Pre-Class Problems 13 for Wednesday, March 14

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem number or letter.

1. Find a polynomial $p$ that has the given zeros (roots) and multiplicity.
a. Degree 2 with zeros (roots) -4 and $\frac{2}{3}$ each of multiplicity 1 .
b. Degree 3 with zeros (roots) $-3,-5$, and 5 each of multiplicity 1 .
c. Degree 3 with zeros (roots) 2 of multiplicity 1 and 4 of multiplicity 2.
d. Degree 4 with zeros (roots) $-1,6,-7 i$, and $7 i$ each of multiplicity 1 .
e. Degree 4 with zeros (roots) $-\frac{5}{4}$ of multiplicity 2 and $2-3 i$ and $2+3 i$ each of multiplicity 1 .
f. Degree 14 with zeros (roots) -8 of multiplicity $5,-\frac{1}{2}$ of multiplicity $3, \frac{11}{7}$ of multiplicity 4 , and 6 of multiplicity 2 .
g. Degree 4 with zeros (roots) $-\frac{7}{3}, \frac{7}{3},-i \sqrt{5}$, and $i \sqrt{5}$ each of multiplicity 1.
h. Degree 8 with zeros (roots) $-3,3,-i$, and $i$ each of multiplicity 2 .

Discussion of the Axiom of Trichotomy and solving nonlinear (polynomial and rational) inequalities.
2. Solve the following nonlinear inequalities.
a. $x^{2}-3 x-40<0$
b. $\frac{6-x}{3 x-14} \leq 0$
c. $4 t^{2}+15 t+14>0$
d. $\frac{7-3 x}{x^{2}+5 x-24}<0$
e. $16-5 y^{2} \geq 0$
f. $\frac{5 w^{3}+20 w^{2}}{2 w-5}>0$
g. $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}} \geq 0$
3. Determine the vertical asymptotes for the graph of the following rational functions (if any).
a. $f(x)=\frac{9-x^{2}}{3 x^{2}+16 x-12}$
b. $g(x)=\frac{2 x^{2}-x-28}{x-4}$

Problems available in the textbook: Page $327 \ldots 71-82$ and Example 10 on page 324. Page $378 \ldots 5-14,21-58,63-84,93-102$ and Examples $1-6$ starting on page 370. Page $362 \ldots 7-12$, 13bcefghi, 14bcefghi, 15bcefghi, $17-24$ and Examples 1bc and 2 starting on page 346.

## SOLUTIONS:

1a. Zero (Root) Multiplicity
Back to Problem 1.
$-4$
1

$$
\frac{2}{3} \quad 1
$$

In order for -4 to be a zero (root) of multiplicity $1, x+4$ must be a factor of $p$. In order for $\frac{2}{3}$ to be a zero (root) of multiplicity $1,3 x-2$ must be a factor of $p$.

Thus, $p(x)=a(x+4)(3 x-2)$, where $a$ is any nonzero real number.

Since $(x+4)(3 x-2)=3 x^{2}-2 x+12 x-8=3 x^{2}+10 x-8$, then $p(x)=a\left(3 x^{2}+10 x-8\right)$

Answer: $p(x)=a\left(3 x^{2}+10 x-8\right)$, where $a$ is any nonzero real number
$\begin{array}{ccc}\text { 1b. Zero (Root) } & \text { Multiplicity } & \text { Back to Problem 1. } \\ -3 & 1 & \\ -5 & 1 & \\ 5 & 1 & \end{array}$

In order for -3 to be a zero (root) of multiplicity $1, x+3$ must be a factor of $p$. In order for -5 to be a zero (root) of multiplicity $1, x+5$ must be a factor of $p$. In order for 5 to be a zero (root) of multiplicity 1 , $x-5$ must be a factor of $p$.

Thus, $\quad p(x)=a(x+3)(x+5)(x-5)$, where $a$ is any nonzero real number.

Using the special product formula $(a+b)(a-b)=a^{2}-b^{2}$, we have that $(x+5)(x-5)=x^{2}-25$.

Thus, $(x+3)(x+5)(x-5)=(x+3)\left(x^{2}-25\right)=$

$$
x^{3}-25 x+3 x^{2}-75=x^{3}+3 x^{2}-25 x-75
$$

Thus, $p(x)=a\left(x^{3}+3 x^{2}-25 x-75\right)$

Answer: $p(x)=a\left(x^{3}+3 x^{2}-25 x-75\right)$, where $a$ is any nonzero real number

1c. Zero (Root) Multiplicity Back to Problem 1.
2

4

1
2

In order for 2 to be a zero (root) of multiplicity $1, x-2$ must be a factor of $p$. In order for 4 to be a zero (root) of multiplicity $2,(x-4)^{2}$ must be a factor of $p$.

Thus, $p(x)=a(x-2)(x-4)^{2}$, where $a$ is any nonzero real number.
Using the special product formula $(a-b)^{2}=a^{2}-2 a b+b^{2}$, we have that $(x-4)^{2}=x^{2}-8 x+16$.

Thus, $(x-2)(x-4)^{2}=(x-2)\left(x^{2}-8 x+16\right)=$
$x^{3}-8 x^{2}+16 x-2 x^{2}+16 x-32=x^{3}-10 x^{2}+32 x-32$

Thus, $p(x)=a\left(x^{3}-10 x^{2}+32 x-32\right)$

Answer: $p(x)=a\left(x^{3}-10 x^{2}+32 x-32\right)$, where $a$ is any nonzero real number

1d.
Zero (Root) Multiplicity Back to Problem 1. 1

In order for -1 to be a zero (root) of multiplicity $1, x+1$ must be a factor of $p$. In order for 6 to be a zero (root) of multiplicity $1, x-6$ must be a factor of $p$. In order for $-7 i$ to be a zero (root) of multiplicity $1, x+7 i$ must be a factor of $p$. In order for $7 i$ to be a zero (root) of multiplicity 1 , $x-7 i$ must be a factor of $p$.

Thus, $\quad p(x)=a(x+1)(x-6)(x+7 i)(x-7 i)$, where $a$ is any nonzero real number.

We will use the special product formula $(a+b)(a-b)=a^{2}-b^{2}$ for $(x+7 i)(x-7 i)$. Thus, $(x+7 i)(x-7 i)=x^{2}-49 i^{2}$. Since $i=\sqrt{-1}$, then $i^{2}=-1$. Thus, $(x+7 i)(x-7 i)=x^{2}-49 i^{2}=x^{2}+49$.

Since $(x+1)(x-6)=x^{2}-6 x+x-6=x^{2}-5 x-6$, then

$$
\begin{aligned}
& (x+1)(x-6)(x+7 i)(x-7 i)=\left(x^{2}-5 x-6\right)\left(x^{2}+49\right)= \\
& \left(x^{2}+49\right)\left(x^{2}-5 x-6\right)=x^{4}-5 x^{3}-6 x^{2}+49 x^{2}-245 x-294= \\
& x^{4}-5 x^{3}+43 x^{2}-245 x-294
\end{aligned}
$$

Thus, $p(x)=a\left(x^{4}-5 x^{3}+43 x^{2}-245 x-294\right)$

Answer: $p(x)=a\left(x^{4}-5 x^{3}+43 x^{2}-245 x-294\right)$, where $a$ is any nonzero real number

1 e.

$$
\begin{array}{ll}
-\frac{5}{4} & 2 \\
2-3 i & 1 \\
2+3 i & 1
\end{array}
$$

In order for $-\frac{5}{4}$ to be a zero (root) of multiplicity $2,(4 x+5)^{2}$ must be a factor of $p$. In order for $2-3 i$ to be a zero (root) of multiplicity 1 , $x-(2-3 i)$ must be a factor of $p$. In order for $2+3 i$ to be a zero (root) of multiplicity $1, x-(2+3 i)$ must be a factor of $p$.

Thus, $\quad p(x)=a(4 x+5)^{2}[x-(2-3 i)][x-(2+3 i)]$, where $a$ is any nonzero real number.

Since $[x-(2-3 i)][x-(2+3 i)]=[(x-2)-3 i][(x-2)+3 i]$, then using the special product formula $(a+b)(a-b)=a^{2}-b^{2}$, we have that $[(x-2)-3 i][(x-2)+3 i]=(x-2)^{2}-9 i^{2}$. Using the special product formula $(a-b)^{2}=a^{2}-2 a b+b^{2}$, we have that $(x-2)^{2}=x^{2}-4 x+4$. Thus, $(x-2)^{2}-9 i^{2}=x^{2}-4 x+4+9$ $=x^{2}-4 x+13$.

Using the special product formula $(a+b)^{2}=a^{2}+2 a b+b^{2}$, we have that $(4 x+5)^{2}=16 x^{2}+40 x+25$.

Thus, $(4 x+5)^{2}[x-(2-3 i)][x-(2+3 i)]=$

$$
\begin{aligned}
& \left(16 x^{2}+40 x+25\right)\left(x^{2}-4 x+13\right)=16 x^{4}-64 x^{3}+208 x^{2}+40 x^{3} \\
& -160 x^{2}+520 x+25 x^{2}-100 x+325= \\
& 16 x^{4}-24 x^{3}+73 x^{2}+420 x+325
\end{aligned}
$$

Thus, $p(x)=a\left(16 x^{4}-24 x^{3}+73 x^{2}+420 x+325\right)$

Answer: $p(x)=a\left(16 x^{4}-24 x^{3}+73 x^{2}+420 x+325\right)$, where $a$ is any nonzero real number

1f.

| Zero (Root) | Multiplicity |
| :---: | :---: |
| -8 | 5 |
| $-\frac{1}{2}$ | 3 |
| $\frac{11}{7}$ | 4 |
| 6 | 2 | Back to Problem 1.

$$
\begin{array}{cc}
-\frac{7}{3} & 1 \\
\frac{7}{3} & 1 \\
-i \sqrt{5} & 1 \\
i \sqrt{5} & 1
\end{array}
$$

In order for $-\frac{7}{3}$ to be a zero (root) of multiplicity $1,3 x+7$ must be a factor of $p$. In order for $\frac{7}{3}$ to be a zero (root) of multiplicity $1,3 x-7$ must be a factor of $p$. In order for $-i \sqrt{5}$ to be a zero (root) of multiplicity $1, x+i \sqrt{5}$ must be a factor of $p$. In order for $i \sqrt{5}$ to be a zero (root) of multiplicity $1, x-i \sqrt{5}$ must be a factor of $p$.

Thus, $p(x)=a(3 x+7)(3 x-7)(x+i \sqrt{5})(x-i \sqrt{5})$, where $a$ is any nonzero real number.

We will use the special product formula $(a+b)(a-b)=a^{2}-b^{2}$ for $(3 x+7)(3 x-7)$. Thus, $(3 x+7)(3 x-7)=9 x^{2}-49$.

We will use the special product formula $(a+b)(a-b)=a^{2}-b^{2}$ for $(x+i \sqrt{5})(x-i \sqrt{5})$. Thus, $(x+i \sqrt{5})(x-i \sqrt{5})=x^{2}-5 i^{2}$. Thus, $(x+i \sqrt{5})(x-i \sqrt{5})=x^{2}-5 i^{2}=x^{2}+5$.

Thus, $(3 x+7)(3 x-7)(x+i \sqrt{5})(x-i \sqrt{5})=\left(9 x^{2}-49\right)\left(x^{2}+5\right)=$ $9 x^{4}+45 x^{2}-49 x^{2}-245=9 x^{4}-4 x^{2}-245$

Thus, $p(x)=a\left(9 x^{4}-4 x^{2}-245\right)$

Answer: $\quad p(x)=a\left(9 x^{4}-4 x^{2}-245\right)$, where $a$ is any nonzero real number

1h. Zero (Root) Multiplicity Back to Problem 1.

| -3 | 2 |
| :---: | :---: |
| 3 | 2 |
| $-i$ | 2 |
| $i$ | 2 |

In order for -3 to be a zero (root) of multiplicity $2,(x+3)^{2}$ must be a factor of $p$. In order for 3 to be a zero (root) of multiplicity $2,(x-3)^{2}$ must be a factor of $p$. In order for $-i$ to be a zero (root) of multiplicity 2 , $(x+i)^{2}$ must be a factor of $p$. In order for $i$ to be a zero (root) of multiplicity $2,(x-i)^{2}$ must be a factor of $p$.

Thus, $\quad p(x)=a(x+3)^{2}(x-3)^{2}(x+i)^{2}(x-i)^{2}$, where $a$ is any nonzero real number.

Since $(a b)^{n}=a^{n} b^{n}$, then $(x+3)^{2}(x-3)^{2}=[(x+3)(x-3)]^{2}=$ $\left(x^{2}-9\right)^{2}=x^{4}-18 x^{2}+81$ and $(x+i)^{2}(x-i)^{2}=$
$[(x+i)(x-i)]^{2}=\left(x^{2}+1\right)^{2}=x^{4}+2 x^{2}+1$
Thus, $(x+3)^{2}(x-3)^{2}(x+i)^{2}(x-i)^{2}=$

$$
\begin{aligned}
& \left(x^{4}-18 x^{2}+81\right)\left(x^{4}+2 x^{2}+1\right)=x^{8}+2 x^{6}+x^{4}-18 x^{6}-36 x^{4} \\
& -18 x^{2}+81 x^{4}+162 x^{2}+81=x^{8}-16 x^{6}+46 x^{4}+144 x^{2}+81
\end{aligned}
$$

Thus, $p(x)=a\left(x^{8}-16 x^{6}+46 x^{4}+144 x^{2}+81\right)$

Answer: $p(x)=a\left(x^{8}-16 x^{6}+46 x^{4}+144 x^{2}+81\right)$, where $a$ is any nonzero real number

2a. $x^{2}-3 x-40<0$
Back to Problem 2.

## Step 1:

Find when the nonlinear expression $x^{2}-3 x-40$ is equal to zero. That is, solve the equation $x^{2}-3 x-40=0$.

$$
x^{2}-3 x-40=0 \Rightarrow(x+5)(x-8)=0 \Rightarrow x=-5, x=8
$$

Find when the nonlinear expression $x^{2}-3 x-40$ is undefined. The expression $x^{2}-3 x-40$ is defined for all real numbers $x$.

Step 2: Plot all the numbers found in Step 1 on the real number line.


Step 3: Use the real number line to identify the open intervals determined by the plotted numbers. Pick a test value for each open interval.

$$
\begin{array}{lcl}
\text { Interval } & \text { Test Value } & \text { Sign of } x^{2}-3 x-40=(x+5)(x-8) \\
(-\infty,-5) & -6 & (-6+5)(-6-8)=(-)(-)=+ \\
(-5,8) & 0 & (0+5)(0-8)=(+)(-)=- \\
(8, \infty) & 9 & (9+5)(9-8)=(+)(+)=+
\end{array}
$$

Answer: (-5, 8)

2b. $\frac{6-x}{3 x-14} \leq 0$
Back to Problem 2.

NOTE: This is a two part problem. One part of the problem is to solve the nonlinear inequality $\frac{6-x}{3 x-14}<0$. The other part of the problem is to solve the equation $\frac{6-x}{3 x-14}=0$.

We will use the three step method to solve the nonlinear inequality $\frac{6-x}{3 x-14}<0$ :

## Step 1:

Find when the nonlinear expression $\frac{6-x}{3 x-14}$ is equal to zero. That is, solve the equation $\frac{6-x}{3 x-14}=0$. The fraction is equal to zero if and only if the numerator of the fraction is equal to zero.

That is, $\frac{6-x}{3 x-14}=0 \Rightarrow 6-x=0 \Rightarrow x=6$

Find when the nonlinear expression $\frac{6-x}{3 x-14}$ is undefined. The fraction is undefined if and only if the denominator of the fraction is equal to zero.

That is, $\frac{6-x}{3 x-14}$ undefined $\Rightarrow 3 x-14=0 \Rightarrow x=\frac{14}{3}$

Step 2: Plot all the numbers found in Step 1 on the real number line.

$$
\xrightarrow{\substack{-\frac{14}{3}}} \xrightarrow{+} \quad \text { Sign of } \frac{6-x}{3 x-14}
$$

Step 3: Use the real number line to identify the open intervals determined by the plotted numbers. Pick a test value for each open interval.
Interval
Test Value
Sign of $\frac{6-x}{3 x-14}$
$\left(-\infty, \frac{14}{3}\right)$
$0 \quad \frac{6-0}{0-14}=\frac{(+)}{(-)}=-$
$\left(\frac{14}{3}, 6\right)$
5

$$
\frac{6-5}{15-14}=\frac{(+)}{(+)}=+
$$

$(6, \infty)$
$7 \quad \frac{6-7}{21-14}=\frac{(-)}{(+)}=-$

Thus, the solution for the nonlinear inequality $\frac{6-x}{3 x-14}<0$ is the set of real numbers given by $\left(-\infty, \frac{14}{3}\right) \cup(6, \infty)$. The solution for $\frac{6-x}{3 x-14}=0$ was found in Step 1 above. Thus, the solution for $\frac{6-x}{3 x-14}=0$ is the set $\{6\}$. Putting these two solutions together, we have that the solution for $\frac{6-x}{3 x-14} \leq 0$ is the set of real numbers $\left(-\infty, \frac{14}{3}\right) \cup[6, \infty)$.

Answer: $\left(-\infty, \frac{14}{3}\right) \cup[6, \infty)$

2c. $4 t^{2}+15 t+14>0$
Step 1:

$$
4 t^{2}+15 t+14=0 \Rightarrow(t+2)(4 t+7)=0 \Rightarrow t=-2, t=-\frac{7}{4}
$$

The expression $4 t^{2}+15 t+14$ is defined for all real numbers $t$.

Step 2:


Sign of $4 t^{2}+15 t+14$

Step 3:
Interval $\quad$ Test Value $\quad$ Sign of $(t+2)(4 t+7)$
$(-\infty,-2) \quad-3 \quad(-3+2)(-12+7)=(-)(-)=+$
$\left(-2,-\frac{7}{4}\right) \quad-1.8 \quad(-1.8+2)(-7.2+7)=(+)(-)=-$
$\left(-\frac{7}{4}, \infty\right) \quad 0 \quad(0+2)(0+7)=(+)(+)=+$

Answer: $(-\infty,-2) \cup\left(-\frac{7}{4}, \infty\right)$

2d. $\frac{7-3 x}{x^{2}+5 x-24}<0$
Back to Problem 2.

Step 1: $\quad \frac{7-3 x}{x^{2}+5 x-24}=0 \Rightarrow 7-3 x=0 \Rightarrow x=\frac{7}{3}$

$$
\begin{aligned}
& \frac{7-3 x}{x^{2}+5 x-24} \text { undefined } \Rightarrow x^{2}+5 x-24=0 \Rightarrow \\
& (x+8)(x-3)=0 \Rightarrow x=-8, x=3
\end{aligned}
$$

Step 2:


Step 3:

| Interval | Test Value | Sign of $\frac{7-3 x}{(x+8)(x-3)}$ |
| :--- | :---: | :--- |
| $(-\infty,-8)$ | -9 | $\frac{(+)}{(-)(-)}=\frac{(+)}{(+)}=+$ |
| $\left(-8, \frac{7}{3}\right)$ | 0 | $\frac{(+)}{(+)(-)}=\frac{(+)}{(-)}=-$ |
| $\left(\frac{7}{3}, 3\right)$ | 2.7 | $\frac{(-)}{(+)(-)}=\frac{(-)}{(-)}=+$ |
| $(3, \infty)$ | 4 | $\frac{(-)}{(+)(+)}=\frac{(-)}{(+)}=-$ |

Answer: $\left(-8, \frac{7}{3}\right) \cup(3, \infty)$

2e. $16-5 y^{2} \geq 0$
Back to Problem 2.

NOTE: This is a two part problem. One part of the problem is to solve the nonlinear inequality $16-5 y^{2}>0$. The other part of the problem is to solve the equation $16-5 y^{2}=0$.

We will use the three step method to solve the nonlinear inequality $16-5 y^{2}>0$ :
Step 1: $16-5 y^{2}=0 \Rightarrow 16=5 y^{2} \Rightarrow y^{2}=\frac{16}{5} \Rightarrow y= \pm \frac{4}{\sqrt{5}}$
The expression $16-5 y^{2}$ is defined for all real numbers $y$.


## Step 3:

Interval
Test Value
Sign of $16-5 y^{2}$
$\left(-\infty,-\frac{4}{\sqrt{5}}\right)$
$16-20=-$
$\left(-\frac{4}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$
0
$16-0=+$
$\left(\frac{4}{\sqrt{5}}, \infty\right)$

$$
\begin{equation*}
16-20=- \tag{2}
\end{equation*}
$$

Thus, the solution for the nonlinear inequality $16-5 y^{2}>0$ is the set of real
numbers given by $\left(-\frac{4}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$. The solution for $16-5 y^{2}=0$ was found in Step 1 above. Thus, the solution for $\frac{6-x}{3 x-14}=0$ is the set $\left\{-\frac{4}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right\}$. Putting these two solutions together, we have that the solution for $16-5 y^{2} \geq 0$ is the set of real numbers $\left[-\frac{4}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right]$.

Answer: $\left[-\frac{4}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right]$

2f. $\frac{5 w^{3}+20 w^{2}}{2 w-5}>0$ Back to Problem 2.

NOTE: Since $5 w^{3}+20 w^{2}=5 w^{2}(w+4)$, then $\frac{5 w^{3}+20 w^{2}}{2 w-5}=\frac{5 w^{2}(w+4)}{2 w-5}$

Step 1: $\quad \frac{5 w^{2}(w+4)}{2 w-5}=0 \Rightarrow w=0, w=-4$

$$
\frac{5 w^{2}(w+4)}{2 w-5} \text { undefined } \Rightarrow w=\frac{5}{2}
$$

Step 2:


Step 3:

$$
\begin{array}{lcl}
\text { Interval } & \text { Test Value } & \text { Sign of } \frac{5 w^{2}(w+4)}{2 w-5} \\
(-\infty,-4) & -5 & \frac{(+)(+)(-)}{(-)}=\frac{(-)}{(-)}=+ \\
(-4,0) & -1 & \frac{(+)(+)(+)}{(-)}=\frac{(+)}{(-)}=- \\
\left(0, \frac{5}{2}\right) & 1 & \frac{(+)(+)(+)}{(-)}=\frac{(+)}{(-)}=- \\
\left(\frac{5}{2}, \infty\right) & 3 & \frac{(+)(+)(+)}{(+)}=\frac{(+)}{(+)}=+
\end{array}
$$

Answer: $(-\infty,-4) \cup\left(\frac{5}{2}, \infty\right)$

2g. $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}} \geq 0$
Back to Problem 2.

NOTE: This is a two part problem. One part of the problem is to solve the nonlinear inequality $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}>0$. The other part of the problem is to solve the equation $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}=0$.

We will use the three step method to solve the nonlinear inequality $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}>0$ :

Step 1: $\quad \frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}=0 \Rightarrow x=0, x=-3, x=\frac{5}{6}$

$$
\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}} \text { undefined } \Rightarrow x=7, x=-\frac{13}{4}
$$

Step 2: $\quad$ Sign of $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}$ :


## Step 3:

Interval
Test Value $\quad$ Sign of $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}$

$$
\begin{array}{lcl}
\left(-\infty,-\frac{13}{4}\right) & -4 & \frac{(-)(+)(-)}{(+)(-)}=\frac{(+)}{(-)}=- \\
\left(-\frac{13}{4},-3\right) & -3.1 & \frac{(-)(+)(-)}{(+)(+)}=\frac{(+)}{(+)}=+ \\
(-3,0) & -1 & \frac{(-)(+)(-)}{(+)(+)}=\frac{(+)}{(+)}=+ \\
\left(0, \frac{5}{6}\right) & 0.1 & \frac{(+)(+)(-)}{(+)(+)}=\frac{(-)}{(+)}=- \\
\left(\frac{5}{6}, 7\right) & 1 & (+)(+)(+) \\
(++) & =+
\end{array}
$$

$(7, \infty)$

$$
8 \quad \frac{(+)(+)(+)}{(+)(+)}=\frac{(+)}{(+)}=+
$$

Thus, the solution for the nonlinear inequality $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}>0$ is the set of real numbers $\left(-\frac{13}{4},-3\right) \cup(-3,0) \cup\left(\frac{5}{6}, 7\right) \cup(7, \infty)$.
The solution for $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}=0$ was found in Step 1 above. Thus, the solution for $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}}=0$ is the set $\left\{-3,0, \frac{5}{6}\right\}$. Putting these two solutions together, we have that the solution for $\frac{x^{3}(x+3)^{2}(6 x-5)}{(7-x)^{4}(4 x+13)^{5}} \geq 0$ is the set of real numbers $\left(-\frac{13}{4}, 0\right] \cup\left[\frac{5}{6}, 7\right) \cup(7, \infty)$.

Answer: $\left(-\frac{13}{4}, 0\right] \cup\left[\frac{5}{6}, 7\right) \cup(7, \infty)$

Earn one bonus point because you read the solution to Problem 1g. Send me an email with PC13 in the Subject box.

3a. $f(x)=\frac{9-x^{2}}{3 x^{2}+16 x-12}$
Back to Problem 3.

$$
f(x)=\frac{(3+x)(3-x)}{(x+6)(3 x-2)}
$$

$$
(x+6)(3 x-2)=0 \Rightarrow x=-6, x=\frac{2}{3}
$$

Answer: $x=-6, x=\frac{2}{3}$

3b. $g(x)=\frac{2 x^{2}-x-28}{x-4}$
Back to Problem 3.

$$
g(x)=\frac{(x-4)(2 x+7)}{x-4}=2 x+7, x \neq 4
$$

The graph of the rational function $g$ is the line $y=2 x+7$ with the point $(4,15)$ missing. The rational function $g$ does not have any vertical asymptotes.

Answer: None

Axiom of Trichotomy A real number can only be one of the following: positive, negative, or zero.

NOTE: When you substitute a real number in for the variable in a nonlinear expression, you will either get another real number (which is either positive, negative, or zero) or something that is undefined as a real number.

For example, when we replace the $x$ in the nonlinear expression $\frac{7-3 x}{x^{2}+5 x-24}$ by 4, we get the real number $-\frac{5}{12}$ obtained by $\frac{7-12}{16+20-24}=\frac{-5}{12}$. The resulting real number is negative. If we replace the $x$ by -9 in the expression, we get the
real number $\frac{17}{6}$ obtained by $\frac{7+27}{81-45-24}=\frac{34}{12}=\frac{17}{6}$. This number is positive. If you replace the $x$ by $\frac{7}{3}$ in the expression, you will get the real number zero obtained by $\frac{7-7}{\frac{49}{9}+\frac{35}{3}-24}=\frac{0}{\frac{49}{9}+\frac{105}{9}-\frac{216}{9}}=\frac{0}{-\frac{62}{9}}=0$. Finally, if we replace the $x$ by 3 , we get an undefined real number since we get division by zero obtained by $\frac{7-9}{9+15-24}=\frac{-2}{0}$.

Thus, to solve a nonlinear inequality, we will find all the real numbers that make a nonlinear expression equal to zero. We will also have to find all the numbers that make the nonlinear expression undefined. Thus, all the remaining real numbers, when substituted for the variable in the nonlinear expression, would make the resulting real number either be positive or negative. Thus, a nonlinear expression has the ability to change signs at the real numbers where the expression is either zero or undefined.

We will determine when a nonlinear expression is positive and negative using the following three steps:

Step 1 Find all the real numbers that make the nonlinear expression equal zero and all the real numbers that make the expression undefined.

Step 2 Plot all the numbers found in Step 1 on the real number line.
Step 3 Using the real number line in Step 2, identify the open intervals determined by the plotted numbers. For each open interval, pick a real number that is in the interval. We will call this number the "test value" for the interval. Substitute the test value for the variable in the nonlinear expression. Whatever sign the expression has for this test value, the expression will have the same sign for any number in the open interval.

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