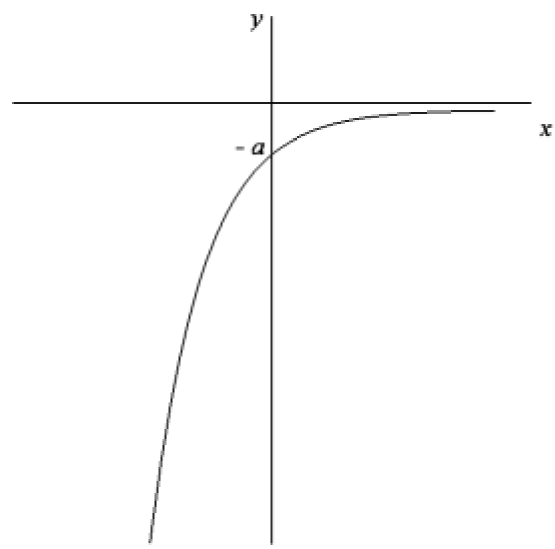
The $y$-intercept of the graph of the function is the point $(0, -a)$.

The horizontal line $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

Let $a > 0$. The sketch of the graph of $y = -a(b^x)$, where $0 < b < 1$ **OR** the sketch of the graph of $y = -a(b^{-x})$, where $b > 1$:

NOTE: $b^{-x} = (b^{-1})^x = \left(\frac{1}{b}\right)^x$

If $b > 1$, then $0 < \frac{1}{b} < 1$.




The $y$-intercept of the graph of the function is the point $(0, -a)$.

The horizontal line $y = 0$, which is the $x$-axis, is a horizontal asymptote of the graph of the function.

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