Solutions for In-Class Problems 6 for Monday, February 12

These problems are from Pre-Class Problems 6.

You can go to the solution for each problem by clicking on the problem letter or problem number.

1. Find the domain of the following functions. Write your answer in interval notation.

   a. $g(x) = \sqrt{3x - 36}$ b. $r(x) = \dfrac{49 - x^2}{12x^2 + 28x - 80}$

   c. $h(x) = \dfrac{\sqrt[4]{5 - x}}{x^2 - 9x + 18}$ d. $f(x) = \sqrt[3]{\dfrac{27x + 1}{x + 8}}$

2. If $f = \{(-6, 9), (-3, 2), (1, 7), (2, 9), (5, -6)\}$, then find the following.

   a. the domain of f b. the range of f c. $f(-6)$

   d. $f(1)$ e. the value(s) of x for which $f(x) = 2$

   f. the value(s) of x for which $f(x) = 9$

3. If $g(x) = 3x^2 - 8x - 12$, then find $\dfrac{g(x + h) - g(x)}{h}$.

4. Betty wishes to fence a rectangular region of area 500 square yards. Express the amount F of fencing that is required as function of x, which is the length of the rectangle.

SOLTUIONS:

1a. $g(x) = \sqrt{3x - 36}$

Want (Need): $3x - 36 \geq 0 \Rightarrow 3x \geq 36 \Rightarrow x \geq 12$

Answer: $[12, \infty)$ Back to Problem 1.

1b. $r(x) = \dfrac{49 - x^2}{12x^2 + 28x - 80}$

Want (Need): $12x^2 + 28x - 80 \neq 0$

$12x^2 + 28x - 80 \neq 0 \Rightarrow 4(3x^2 + 7x - 20) \neq 0 \Rightarrow$

$4(x + 4)(3x - 5) \neq 0 \Rightarrow x + 4 \neq 0$ and $3x - 5 \neq 0 \Rightarrow$

$x \neq -4$ and $x \neq \dfrac{5}{3}$

Answer: $(-\infty, -4) \cup \left(-4, \dfrac{5}{3}\right) \cup \left(\dfrac{5}{3}, \infty\right)$ Back to Problem 1.

1c. $h(x) = \dfrac{\sqrt[4]{5 - x}}{x^2 - 9x + 18}$

Want (Need): $5 - x \geq 0$ **and** $x^2 - 9x + 18 \neq 0$

$5 - x \geq 0 \Rightarrow 5 \geq x \Rightarrow x \leq 5$

$x^2 - 9x + 18 \neq 0 \Rightarrow (x - 3)(x - 6) \neq 0 \Rightarrow x \neq 3$ **and** $x \neq 6$

Thus, we need that $x \leq 5$ **and** $x \neq 3$ **and** $x \neq 6$. Since $x \leq 5$ **and** $x \neq 6$ is equilavent to $x \leq 5$, then $x \leq 5$ **and** $x \neq 3$ **and** $x \neq 6$ implies that $x \leq 5$ **and** $x \neq 3$.

Answer: $(-\infty, 3) \cup (3, 5]$ Back to Problem 1.

1d. $f(x) = \sqrt[3]{\dfrac{27x + 1}{x + 8}}$

NOTE: Cube roots are defined for all real numbers. We just can't do division by zero.

Want (Need): $x + 8 \neq 0 \Rightarrow x \neq -8$

Answer: $(-\infty, -8) \cup (-8, \infty)$ Back to Back to Problem 1.

2a. the domain of *f*

$f = \{(-6, 9), (-3, 2), (1, 7), (2, 9), (5, -6)\}$

NOTE: The points of the function *f* are of the form $(x, f(x))$.

The domain of the function is set $\{-6, -3, 1, 2, 5\}$.

Answer: $\{-6, -3, 1, 2, 5\}$ Back to Problem 2.

2b. the range of *f*

$f = \{(-6, 9), (-3, 2), (1, 7), (2, 9), (5, -6)\}$

NOTE: The points of the function *f* are of the form $(x,\ f(x))$.

The range of the function is set $\{9, 2, 7, -6\}$. NOTE: We don't list the value of -7 for a second time. We only list it once.

Answer: $\{9, 2, 7, -6\}$ **or** $\{-6, 2, 7, 9\}$ Back to Problem 2.

2c. $f(-6)$

$f = \{(-6, 9), (-3, 2), (1, 7), (2, 9), (5, -6)\}$

NOTE: The points of the function *f* are of the form $(x,\ f(x))$.

$(-6, 9) \Rightarrow f(-6) = 9$

Answer: 9 Back to Problem 2.

2d. $f(1)$

$f = \{(-6, 9), (-3, 2), (1, 7), (2, 9), (5, -6)\}$

NOTE: The points of the function *f* are of the form $(x,\ f(x))$.

$(1, 7) \Rightarrow f(1) = 7$

Answer: 7 Back to Problem 2.

2e. the value(s) of *x* for which $f(x) = 2$

$f = \{(-6, 9), (-3, 2), (1, 7), (2, 9), (5, -6)\}$

NOTE: The points of the function *f* are of the form $(x,\ f(x))$.

$(-3, 2) \Rightarrow f(-3) = 2$ Thus, $f(x) = 2$ when $x = -3$

Answer: -3 Back to Problem 2.

2f. the value(s) of *x* for which $f(x) = 9$

$f = \{(-6, 9), (-3, 2), (1, 7), (2, 9), (5, -6)\}$

NOTE: The points of the function *f* are of the form $(x,\ f(x))$.

$(-6, 9) \Rightarrow f(-6) = 9$ Thus, $f(x) = 9$ when $x = -6$

$(2, 9) \Rightarrow f(2) = 9$ Thus, $f(x) = 9$ when $x = 2$

Answer: $-6,\ 2$ Back to Problem 2.

3. If $g(x) = 3x^2 - 8x - 12$, then find $\dfrac{g(x+h) - g(x)}{h}$.

$g(x+h) = 3(x+h)^2 - 8(x+h) - 12 =$

$3(x^2 + 2xh + h^2) - 8x - 8h - 12 = 3x^2 + 6xh + 3h^2 - 8x - 8h - 12$

$g(x) = 3x^2 - 8x - 12$

NOTE: In the subtraction of $g(x)$ from $g(x+h)$, the $3x^2$ terms will cancel, the $-8x$ terms will cancel, and the -12's will cancel. Thus,

$g(x+h) - g(x) = 6xh + 3h^2 - 8h = h(6x + 3h - 8)$

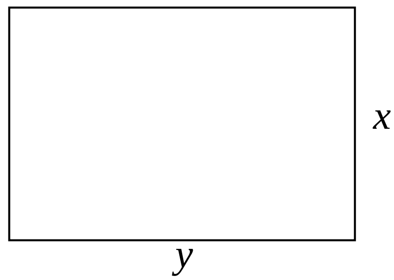
NOTE: In the division of $g(x+h) - g(x)$ by *h*, the *h*'s will cancel. Thus,

$\dfrac{g(x+h) - g(x)}{h} = 6x + 3h - 8$ provided that $h \neq 0$.

Answer: $6x + 3h - 8$ Back to Problem 3.

4. Betty wishes to fence a rectangular region of area 500 square yards. Express the amount F of fencing that is required as function of *x*, which is the length of the rectangle.

We will need to identify the width of the rectangle: Let *y* be the width of the rectangle.



NOTE: The amount of fencing, which is required to fence this rectangular region, is the perimeter of the rectangle. Thus, $F = 2x + 2y$.

NOTE: F is a function of two variables x and y. In order to get F as a function of one variable x, we will need to get a relationship between x and y. A relationship between x and y is an equation containing only the variables of x and y. We haven't used the information that the area of the rectangular enclosure is to 500 square yards. Since the area of a rectangle is given by the formula $A = l\,w$, then the area of our rectangular enclosure is $A = xy$. Thus, in order for the area of our enclosure to be 500 cubic feet, we need that $xy = 500$. This is our relationship between x and y.

Now, we can solve for y in terms of x. Thus, $xy = 500 \Rightarrow y = \dfrac{500}{x}$.

Since $F = 2x + 2y$ and $y = \dfrac{500}{x}$, then $F = 2x + 2y = 2x + 2\left(\dfrac{500}{x}\right)$

$= 2x + \dfrac{1000}{x}$.

Answer: $F = 2x + \dfrac{1000}{x}$ (in yards)

Back to Problem 4.