Pre-Class Problems 10 for Monday, February 26

These are the type of problems that you will be working on in class.

You can go to the solution for each problem by clicking on the problem number or letter.

Definition The composition of the function f with the function g, denoted by $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$, where the domain of the function $f \circ g$ is the set of real number x in the domain of the function g such that $g(x)$ is in the domain of the function f.

1. If $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$, then find the following.

   a. $(f \circ g)(-3)$ b. $(g \circ f)(-3)$ c. $(f \circ f)(5)$ d. $(g \circ g)(0)$

2. If $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$, then find the following.

   a. $(f \circ g)(x)$ b. $(g \circ f)(x)$

3. If $h(x) = 8 - 5x$ and $k(x) = x^2 + 9x - 36$, then find the following.

   a. $(h \circ k)(x)$ b. $(k \circ h)(x)$

4. Given the function h, find functions f and g such that $h = f \circ g$.

   a. $h(x) = \sqrt{3x + 25}$ b. $h(x) = (x^2 + 5x - 9)^3$

   c. $h(x) = |2x - 7|$ d. $h(x) = \sqrt[4]{\dfrac{x - 3}{6x + 11}}$

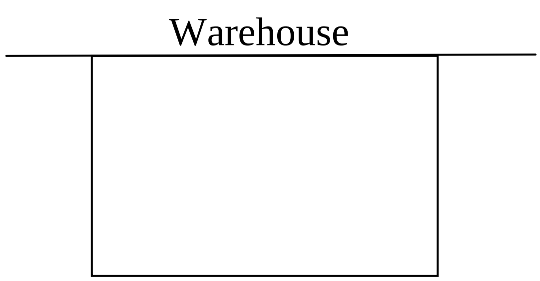
[Discussion](#) of quadratic functions.

5. Write the following quadratic functions in vertex form by completing the square. Then a. identify the vertex, b. determine whether the graph of the parabola opens upward or downward, c. determine the *x*-intercept(s), d. determine the *y*-intercept, e. sketch the graph of the function, f. determine the axis of symmetry, g. determine the maximum or minimum value of the function, h. determine the range of the function.

   a. $f(x) = x^2 + 12x + 45$ b. $g(x) = 2x^2 - 20x + 15$

   c. $h(x) = -3x^2 + 8x - 2$

6. A company wants to use fencing to enclose a rectangular region next to a warehouse. If they have 600 feet of fencing and they do not fence in the side next to the warehouse, what is the largest area that they can enclose?



[Discussion](#) of polynomials and their behavior at $x \to \infty$ and $x \to -\infty$.

7. Determine the behavior of the following functions as $x \to \infty$ and $x \to -\infty$.

   a. $f(x) = 2x^3 - 72x$ b. $g(x) = 48 + 32x - 3x^2 - 2x^3$

   c. $h(x) = 3x^4 + 5x^3 - 12x^2$ d. $f(x) = -16x^4 + 56x^2 - 49$

e. $g(x) = x^5 - 4x^3 - 8x^2 + 32$

f. $h(x) = -4x^5 + 9x^4 - 7$

g. $f(x) = 5x^2(8 - x)(2x + 7)^3$

h. $g(x) = \frac{2}{3}(x - 1)^4(x + 2)(3x - 4)^3$

Problems available in the textbook: Page 271 … 47 – 81, 91 – 98, 99efg – 102efg, 103 – 110 and Examples 6 – 9, 11, 12def starting on page 266. Page 295 … 7 – 30, 49, 51 – 55 and Examples 1 – 3, 5 starting on page 287. Page 311 … 13 – 20 and Example 1 on page 303.

SOLUTIONS:

1a. $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$

$(f \circ g)(-3) = f(g(-3))$

$g(-3) = \sqrt{-9 + 25} = \sqrt{16} = 4$

$(f \circ g)(-3) = f(g(-3)) = f(4) = 64 - 28 - 75 = 64 - 103 = -39$

Answer: $(f \circ g)(-3) = -39$

Back to Problem 1.

1b. $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$

$(g \circ f)(-3) = g(f(-3))$

$f(-3) = 36 + 21 - 75 = -18$

$(g \circ f)(-3) = g(f(-3)) = g(-18) = \sqrt{-54 + 25} = \sqrt{-29} = i\sqrt{29}$

The value of $(g \circ f)(-3)$ is a complex number. The number -3 is not in the domain of the function $g \circ f$ because the number $f(-3)$, which is the number -18, is not in the domain of the function g. The domain of the function g is the interval of real numbers given by $\left[-\frac{25}{3}, \infty\right)$.

Answer: $(g \circ f)(-3)$ is undefined as a real number

Earn one bonus point because you checked this Pre-Class problem. Send me an email with PC10-1b in the Subject line.

1c. $f(x) = 4x^2 - 7x - 75$ Back to Problem 1.

$(f \circ f)(5) = f(f(5))$

$f(5) = 100 - 35 - 75 = 100 - 110 = -10$

$(f \circ f)(5) = f(f(5)) = f(-10) = 400 - 70 - 75 = 400 - 145 = 255$

Answer: $(f \circ f)(5) = 255$

1d. $g(x) = \sqrt{3x + 25}$ Back to Problem 1.

$(g \circ g)(0) = g(g(0))$

$g(0) = \sqrt{0 + 25} = \sqrt{25} = 5$

$(g \circ g)(0) = g(g(0)) = g(5) = \sqrt{15 + 25} = \sqrt{40} = 2\sqrt{10}$

Answer: $(g \circ g)(0) = 2\sqrt{10}$ Back to Problem 1.

2a. $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{3x + 25}) =$

$4(\sqrt{3x + 25})^2 - 7\sqrt{3x + 25} - 75 =$

$4(3x + 25) - 7\sqrt{3x + 25} - 75 = 12x + 100 - 7\sqrt{3x + 25} - 75 =$

$12x + 25 - 7\sqrt{3x + 25}$

Answer: $(f \circ g)(x) = 12x + 25 - 7\sqrt{3x + 25}$

Back to Problem 2.

2b. $f(x) = 4x^2 - 7x - 75$ and $g(x) = \sqrt{3x + 25}$

$(g \circ f)(x) = g(f(x)) = g(4x^2 - 7x - 75) =$

$\sqrt{3(4x^2 - 7x - 75) + 25} = \sqrt{12x^2 - 21x - 225 + 25} =$

$\sqrt{12x^2 - 21x - 200}$

Answer: $(g \circ f)(x) = \sqrt{12x^2 - 21x - 200}$ Back to Problem 2.

3a. $h(x) = 8 - 5x$ and $k(x) = x^2 + 9x - 36$

$(h \circ k)(x) = h(k(x)) = h(x^2 + 9x - 36) = 8 - 5(x^2 + 9x - 36) =$

$8 - 5x^2 - 45x + 180 = 188 - 45x - 5x^2$

Answer: $(h \circ k)(x) = 188 - 45x - 5x^2$ Back to Problem 3.

3b. $h(x) = 8 - 5x$ and $k(x) = x^2 + 9x - 36$

$(k \circ h)(x) = k(h(x)) = k(8 - 5x) = (8 - 5x)^2 + 9(8 - 5x) - 36 =$

$64 - 80x + 25x^2 + 72 - 45x - 36 = 25x^2 - 125x + 100$

Answer: $(k \circ h)(x) = 25x^2 - 125x + 100$ Back to Problem 3.

4a. $h(x) = \sqrt{3x + 25}$ Back to Problem 4.

Let $f(x) = \sqrt{x}$ and let $g(x) = 3x + 25$.

Then $(f \circ g)(x) = f(g(x)) = f(3x + 25) = \sqrt{3x + 25} = h(x)$

Answer: $f(x) = \sqrt{x}$, $g(x) = 3x + 25$

4b. $h(x) = (x^2 + 5x - 9)^3$ Back to Problem 4.

Let $f(x) = x^3$ and let $g(x) = x^2 + 5x - 9$

Then $(f \circ g)(x) = f(g(x)) = f(x^2 + 5x - 9) = (x^2 + 5x - 9)^3 = h(x)$

Answer: $f(x) = x^3$, $g(x) = x^2 + 5x - 9$

4c. $h(x) = |2x - 7|$ Back to Problem 4.

Let $f(x) = |x|$ and let $g(x) = 2x - 7$

Then $(f \circ g)(x) = f(g(x)) = f(2x - 7) = |2x - 7| = h(x)$

Answer: $f(x) = |x|$, $g(x) = 2x - 7$

4d. $h(x) = \sqrt[4]{\dfrac{x - 3}{6x + 11}}$ Back to Problem 4.

Let $f(x) = \sqrt[4]{x}$ and let $g(x) = \dfrac{x - 3}{6x + 11}$

Then $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{x - 3}{6x + 11}\right) = \sqrt[4]{\dfrac{x - 3}{6x + 11}} = h(x)$

Answer: $f(x) = \sqrt[4]{x}$, $g(x) = \dfrac{x - 3}{6x + 11}$

5a. $f(x) = x^2 + 12x + 45$ Back to Problem 5.

$y = x^2 + 12x + 45$

$$\underset{\substack{\downarrow\ \text{Half} \\ 6 \\ \downarrow\ \text{Square} \\ 36}}{y - 45 \underline{\ + 36\ } = x^2 + 12x + \underline{\ 36\ }} \Rightarrow y - 9 = (x + 6)^2$$

a. **Vertex:** $(-6, 9)$ Back to Problem 5.

b. $ax^2 = x^2 \Rightarrow a = 1 > 0 \Rightarrow$ **parabola opens upward**

c. x-intercept(s): Set $y = 0 \Rightarrow -9 = (x + 6)^2$

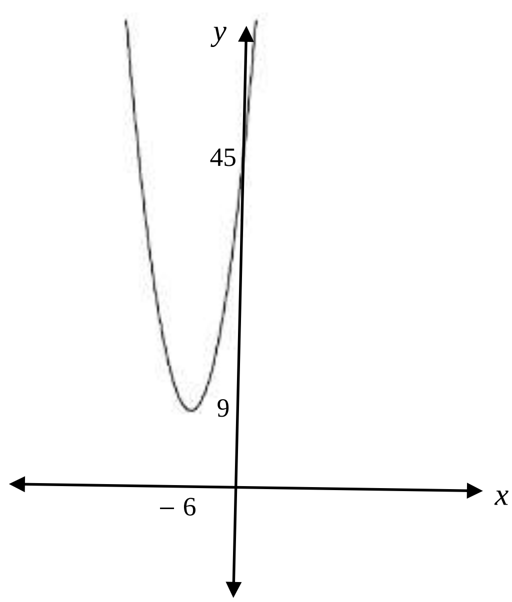
The solutions to the equation $-9 = (x + 6)^2$ are complex numbers. Thus, there are no x-intercepts.

Answer: None

d. y-intercept: Set $x = 0 \Rightarrow y = 45$

Answer: $(0,\ 45)$ Back to Problem 5.

e.



f. axis of symmetry: Since the vertex of the parabola is the point $(-6, 9)$, then the axis of symmetry is the vertical line $x = -6$.

Answer: $x = -6$ Back to Problem 5.

g. Since the parabola opens upward and the vertex of the parabola is the point $(-6, 9)$, then the function only has a minimum value of 9.

Answer: 9 (occurring at $x = -6$)

h. **Range:** $[9, \infty)$

5b. $g(x) = 2x^2 - 20x + 15$ Back to Problem 5.

$$y = 2x^2 - 20x + 15$$

$$y - 15 \underline{\quad} = 2(x^2 - 10x + \underline{\quad})$$

$$y - 15 \underline{+\ 50} = 2(x^2 - 10x + \underline{25}) \Rightarrow y + 35 = 2(x - 5)^2$$

$$\downarrow \textit{Half}$$

$$5$$

$$\downarrow \textit{Square}$$

$$25$$

NOTE: $2 \cdot 25 = 50$ Thus, we added 50 to the right side of the first equation above. So, to keep the equation equivalent, we add 50 to the left side of this equation.

a. **Vertex:** $(5, -35)$ Back to Problem 5.

b. $ax^2 = 2x^2 \Rightarrow a = 2 > 0 \Rightarrow$ **parabola opens upward**

c. x-intercept(s): Set $y = 0 \Rightarrow 35 = 2(x - 5)^2 \Rightarrow$

$$\frac{35}{2} = (x - 5)^2 \Rightarrow x - 5 = \pm\sqrt{\frac{35}{2}} \Rightarrow x - 5 = \pm\frac{\sqrt{70}}{2} \Rightarrow$$

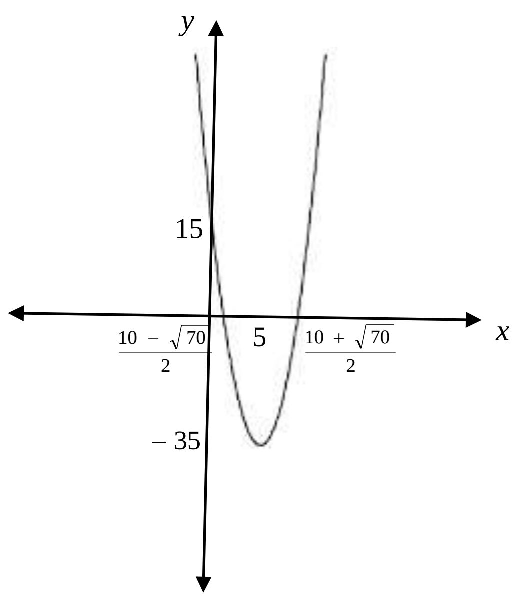
$$x = \frac{10}{2} \pm \frac{\sqrt{70}}{2} = \frac{10 \pm \sqrt{70}}{2}$$

Answer: $\left(\dfrac{10 \pm \sqrt{70}}{2}, 0\right)$ Back to Problem 5.

d. y-intercept: Set $x = 0 \Rightarrow y = 15$

Answer: $(0, 15)$ Back to Problem 5.

e.



f. axis of symmetry: Since the vertex of the parabola is the point $(5, -35)$, then the axis of symmetry is the vertical line $x = 5$.

Answer: $x = 5$

g. Since the parabola opens upward and the vertex of the parabola is the point $(5, -35)$, then the function only has a minimum value of -35.

Answer: -35 (occurring at $x = 5$)

h. **Range:** $[-35, \infty)$

5c. $h(x) = -3x^2 + 8x - 2$

Back to Problem 5.

$$y = -3x^2 + 8x - 2$$

$$y + 2 \underline{\quad\quad} = -3\left(x^2 - \frac{8}{3}x + \underline{\quad\quad}\right)$$

$$y + 2 \underline{- \frac{16}{3}} = -3\left(x^2 - \frac{8}{3}x + \underline{\frac{16}{9}}\right) \Rightarrow y - \frac{10}{3} = -3\left(x - \frac{4}{3}\right)^2$$

$\downarrow$ *Half*

$\frac{4}{3}$

$\downarrow$ *Square*

$\frac{16}{9}$

NOTE: $-3 \cdot \dfrac{16}{9} = -\dfrac{16}{3}$. Thus, we subtracted $\dfrac{16}{3}$ from the right side of the first equation above. So, to keep the equation equivalent, we subtract $\dfrac{16}{3}$ from the left side of this equation.

NOTE: $2 - \dfrac{16}{3} = \dfrac{6}{3} - \dfrac{16}{3} = -\dfrac{10}{3}$

a. **Vertex:** $\left(\dfrac{4}{3}, \dfrac{10}{3} \right)$ Back to Problem 5.

b. $a x^2 = -3x^2 \Rightarrow a = -3 < 0 \Rightarrow$ **parabola opens downward**

c. x-intercept(s): Set $y = 0 \Rightarrow -\dfrac{10}{3} = -3\left(x - \dfrac{4}{3} \right)^2 \Rightarrow$

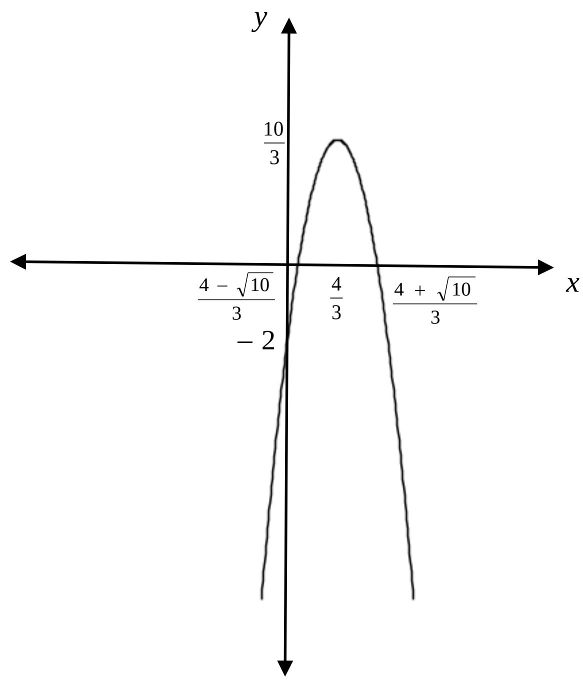
$$\frac{10}{9} = \left(x - \frac{4}{3} \right)^2 \Rightarrow x - \frac{4}{3} = \pm \frac{\sqrt{10}}{3} \Rightarrow x = \frac{4}{3} \pm \frac{\sqrt{10}}{3} = \frac{4 \pm \sqrt{10}}{3}$$

Answer: $\left(\dfrac{4 \pm \sqrt{10}}{3}, 0 \right)$

d. y-intercept: Set $x = 0 \Rightarrow y = -2$

Answer: $(0, -2)$ Back to Problem 5.

e.



f. axis of symmetry: Since the vertex of the parabola is the point $\left(\frac{4}{3}, \frac{10}{3}\right)$, then the axis of symmetry is the vertical line $x = \frac{4}{3}$.

Answer: $x = \frac{4}{3}$

Back to Problem 5.

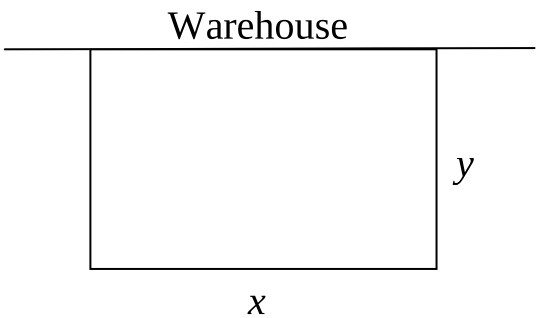
g. Since the parabola opens downward and the vertex of the parabola is the point $\left(\frac{4}{3}, \frac{10}{3}\right)$, then the function only has a maximum value of $\frac{10}{3}$.

Answer: $\frac{10}{3}$ (occurring at $x = \frac{4}{3}$)

h. **Range:** $\left(-\infty, \frac{10}{3}\right]$

Back to Problem 5.

6. A company wants to use fencing to enclose a rectangular region next to a warehouse. If they have 600 feet of fencing and they do not fence in the side next to the warehouse, what is the largest area that they can enclose?



$A = xy$

NOTE: The amount of fencing that would be used to make the enclosure according to the diagram is $x + 2y$.

Since the company has 600 feet of fencing, then $x + 2y = 600$. Solving for x, we have that $x = 600 - 2y$.

Thus, $A = xy$ and $x = 600 - 2y \Rightarrow A = (600 - 2y)y$

$A = (600 - 2y)y = 600y - 2y^2 = -2y^2 + 600y$

Completing the square on $A = -2y^2 + 600y$:

$A \underline{- \ 45000} = -2(y^2 - 300y + \underline{22500}) \Rightarrow A - 45000 = -2(y - 150)^2$

$\downarrow$ *Half*

150

$\downarrow$ *Square*

22500

NOTE: $150^2 = (15 \cdot 10)^2 = (15)^2(10)^2 = 225(100) = 22500$

NOTE: $-2 \cdot 22500 = -45000$. Thus, we subtracted 45000 from the right side of the first equation above. So, to keep the equation equivalent, we subtract 45000 from the left side of this equation.

The vertex of this parabola is of the form (y, A) and the vertex is $(150, 45000)$. Since $a y^2 = -2y^2 \Rightarrow a = -2 < 0$, then the parabola opens downward. Thus, A has a maximum of 45,000 which occurs when $y = 150$. That is, the area of the rectangular enclosure has a maximum area of 45,000 square feet occurring when $y = 150$ feet.

Answer: 45,000 square feet Back to Problem 6.

NOTE: Since $x = 600 - 2y$ and $y = 150$, then $x = 600 - 300 = 300$. Thus, the dimensions of the rectangular enclosure are 300 feet by 150 feet.

7a. $f(x) = 2x^3 - 72x$ Back to Problem 7.

For infinitely large values of x, $f(x) \approx 2x^3$

As $x \to \infty$, $x^3 \to \infty$. Thus, since 2 > 0, then $f(x) \approx 2x^3 \to \infty$.

As $x \to -\infty$, $x^3 \to -\infty$. Thus, since 2 > 0, then $f(x) \approx 2x^3 \to -\infty$.

Answer: As $x \to \infty$, $f(x) \to \infty$.

As $x \to -\infty$, $f(x) \to -\infty$.

7b. $g(x) = 48 + 32x - 3x^2 - 2x^3$ Back to Problem 7.

For infinitely large values of x, $g(x) \approx -2x^3$

As $x \to \infty$, $x^3 \to \infty$. Thus, since $-2 < 0$, then $g(x) \approx -2x^3 \to -\infty$.

As $x \to -\infty$, $x^3 \to -\infty$. Thus, since $-2 < 0$, then $g(x) \approx -2x^3 \to \infty$.

Answer: As $x \to \infty$, $g(x) \to -\infty$.

As $x \to -\infty$, $g(x) \to \infty$.

7c. $h(x) = 3x^4 + 5x^3 - 12x^2$ Back to Problem 7.

For infinitely large values of x, $h(x) \approx 3x^4$

As $x \to \infty$, $x^4 \to \infty$. Thus, since $3 > 0$, then $h(x) \approx 3x^4 \to \infty$.

As $x \to -\infty$, $x^4 \to \infty$. Thus, since $3 > 0$, then $h(x) \approx 3x^4 \to \infty$.

Answer: As $x \to \infty$, $h(x) \to \infty$.

As $x \to -\infty$, $h(x) \to \infty$.

7d. $f(x) = -16x^4 + 56x^2 - 49$ Back to Problem 7.

For infinitely large values of x, $f(x) \approx -16x^4$

As $x \to \infty$, $x^4 \to \infty$. Thus, since $-16 < 0$, then $f(x) \approx -16x^4 \to -\infty$.

As $x \to -\infty$, $x^4 \to \infty$. Thus, since $-16 < 0$, then $f(x) \approx -16x^4 \to -\infty$.

Answer: As $x \to \infty,\ f(x) \to -\infty$.

As $x \to -\infty,\ f(x) \to -\infty$.

7e. $g(x) = x^5 - 4x^3 - 8x^2 + 32$ Back to Problem 7.

For infinitely large values of x, $g(x) \approx x^5$

As $x \to \infty,\ x^5 \to \infty$. Thus, $g(x) \approx x^5 \to \infty$.

As $x \to -\infty,\ x^5 \to -\infty$. Thus, $g(x) \approx x^5 \to -\infty$.

Answer: As $x \to \infty,\ g(x) \to \infty$.

As $x \to -\infty,\ g(x) \to -\infty$.

7f. $h(x) = -4x^5 + 9x^4 - 7$ Back to Problem 7.

For infinitely large values of x, $h(x) \approx -4x^5$

As $x \to \infty,\ x^5 \to \infty$. Thus, since $-4 < 0$, then $h(x) \approx -4x^5 \to -\infty$.

As $x \to -\infty,\ x^5 \to -\infty$. Thus, since $-4 < 0$, then $h(x) \approx -4x^5 \to \infty$.

Answer: As $x \to \infty,\ h(x) \to -\infty$.

As $x \to -\infty,\ h(x) \to \infty$.

7g. $f(x) = 5x^2(8 - x)(2x + 7)^3$ Back to Problem 7.

For infinitely large values of x, $f(x) \approx 5x^2(-x)(2x)^3 = -40x^6$

As $x \to \infty$, $x^6 \to \infty$. Thus, since $-40 < 0$, then $f(x) \approx -40x^6 \to -\infty$.

As $x \to -\infty$, $x^6 \to \infty$. Thus, since $-40 < 0$, then $f(x) \approx -40x^6 \to -\infty$.

Answer: As $x \to \infty$, $f(x) \to -\infty$.

As $x \to -\infty$, $f(x) \to -\infty$.

7h. $g(x) = \dfrac{2}{3}(x - 1)^4(x + 2)(3x - 4)^3$ Back to Problem 7.

For infinitely large values of x, $g(x) \approx \dfrac{2}{3}x^4(x)(3x)^3 = 18x^8$

As $x \to \infty$, $x^8 \to \infty$. Thus, since $\dfrac{2}{3} > 0$, then $g(x) \approx 18x^8 \to \infty$.

As $x \to -\infty$, $x^8 \to \infty$. Thus, since $\dfrac{2}{3} > 0$, then $g(x) \approx 18x^8 \to \infty$.

Answer: As $x \to \infty$, $g(x) \to \infty$.

As $x \to -\infty$, $g(x) \to \infty$.

A function f defined by $f(x) = ax^2 - bx + c$, where $a \neq 0$, is called a quadratic function. The graph of a quadratic function is a parabola which opens upward from its vertex if $a > 0$ or downward from its vertex if $a < 0$.

In order to find the vertex and the axis of symmetry of the parabola, we first set $f(x) = y$ obtaining $y = ax^2 - bx + c$. By completing the square, we can write the function $y = ax^2 - bx + c$ in vertex form as $y - k = a(x - h)^2$. We can identify the vertex and the axis of symmetry from this vertex form of the function.

The vertex of the parabola is the point (h, k) and the axis of symmetry is the vertical line $x = h$.

NOTE: The graph of the equation $y - k = a(x - h)^2$ is the graph of the equation $y = ax^2$ shifted horizontally h units to the right if $h > 0$ or to the left if $h < 0$ and shifted vertically k units upward if $k > 0$ or downward if $k < 0$.

If $a > 0$, then the parabola opens upward and the vertex is the minimum point. The minimum value of the quadratic function is k occurring when $x = h$.

If $a < 0$, then the parabola opens downward and the vertex is the maximum point. The maximum value of the quadratic function is k occurring when $x = h$.

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Definition A polynomial p is a function of the form where

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots\cdot + a_2 x^2 + a_1 x + a_0,$$

where n is a positive integer. The a_i's can be real or complex numbers for $i = 0, 1, 2, 3, \ldots., n$ and $a_n \neq 0$.

NOTE: In this class, we will restrict our discussion to polynomials with real (number) coefficients.

Notation: The coefficient a_n is called the leading coefficient and n is called the degree of the polynomial.

COMMENT: Any polynomial is a "continuous" function. The concept of continuity will be defined in calculus. The continuity of a polynomial implies that its graph can be drawn without lifting your pencil. Thus, there are no points missing in the graph, and there are no jumps or breaks in the graph. In general, the graph of any continuous function can be drawn without lifting your pencil.

COMMENT: The graph of any polynomial is "smooth." The concept of smoothness will also be defined in calculus. The smoothness of the graph means that all the "turns" in the graph are "smooth."

Now, we will look at the behavior of a polynomial for numerically large numbers.

The mathematical symbol $\rightarrow$ means approaches. For example, $x \rightarrow 5$ means that x approaches 5.

Real Number Line: $-\infty \longleftrightarrow \infty$

Then $x \rightarrow \infty$ means x approaches positive infinity. That is, x grows positively without bound. Note, that you can only approach ∞ (positive infinity) from the left-hand side. Also, $x \rightarrow -\infty$ means x approaches negative infinity. That is, x grows negatively without bound. Note, that you can only approach $-\infty$ (negative infinity) from the right-hand side. In calculus, these would be called one-sided limits as $x \rightarrow \infty$ or $x \rightarrow -\infty$.

Now, consider

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

for numerically large values of x positive or negative. In order to do this, we will need to rewrite $p(x)$. Factoring out x^n, we obtain that

$$p(x) = x^n \left(\frac{a_n x^n}{x^n} + \frac{a_{n-1} x^{n-1}}{x^n} + \frac{a_{n-2} x^{n-2}}{x^n} + \cdots + \frac{a_2 x^2}{x^n} + \frac{a_1 x}{x^n} + \frac{a_0}{x^n} \right) =$$

$$x^n \left(a_n + \frac{a_{n-1}}{x} + \frac{a_{n-2}}{x^2} + \cdots + \frac{a_2}{x^{n-2}} + \frac{a_1}{x^{n-1}} + \frac{a_0}{x^n} \right)$$

As $x \to \infty$ or $x \to -\infty$, we have that $\dfrac{a_{n-1}}{x} \to 0$, $\dfrac{a_{n-2}}{x^2} \to 0, \ldots,$

$\dfrac{a_2}{x^{n-2}} \to 0$, $\dfrac{a_1}{x^{n-1}} \to 0$, and $\dfrac{a_0}{x^n} \to 0$.

Thus, we have that $p(x) \approx a_n x^n$ for numerically large values of x positive or negative.

Thus, as $x \to \infty$, $p(x) \to \begin{cases} \infty, & a_n > 0 \\ -\infty, & a_n < 0 \end{cases}$

and as $x \to -\infty$, $p(x) \to \begin{cases} -\infty, & n \text{ is odd and } a_n > 0 \\ -\infty, & n \text{ is even and } a_n < 0 \\ \infty, & n \text{ is odd and } a_n < 0 \\ \infty, & n \text{ is even and } a_n > 0 \end{cases}$

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