Solutions for In-Class Problems 8 for Monday, February 19

These problems are from Pre-Class Problems 8.

You can go to the solution for each problem by clicking on the problem letter.

1. Sketch the graph of the following functions. State the domain of the function and use the sketch to state the range of the function. Find the x-intercept(s) or t-intercept(s) and the y-intercept.

   a. $f(x) = 2(x - 6)^2$

   b. $g(t) = -(t + 4)^3 + 9$

   c. $y = \frac{3}{4}\sqrt{5x - 8} - 7$

   d. $h(x) = -\sqrt[3]{3x} + 2$

   e. $f(t) = -|3t + 11| - 4$

   f. $g(x) = \frac{5}{2x - 8} + 3$

SOLUTIONS:

1a. $f(x) = 2(x - 6)^2$

Back to Problem 1.

Domain: $(-\infty, \infty)$

Set $y = f(x)$: $y = 2(x - 6)^2$

Graph Being Shifted: $y = 2x^2$

Parent Function: $y = x^2$

Horizontal Shift: 6 units to the right

Vertical Shift: None

Sketch: I owe you the sketch.

Range: $[0, \infty)$

x-intercept: Set $y = 0$

$y = 2(x - 6)^2 \Rightarrow 0 = 2(x - 6)^2 \Rightarrow (x - 6)^2 = 0 \Rightarrow$
$x - 6 = 0 \Rightarrow x = 6$

***x*-intercept:** $(6, 0)$

y-intercept: Set $x = 0$

$y = 2(x - 6)^2 \Rightarrow y = 2(-6)^2 = 2(36) = 72$

***y*-intercept:** $(0, 72)$

1b. $g(t) = -(t + 4)^3 + 9$ Back to Problem 1.

Domain: $(-\infty, \infty)$

Set $y = g(t)$: $y = -(t + 4)^3 + 9 \Rightarrow y - 9 = -(t + 4)^3$

Graph Being Shifted: $y = -t^3$ **Parent Function:** $y = -x^3$

Horizontal Shift: 4 units to the left

Vertical Shift: 9 units upward

Sketch: I owe you the sketch.

Range: $(-\infty, \infty)$

t-intercept: Set $y = 0$

$$y = -(t+4)^3 + 9 \Rightarrow 0 = -(t+4)^3 + 9 \Rightarrow$$
$$(t+4)^3 = 9 \Rightarrow t + 4 = \sqrt[3]{9} \Rightarrow t = \sqrt[3]{9} - 4$$

***t*-intercept:** $(\sqrt[3]{9} - 4,\ 0)$

y-intercept: Set $t = 0$

$$y = -(t+4)^3 + 9 \Rightarrow y = -(4)^3 + 9 = -64 + 9 = -55$$

***y*-intercept:** $(0,\ -55)$

1c. $y = \frac{3}{4}\sqrt{5x - 8} - 7$ Back to Problem 1.

Domain: $5x - 8 \geq 0 \Rightarrow 5x \geq 8 \Rightarrow x \geq \frac{8}{5}$: $\left[\frac{8}{5},\ \infty\right)$

$$y = \frac{3}{4}\sqrt{5x - 8} - 7 \Rightarrow y + 7 = \frac{3}{4}\sqrt{5\left(x - \frac{8}{5}\right)}$$

Graph Being Shifted: $y = \frac{3}{4}\sqrt{5x}$ **Parent Function:** $y = \sqrt{x}$

Horizontal Shift: $\frac{8}{5}$ units to the right

Vertical Shift: 7 units downward

Sketch: I owe you the sketch.

Range: $[-7,\ \infty)$

x-intercept: Set $y = 0$

$$y = \frac{3}{4}\sqrt{5x - 8} - 7 \Rightarrow 0 = \frac{3}{4}\sqrt{5x - 8} - 7 \Rightarrow 7 = \frac{3}{4}\sqrt{5x - 8} \Rightarrow$$

$$\frac{28}{3} = \sqrt{5x - 8} \Rightarrow 5x - 8 = \frac{784}{9} \Rightarrow 5x = \frac{784}{9} + \frac{72}{9} \Rightarrow 5x = \frac{856}{9} \Rightarrow$$

$$x = \frac{856}{45}$$

NOTE: $28^2 = 28 \cdot 28 = 28(30 - 2) = 840 - 56 = 784$

NOTE: You find $28 \cdot 30$ by multiplying 28 and 3 and tacking a zero on the end of this product.

***x*-intercept:** $\left(\frac{856}{45}, 0\right)$

y-intercept: Set $x = 0$

$$y = \frac{3}{4}\sqrt{5x - 8} - 7 \Rightarrow y = \frac{3}{4}\sqrt{-8} - 7$$

Since $\sqrt{-8} = 2\sqrt{2}\, i$ is a complex number, then the function does not have a *y*-intercept.

***y*-intercept:** None

1d. $h(x) = -\sqrt[3]{3x} + 2$ Back to Problem 1.

Domain: $(-\infty, \infty)$

Set $y = h(x)$: $y = -\sqrt[3]{3x} + 2 \Rightarrow y - 2 = -\sqrt[3]{3x}$

Graph Being Shifted: $y = -\sqrt[3]{3x}$ **Parent Function:** $y = -\sqrt[3]{x}$

Horizontal Shift: None

Vertical Shift: 2 units upward

Sketch: I owe you the sketch.

Range: $(-\infty, \infty)$

x-intercept: Set $y = 0$

$y = -\sqrt[3]{3x} + 2 \Rightarrow 0 = -\sqrt[3]{3x} + 2 \Rightarrow \sqrt[3]{3x} = 2 \Rightarrow 3x = 8 \Rightarrow$
$x = \frac{8}{3}$

***x*-intercept:** $\left(\frac{8}{3}, 0\right)$

y-intercept: Set $x = 0$

$y = -\sqrt[3]{3x} + 2 \Rightarrow y = -\sqrt[3]{0} + 2 = 0 + 2 = 2$

***y*-intercept:** $(0, 2)$

1e. $f(t) = -|3t + 11| - 4$

Back to Problem 1.

Domain: $(-\infty, \infty)$

Set $y = f(t)$: $y = -|3t + 11| - 4 \Rightarrow y + 4 = -\left|3\left(t + \frac{11}{3}\right)\right|$

Graph Being Shifted: $y = -|3t|$

Parent Function: $y = -|x|$

Horizontal Shift: $\frac{11}{3}$ units to the left

Vertical Shift: 4 units downward

Sketch: I owe you the sketch.

Range: $(-\infty, -4]$

t-intercept: Set $y = 0$

$y = -|3t + 11| - 4 \Rightarrow 0 = -|3t + 11| - 4 \Rightarrow |3t + 11| = -4$

NOTE: The absolute value of the expression $3t + 11$ must be nonnegative (zero or positive). Thus, the equation $|3t + 11| = -4$ has no solution. Thus, the function has no *t*-intercept.

***t*-intercept:** None

y-intercept: Set $t = 0$

$y = -|3t + 11| - 4 \Rightarrow y = -|11| - 4 = -11 - 4 = -15$

***y*-intercept:** $(0, -15)$

1f. $g(x) = \dfrac{5}{2x - 8} + 3$

Back to Problem 1.

Domain: $2x - 8 \neq 0 \Rightarrow 2x \neq 8 \Rightarrow x \neq 4$: $(-\infty, 4) \cup (4, \infty)$

Set $y = g(x)$: $y = \dfrac{5}{2x - 8} + 3 \Rightarrow y - 3 = \dfrac{5}{2(x - 4)}$

Graph Being Shifted: $y = \dfrac{5}{2x} = \dfrac{5}{2} \cdot \dfrac{1}{x}$ **Parent Function:** $y = \dfrac{1}{x}$

Horizontal Shift: 4 units to the right

Vertical Shift: 3 units upward

Sketch: I owe you the sketch.

Range: $y \neq 3$: $(-\infty, 3) \cup (3, \infty)$

x-intercept: Set $y = 0$

$y = \dfrac{5}{2x - 8} + 3 \Rightarrow 0 = \dfrac{5}{2x - 8} + 3 \Rightarrow -3 = \dfrac{5}{2x - 8} \Rightarrow$

$-3(2x - 8) = 5 \Rightarrow -6x + 24 = 5 \Rightarrow 19 = 6x \Rightarrow x = \dfrac{19}{6}$

***x*-intercept:** $\left(\dfrac{19}{6}, 0\right)$

y-intercept: Set $x = 0$

$$y = \frac{5}{2x - 8} + 3 \Rightarrow y = \frac{5}{-8} + 3 = -\frac{5}{8} + \frac{24}{8} = \frac{19}{8}$$

***y*-intercept:** $\left(0, \frac{19}{8}\right)$