Solutions for In-Class Problems 5 for Wednesday, February 7

These problems are from Pre-Class Problems 5.

You can go to the solution for each problem by clicking on the problem letter or problem number.

1. Identify the set of values of *x* for which *y* will be a real number. Use interval notation to write your answer.

   a. $y = \dfrac{4}{5x + 6}$ b. $y = \sqrt{25 - 49x}$

2. Find the *x*-intercept(s) and the *y*-intercept(s) of the graph of the following equations.

   a. $9x^2 - y^2 = 81$ b. $y = 2|3x - 7| + 11$

3. Write the equation of the circle in standard form given the following information.

   a. Center: $(-3, 7)$; Radius: 4

   b. The endpoints of a diameter are $(-2, -5)$ and $(6, -11)$.

   c. The center is $(9, 4)$ and the point $(1, -3)$ is a point on the circle.

   d. Write an equation that represents the set of points that are 7 units from the point $(-6, 0)$.

   e. The center is $(5, -8)$ and the circle is tangent to the *x*-axis.

4. Write the following equation of a circle in standard form. Then find the center and radius of the circle.
   $x^2 + y^2 - 6x + 14y + 30 = 0$.

5. If $f(x) = 3x^2 - 4x - 12$, then find

a. $f(-2)$ b. $f(3)$ c. $f(x+h)$

SOLUTIONS:

1a. $y = \dfrac{4}{5x + 6}$

$$5x + 6 \neq 0 \Rightarrow 5x \neq -6 \Rightarrow x \neq -\frac{6}{5}$$

Answer: $\left(-\infty, -\dfrac{6}{5}\right) \cup \left(-\dfrac{6}{5}, \infty\right)$ Back to Problem 1.

1b. $y = \sqrt{25 - 49x}$

$$25 - 49x \geq 0 \Rightarrow 25 \geq 49x \Rightarrow \frac{25}{49} \geq x \Rightarrow x \leq \frac{25}{49}$$

Answer: $\left(-\infty, \dfrac{25}{49}\right]$ Back to Problem 1.

2a. $9x^2 - y^2 = 81$ Back to Problem 2.

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$9x^2 - y^2 = 81$ and $y = 0 \Rightarrow 9x^2 = 81 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

***x*-intercept(s):** $(-3,\ 0)$; $(3,\ 0)$

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$9x^2 - y^2 = 81$ and $x = 0 \Rightarrow -y^2 = 81 \Rightarrow y^2 = -81 \Rightarrow y = \pm 9i$

***y*-intercept(s):** None

2b. $y = 2|3x - 7| + 11$ Back to Problem 2.

x-intercept(s): To find the *x*-intercept(s), set $y = 0$.

$y = 2|3x - 7| + 11$ and $y = 0 \Rightarrow 0 = 2|3x - 7| + 11 \Rightarrow$

$-11 = 2|3x - 7| \Rightarrow |3x - 7| = -\frac{11}{2}$

The equation $|3x - 7| = -\frac{11}{2}$ had no solutions.

***x*-intercept(s):** None

y-intercept(s): To find the *y*-intercept(s), set $x = 0$.

$y = 2|3x - 7| + 11$ and $x = 0 \Rightarrow y = 2|-7| + 11 = 2(7) + 11 =$

$14 + 11 = 25$

***y*-intercept(s):** $(0,\ 25)$

3a. Center: $(-3, 7)$; Radius: 4 Back to Problem 3.

Standard form: $(x - h)^2 + (y - k)^2 = r^2$

$h = -3,\ k = 7,\ r = 4 \Rightarrow (x + 3)^2 + (y - 7)^2 = 16$

Answer: $(x + 3)^2 + (y - 7)^2 = 16$ Back to Problem 3.

3b. The endpoints of a diameter are $(-2,\ -5)$ and $(6,\ -11)$.

Use the midpoint formula to find the center of the circle:

$$\left(\frac{-2 + 6}{2}, \frac{-5 - 11}{2} \right) = \left(\frac{4}{2}, \frac{-16}{2} \right) = (2,\ -8)$$

Use the distance formula to find the length of the diameter in order to find the radius:

$$d = \sqrt{(-2-6)^2 + (-5+11)^2} = \sqrt{(-8)^2 + 6^2} = \sqrt{64+36} = \sqrt{100}$$
$$= 10$$

Since the diameter is 10, then the radius is 5.

Center: $(2,\ -8)$; Radius: 5

Standard form: $(x - h)^2 + (y - k)^2 = r^2$

$h = 2,\ k = -8,\ r = 5 \Rightarrow (x - 2)^2 + (y + 8)^2 = 25$

Answer: $(x - 2)^2 + (y + 8)^2 = 25$ Back to Problem 3.

3c. The center is $(9, 4)$ and the point $(1, -3)$ is a point on the circle.

Standard form: $(x - h)^2 + (y - k)^2 = r^2$

$h = 9,\ k = 4,\ r = ? \Rightarrow (x - 9)^2 + (y - 4)^2 = r^2$

In order for the point $(1, -3)$ to be on the circle, it must satisfy the equation for the circle. Thus,

$(1 - 9)^2 + (-3 - 4)^2 = r^2 \Rightarrow r^2 = (-8)^2 + (-7)^2 = 64 + 49 = 113$

$r^2 = 113 \Rightarrow (x - 9)^2 + (y - 4)^2 = 113$

Answer: $(x - 9)^2 + (y - 4)^2 = 113$ Back to Problem 3.

3d. Write an equation that represents the set of points that are 7 units from the point $(-6, 0)$.

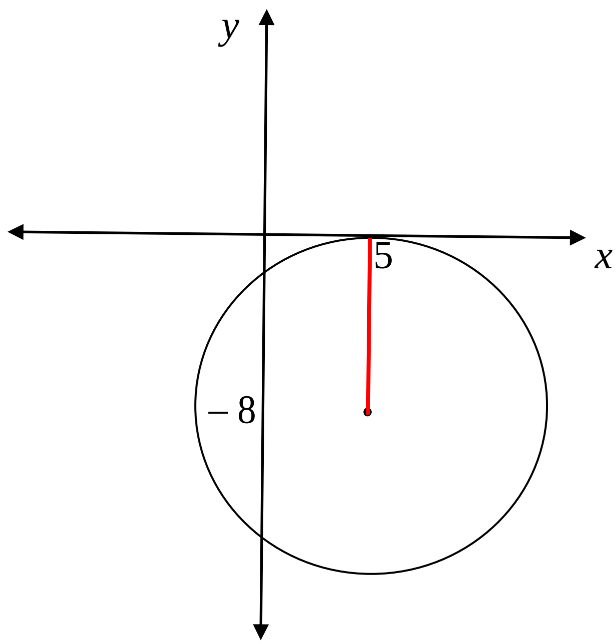
Standard form: $(x - h)^2 + (y - k)^2 = r^2$

$h = -6,\ k = 0,\ r = 7 \Rightarrow (x+6)^2 + (y-0)^2 = 49 \Rightarrow$

$(x+6)^2 + y^2 = 49$

Answer: $(x+6)^2 + y^2 = 49$ Back to Problem 3.

3e. The center is $(5, -8)$ and the circle is tangent to the *x*-axis.



Standard form: $(x-h)^2 + (y-k)^2 = r^2$

NOTE: The radius of the center is the length of the red line segment shown above. This length is 8.

$h = 5,\ k = -8,\ r = 8 \Rightarrow (x-5)^2 + (y+8)^2 = 64$

Answer: $(x-5)^2+(y+8)^2=64$ Back to Problem 3.

4. We will need to complete the squares for both x and y in order to put the equation in standard form.

$$x^2+y^2-6x+14y+30=0$$

$$x^2-6x+\underline{\ 9\ }+y^2+14y+\underline{\ 49\ }=-30+9+49$$

$\downarrow$ *Half* $\quad\quad$ $\downarrow$ *Half*

3 $\quad\quad$ 7

$\downarrow$ *Square* $\quad\quad$ $\downarrow$ *Square*

9 $\quad\quad$ 49

$$(x-3)^2+(y+7)^2=28$$

Center: $(3,-7)$ **Radius:** $\sqrt{28}=2\sqrt{7}$

Back to Problem 4.

5a. $f(x)=3x^2-4x-12$

$f(-2)=12+8-12=8$

Answer: 8 Back to Problem 5.

5b. $f(x)=3x^2-4x-12$

$f(3)=27-12-12=3$

Answer: 3 Back to Problem 5.

5c. $f(x) = 3x^2 - 4x - 12$

$f(x+h) = 3(x+h)^2 - 4(x+h) - 12 =$

$3(x^2 + 2xh + h^2) - 4x - 4h - 12 =$

$3x^2 + 6xh + 3h^2 - 4x - 4h - 12$

Answer: $3x^2 + 6xh + 3h^2 - 4x - 4h - 12$