Solutions for In-Class Problems 9 for Wednesday, February 21

These problems are from Pre-Class Problems 9.

You can go to the solution for each problem by clicking on the problem letter or problem number.

1. Determine if the graph of the following equations is symmetric with respect to the x-axis, y-axis, origin, or none of these.

   a. $y = x^4 - |x| + 3$ b. $y^2 = 4x^3 - 7x$ c. $y = -6x^5$

2. Determine if the following functions are even, odd, or neither.

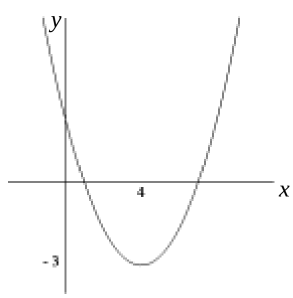
   a. $f(x) = x^6 + 8x^2 - 5$ b. $g(x) = \dfrac{4x^3}{x^2 - 16}$

3. If $h(x) = \begin{cases} x^2 + 6x + 9, & x \le -5 \\ \sqrt[3]{4x + 9}, & -5 < x \le -2 \\ 3x^2 + \dfrac{11}{2}x, & x > -2 \end{cases}$, then find

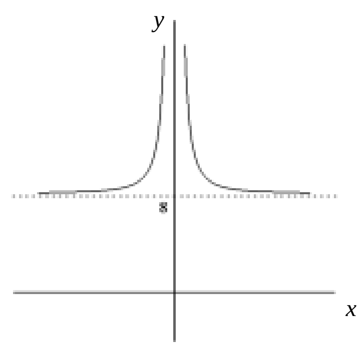
   a. $h(-5)$ b. $h(-3)$ c. $h(-1)$

4. Determine the interval(s) where the following functions are increasing and decreasing. Determine the location and the value of any relative (local) maximum and minimum of the functions.

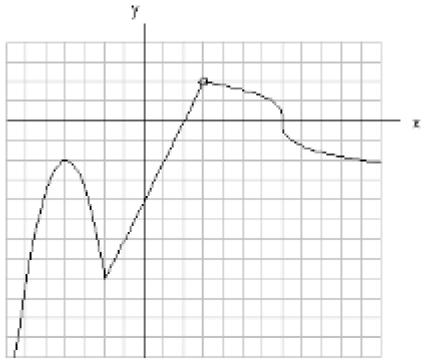
   a.



   b.



c.



5. If $f(x) = \sqrt{2x + 13}$ and $g(x) = 3x^2 - 5x - 27$, then find

a. $(f + g)(-2)$ b. $(f - g)(-2)$ c. $(fg)(-2)$

d. $\left(\frac{f}{g}\right)(-2)$

6. If $f(x) = 9 - x^2$ and $g(x) = 4x - 7$, then find

a. $(f + g)(x)$ b. $(f - g)(x)$ c. $(fg)(x)$

d. $\left(\frac{f}{g}\right)(x)$

SOLUTIONS:

1a. $y = x^4 - |x| + 3$

Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$-y = x^4 - |x| + 3$.

Solving for *y*: $-y = x^4 - |x| + 3 \Rightarrow y = -x^4 + |x| - 3$

The equations $y = x^4 - |x| + 3$ and $-y = x^4 - |x| + 3$ are NOT the same. Thus, there is no symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$y = (-x)^4 - |-x| + 3 = x^4 - |x| + 3$.

The equations $y = x^4 - |x| + 3$ and $y = (-x)^4 - |-x| + 3$ ARE the same. Thus, there is symmetry with respect to the *y*-axis.

To check for symmetry with respect to the origin, replace *x* by $-x$ and replace *y* by $-y$:

$-y = (-x)^4 - |-x| + 3 = x^4 - |x| + 3$

Solving for *y*: $y = -x^4 + |x| - 3$

The equations $y = x^4 - |x| + 3$ and $-y = (-x)^4 - |-x| + 3$ are NOT the same. Thus, there is no symmetry with respect to the origin.

Answer: *y*-axis

1b. $y^2 = 4x^3 - 7x$ Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$(-y)^2 = 4x^3 - 7x \Rightarrow y^2 = 4x^3 - 7x$.

The equations $y^2 = 4x^3 - 7x$ and $(-y)^2 = 4x^3 - 7x$ ARE the same. Thus, there is symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:

$y^2 = 4(-x)^3 - 7(-x) = -4x^3 + 7x$.

The equations $y^2 = 4x^3 - 7x$ and $y^2 = 4(-x)^3 - 7(-x)$ are NOT the same. Thus, there is no symmetry with respect to the *y*-axis.

To check for symmetry with respect to the origin, replace *x* by $-x$ and replace *y* by $-y$:

$(-y)^2 = 4(-x)^3 - 7(-x) \Rightarrow y^2 = -4x^3 + 7x$

The equations $y^2 = 4x^3 - 7x$ and $(-y)^2 = 4(-x)^3 - 7(-x)$ are NOT the same. Thus, there is no symmetry with respect to the origin.

Answer: *x*-axis

1c. $y = -6x^5$ Back to Problem 1.

To check for symmetry with respect to the *x*-axis, replace *y* by $-y$:
$-y = -6x^5$.

Solving for *y*: $-y = -6x^5 \Rightarrow y = 6x^5$

The equations $y = -6x^5$ and $-y = -6x^5$ are NOT the same. Thus, there is no symmetry with respect to the *x*-axis.

To check for symmetry with respect to the *y*-axis, replace *x* by $-x$:
$y = -6(-x)^5 = -6(-x^5) = 6x^5$.

The equations $y = -6x^5$ and $y = -6(-x)^5$ are NOT the same. Thus, there is no symmetry with respect to the *y*-axis.

To check for symmetry with respect to the origin, replace x by $-x$ and replace y by $-y$: $-y = -6(-x)^5$.

Solving for y: $-y = -6(-x)^5 \Rightarrow -y = 6x^5 \Rightarrow y = -6x^5$

The equations $y = -6x^5$ and $-y = -6(-x)^5$ ARE the same. Thus, there is symmetry with respect to the origin.

Answer: origin

2a. $f(x) = x^6 + 8x^2 - 5$ Back to Problem 2.

$$f(-x) = (-x)^6 + 8(-x)^2 - 5 = x^6 + 8x^2 - 5 = f(x)$$

Answer: Even

2b. $g(x) = \dfrac{4x^3}{x^2 - 16}$ Back to Problem 2.

$$g(-x) = \frac{4(-x)^3}{(-x)^2 - 16} = \frac{-4x^3}{x^2 - 16} = -\frac{4x^3}{x^2 - 16} = -g(x)$$

Answer: Odd

3. If $h(x) = \begin{cases} x^2 + 6x + 9, & x \le -5 \\ \sqrt[3]{4x + 9}, & -5 < x \le -2 \\ 3x^2 + \dfrac{11}{2}x, & x > -2 \end{cases}$, then find

3a. To find $h(-5)$: Since $-5 \le -5$ and $h(x) = x^2 + 6x + 9$ when $x \le -5$, then $h(-5) = 25 - 30 + 9 = 4$. NOTE: Since $x^2 + 6x + 9 = (x + 3)^2$, then we could use this information in order to find $h(-5)$. Thus, $h(-5) = (-2)^2 = 4$.

Answer: 4 Back to Problem 3.

3b. To find $h(-3)$: Since $-5 < -3 \le -2$ and $h(x) = \sqrt[3]{4x + 9}$ when $-5 < x \le -2$, then $h(-3) = \sqrt[3]{-12 + 9} = \sqrt[3]{-3} = -\sqrt[3]{3}$.

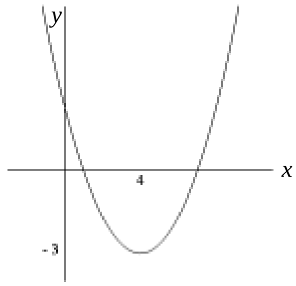
Answer: $-\sqrt[3]{3}$ Back to Problem 3.

3c. To find $h(-1)$: Since $-1 > -2$ and $h(x) = 3x^2 + \dfrac{11}{2}x$ when $x > -2$, then $h(-1) = 3 - \dfrac{11}{2} = \dfrac{6}{2} - \dfrac{11}{2} = -\dfrac{5}{2}$. NOTE: Since $3x^2 + \dfrac{11}{2}x = \dfrac{1}{2}x(6x + 11)$, then we could use this information in order to find $h(-1)$. Thus, $h(-1) = \dfrac{1}{2}(-1)(5) = -\dfrac{5}{2}$.

Answer: $-\dfrac{5}{2}$ Back to Problem 3.

4. Determine the interval(s) where the following functions are increasing and decreasing. Determine the location and the value of any relative maximum and minimum of the functions.

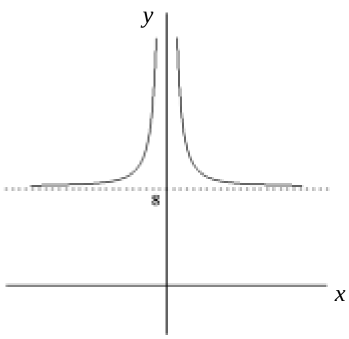
a.



Answer: Increasing: $(4, \infty)$
Decreasing: $(-\infty, 4)$
Rel. Max: None
Rel. Min: -3 at $x = 4$

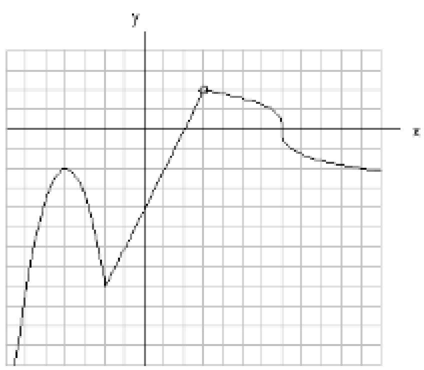
Back to Problem 4.

b.



Answer: Increasing: $(-\infty, 0)$
Decreasing: $(0, \infty)$
Rel. Max: None
Rel. Min: None

c.



Answer: Increasing: $(-\infty, -4) \cup (-2,\ 3)$
Decreasing: $(-4, -2) \cup (3,\ \infty)$
Rel. Max: -2 at $x = -4$
Rel. Min: -8 at $x = -2$

Back to Problem 4.

5. $f(x) = \sqrt{2x + 13}$ and $g(x) = 3x^2 - 5x - 27$

$f(-2) = \sqrt{9} = 3$

$g(-2) = 12 + 10 - 27 = -5$

5a. $(f + g)(-2) = f(-2) + g(-2) = 3 + (-5) = -2$

Answer: -2 Back to Problem 5.

5b. $(f - g)(-2) = f(-2) - g(-2) = 3 - (-5) = 8$

Answer: 8 Back to Problem 5.

5c. $(fg)(-2) = [f(-2)][g(-2)] = 3(-5) = -15$

Answer: -15 Back to Problem 5.

5d. $\left(\frac{f}{g}\right)(-2) = \frac{f(-2)}{g(-2)} = \frac{3}{-5} = -\frac{3}{5}$

Answer: $-\frac{3}{5}$ Back to Problem 5.

6. $f(x) = 9 - x^2$ and $g(x) = 4x - 7$

6a. $(f + g)(x) = f(x) + g(x) = 9 - x^2 + 4x - 7 =$

$2 + 4x - x^2$

Answer: $2 + 4x - x^2$ Back to Problem 6.

6b. $(f - g)(x) = f(x) - g(x) = 9 - x^2 - (4x - 7) =$

$9 - x^2 - 4x + 7 = 16 - 4x - x^2$

Answer: $16 - 4x - x^2$ Back to Problem 6.

6c. $(fg)(x) = [f(x)][g(x)] = (9 - x^2)(4x - 7) =$

$36x - 63 - 4x^3 + 7x^2 = -4x^3 + 7x^2 + 36x - 63$

Answer: $-4x^3 + 7x^2 + 36x - 63$ Back to Problem 6.

6d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{9 - x^2}{4x - 7}$

Answer: $\frac{9 - x^2}{4x - 7}$; $x \neq \frac{7}{4}$ Back to Problem 6.