Solutions for In-Class Problems 7 for Wednesday, February 14

Earn one bonus point because you checked the solutions for these problems. Send me an email with the following in the Subject line: ICPS7.

These problems are from Pre-Class Problems 7.

You can go to the solution for each problem by clicking on the problem letter.

1. If $f(x) = 2x^2 - 7x$, then find the average rate of change of the function f on the intervals a. $[0, 2]$ b. $[2, 5]$ c. $[5, 5 + h]$

2. Find the point-slope form and the slope-intercept form for the equation of the line if given the following.

   a. passes through $(6, -8)$ and $(4, -2)$
   b. passes through $(0, -5)$ and $(3, 0)$
   c. passes through $(-4, -7)$ and is perpendicular to the line $4x - 3y = 12$

3. Mike makes a base salary of $600 per week plus 8% commission on all his sales.
   a. Write a linear function for Mike's weekly salary $S(x)$, where x represents his weekly sales. Find the domain and range of this function.
   b. Find $S(5000)$ and interpret its meaning.
   c. Determine the amount of sales Mike will need to make in order to have a salary of $2000 for one week.

4. Graph the function $y = \sqrt[3]{x}$ by plotting at least five points.

SOLUTIONS:

1a. $[0, 2]$ Back to Problem 1.

$f(x) = 2x^2 - 7x$

NOTE: You need to calculate the slope of the line passing through the points $(0,\ f(0))$ and $(2,\ f(2))$.

$f(0) = 0 - 0 = 0 \Rightarrow (0,\ f(0)) = (0,\ 0)$

$f(2) = 8 - 14 = -6 \Rightarrow (2,\ f(2)) = (2,\ -6)$

$$m_{\text{sec}} = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{0 + 6}{0 - 2} = \frac{6}{-2} = -3$$

NOTE: $\dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{f(0) - f(2)}{0 - 2}$

Answer: -3

1b. $[2,\ 5]$ Back to Problem 1.

$f(x) = 2x^2 - 7x$

NOTE: You need to calculate the slope of the line passing through the points $(2,\ f(2))$ and $(5,\ f(5))$.

$f(2) = 8 - 14 = -6 \Rightarrow (2,\ f(2)) = (2,\ -6)$

$f(5) = 50 - 35 = 15 \Rightarrow (5,\ f(5)) = (5,\ 15)$

$$m_{\sec} = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-6 - 15}{2 - 5} = \frac{-21}{-3} = 7$$

NOTE: $\dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{f(2) - f(5)}{2 - 5}$

Answer: 7

1c. $[5, 5 + h]$ Back to Problem 1.

$f(x) = 2x^2 - 7x$

NOTE: You need to calculate the slope of the line passing through the points $(5, f(5))$ and $(5 + h, f(5 + h))$.

$f(5) = 50 - 35 = 15 \Rightarrow (5, f(5)) = (5, 15)$

$f(5 + h) = 2(5 + h)^2 - 7(5 + h) = 2(25 + 10h + h^2) - 35 - 7h$
$= 50 + 20h + 2h^2 - 35 - 7h = 15 + 13h + 2h^2$

$f(5 + h) = 15 + 13h + 2h^2 \Rightarrow$
$(5 + h, f(5 + h)) = (5 + h, 15 + 13h + 2h^2)$

$$m_{\sec} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 + 13h + 2h^2 - 15}{5 + h - 5} = \frac{13h + 2h^2}{h} =$$

$$= \frac{h(13 + 2h)}{h} = 13 + 2h$$

NOTE: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{f(5+h) - f(5)}{5 + h - 5}$

Answer: $13 + 2h$

2a. passes through $(6, -8)$ and $(4, -2)$ Back to Problem 2.

$$m = \frac{-8 + 2}{6 - 4} = \frac{-6}{2} = -3$$

Using the point $(4, -2)$, the point-slope form of the line is $y + 2 = -3(x - 4)$.

The slope-intercept form of the line can be found by solving the equation $y + 2 = -3(x - 4)$ for y:

$$y + 2 = -3(x - 4) \Rightarrow y + 2 = -3x + 12 \Rightarrow y = -3x + 10$$

Answer: Point-Slope Form: $y + 2 = -3(x - 4)$ or $y + 8 = -3(x - 6)$

Slope-Intercept Form: $y = -3x + 10$

2b. passes through $(0, -5)$ and $(3, 0)$ Back to Problem 2.

$$m = \frac{-5 - 0}{0 - 3} = \frac{-5}{-3} = \frac{5}{3}$$

Using the point $(3,\ 0)$, the point-slope form of the line is

$$y - 0 = \frac{5}{3}(x - 3) \Rightarrow y = \frac{5}{3}(x - 3).$$

The slope-intercept form of the line:

$$y = \frac{5}{3}(x - 3) \Rightarrow y = \frac{5}{3}x - 5$$

Answer: Point-Slope Form: $y = \frac{5}{3}(x - 3)$ or $y + 5 = \frac{5}{3}x$

Slope-Intercept Form: $y = \frac{5}{3}x - 5$

2c. passes through $(-4,\ -7)$ and is perpendicular to the line $4x - 3y = 12$
Back to Problem 2.

$$4x - 3y = 12 \Rightarrow 4x - 12 = 3y \Rightarrow y = \frac{4}{3}x - 4$$

The slope of the line $4x - 3y = 12$ is $\frac{4}{3}$. Thus, the slope of a line which is perpendicular to the line $4x - 3y = 12$ is $-\frac{3}{4}$.

The point-slope form of the line: $y + 7 = -\frac{3}{4}(x + 4)$.

The slope-intercept form of the line can be found by solving the equation $y + 7 = -\frac{3}{4}(x + 4)$ for *y*:

$$y + 7 = -\frac{3}{4}(x + 4) \Rightarrow y + 7 = -\frac{3}{4}x - 3 \Rightarrow y = -\frac{3}{4}x - 10$$

Answer: Point-Slope Form: $y + 7 = -\frac{3}{4}(x + 4)$

Slope-Intercept Form: $y = -\frac{3}{4}x - 10$

3a. Mike makes a base salary of \$600 per week plus 8% commission on all his sales.

$$S(x) = 0.08x + 600$$

Domain of S: $[0, \infty)$

Range of S: $[600, \infty)$

Back to Problem 3.

3b. $S(5000) = 0.08(5000) + 600 = 400 + 600 = \$1000$

Mike had a weekly salary of \$1000 for a week in which he had \$5000 in sales.

Back to Problem 3.

3c. $S(x) = 2000 \Rightarrow 2000 = 0.08x + 600 \Rightarrow 1400 = 0.08x \Rightarrow$

$$0.08x = 1400 \Rightarrow x = \frac{1400}{0.08} = \frac{140000}{8} = \frac{70000}{4} = \frac{35000}{2} = 17500$$

or $x = \frac{1400}{0.08} = \frac{700}{0.04} = \frac{350}{0.02} = \frac{175}{0.01} = 17500$

Mike will need to have weekly sales of $17,500 in order to have a weekly salary of $2000.

Answer: $17,500 Back to Problem 3.

4. I will graph this function in class. Back to Problem 4.