Solutions for In-Class Problems 4 for Wednesday, January 31

These problems are from Pre-Class Problems 4.

1. A motorboat travels 72 miles with a current of 3 mph. The return trip against the current takes 2 hours longer. Find the average rate of the motorboat in still water.

2. Solve the following equations.

   a. $3|6x - 11| - 8 = 13$ b. $|9 - 4t| = |t + 3|$

3. Solve $\sqrt{3x + 16} + \sqrt{x + 13} = 5$

4. Solve the following inequalities. Graph the solution set. Write the solution set in interval notation.

   a. $8x + 9 < -15$ b. $\frac{y + 4}{4} - \frac{3y - 8}{6} \leq -\frac{7}{3}$

5. Solve the following inequalities. Graph the solution set. Write the solution set in interval notation.

   a. $-3 < 5 - 4t \leq 25$ b. $5 \leq \frac{6w + 11}{3} < 7$

6. Solve the following inequalities. Graph the solution set. Write the solution set in interval notation.

   a. $|2x - 7| < 15$ b. $6|3y + 8| - 7 \geq 23$

SOLUTIONS:

1. A motorboat travels 72 miles with a current of 3 mph. The return trip against the current takes 2 hours longer. Find the average rate of the motorboat in still water. Back to Problem 1.

Let m = the rate of the motorboat in still water

	R	$T = \frac{D}{R}$	D
With the current	$m + 3$	$\frac{72}{m + 3}$	72
Against the current	$m - 3$	$\frac{72}{m - 3}$	72

NOTE: The difference between the time to travel against the current and the time to travel with the current is 2 hours. That is, $\frac{72}{m - 3} - \frac{72}{m + 3} = 2$.

$$\frac{72}{m - 3} - \frac{72}{m + 3} = 2 \Rightarrow$$

$$(m + 3)(m - 3)\left(\frac{72}{m - 3} - \frac{72}{m + 2}\right) = 2(m + 3)(m - 3) \Rightarrow$$

$$72(m + 3) - 72(m - 3) = 2(m^2 - 9) \Rightarrow$$

$$72m + 216 - 72m + 216 = 2m^2 - 18 \Rightarrow 432 = 2m^2 - 18 \Rightarrow$$

$$450 = 2m^2 \Rightarrow m^2 = 225 \Rightarrow m = \pm 15$$

Speed can't be negative. Thus, $m = 15$.

Answer: 15 mph Back to Problem 1.

2a. $3|6x - 11| - 8 = 13$ Back to Problem 2.

$$3|6x - 11| - 8 = 13 \Rightarrow 3|6x - 11| = 21 \Rightarrow |6x - 11| = 7 \Rightarrow$$

$$6x - 11 = \pm 7 \Rightarrow 6x = 11 \pm 7 \Rightarrow x = \frac{11 \pm 7}{6}$$

$$x = \frac{11 - 7}{6} = \frac{4}{6} = \frac{2}{3}, \quad x = \frac{11 + 7}{6} = \frac{18}{6} = 3$$

Answer: $x = \frac{2}{3}, 3$ **or** $\left\{\frac{2}{3}, 3\right\}$

2b. $|9 - 4t| = |t + 3|$ Back to Problem 2.

$$|9 - 4t| = |t + 3| \Rightarrow 9 - 4t = \pm(t + 3)$$

$9 - 4t = t + 3$ or $9 - 4t = -(t + 3)$

$6 = 5t$ $9 - 4t = -t - 3$

$t = \frac{6}{5}$ $12 = 3t$

$t = 4$

Answer: $t = 4, \frac{6}{5}$ **or** $\left\{4, \frac{6}{5}\right\}$

3. $\sqrt{3x+16} + \sqrt{x+13} = 5$ Back to Problem 3.

$\sqrt{3x+16} + \sqrt{x+13} = 5 \Rightarrow \sqrt{3x+16} = 5 - \sqrt{x+13} \Rightarrow$

$3x + 16 = 25 - 10\sqrt{x+13} + x + 13 \Rightarrow 2x - 22 = -10\sqrt{x+13} \Rightarrow$

$x - 11 = -5\sqrt{x+13} \Rightarrow x^2 - 22x + 121 = 25(x+13) \Rightarrow$

$x^2 - 22x + 121 = 25x + 325 \Rightarrow x^2 - 47x - 204 = 0 \Rightarrow$

$(x+4)(x-51) = 0 \Rightarrow x = -4, x = 51$

Check for $x = -4$: $\sqrt{-12+16} + \sqrt{-4+13} \stackrel{?}{=} 5 \Rightarrow \sqrt{4} + \sqrt{9} \stackrel{?}{=} 5$

$\Rightarrow 2 + 3 \stackrel{?}{=} 5 \Rightarrow 5 \stackrel{?}{=} 5$ True

Check for $x = 51$: $\sqrt{153+16} + \sqrt{51+13} \stackrel{?}{=} 5 \Rightarrow \sqrt{169} + \sqrt{64} \stackrel{?}{=} 5$

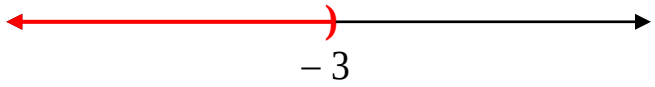
$\Rightarrow 13 + 8 \stackrel{?}{=} 5 \Rightarrow 21 \stackrel{?}{=} 5$ False

Answer: $x = -4$ **or** $\{-4\}$ Back to Problem 3.

4a. $8x + 9 < -15$ Back to Problem 4.

$8x + 9 < -15 \Rightarrow 8x < -24 \Rightarrow x < -3$

Answer:



Answer: $(-\infty, -3)$

4b. $\dfrac{y+4}{4} - \dfrac{3y-8}{6} \le -\dfrac{7}{3}$

Back to Problem 4.

LCD = 12

$$\frac{y+4}{4} - \frac{3y-8}{6} \le -\frac{7}{3} \Rightarrow 12\left(\frac{y+4}{4} - \frac{3y-8}{6}\right) \le \left(-\frac{7}{3}\right)12 \Rightarrow$$

$$3(y+4) - 2(3y-8) \le -28 \Rightarrow 3y + 12 - 6y + 16 \le -28 \Rightarrow$$

$$-3y + 28 \le -28 \Rightarrow -3y \le -56 \Rightarrow y \ge \frac{56}{3}$$

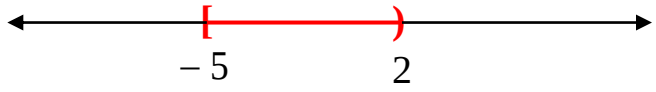
Answer:

Answer: $\left[\dfrac{56}{3}, \infty\right)$

5a. $-3 < 5 - 4t \le 25$

Back to Problem 5.

$-3 < 5 - 4t \le 25 \Rightarrow -8 < -4t \le 20 \Rightarrow 2 > t \ge -5 \Rightarrow -5 \le t < 2$

Answer: 

Answer: $[-5,\ 2)$

5b. $5 \le \dfrac{6w + 11}{3} < 7$ Back to Problem 5.

$$5 \le \frac{6w + 11}{3} < 7 \Rightarrow 15 \le 6w + 11 < 21 \Rightarrow 4 \le 6w < 10 \Rightarrow$$

$$\frac{4}{6} \le w < \frac{10}{6} \Rightarrow \frac{2}{3} \le w < \frac{5}{3}$$

Answer: 

Answer: $\left[\dfrac{2}{3}, \dfrac{5}{3}\right)$

6a. $|2x - 7| < 15$ Back to Problem 6.

$$|2x - 7| < 15 \Rightarrow -15 < 2x - 7 < 15 \Rightarrow -8 < 2x < 22 \Rightarrow -4 < x < 11$$

Answer: 

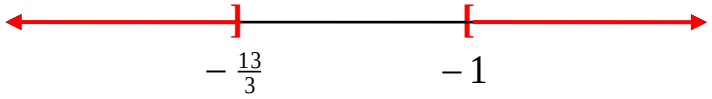
Answer: $(-4,\ 11)$

6b. $6|3y + 8| - 7 \geq 23$ Back to Problem 6.

$$6|3y + 8| - 7 \geq 23 \Rightarrow 6|3y + 8| \geq 30 \Rightarrow |3y + 8| \geq 5$$

$$|3y + 8| \geq 5 \Rightarrow 3y + 8 \geq 5 \quad \text{or} \quad 3y + 8 \leq -5$$
$$3y \geq -3 \qquad 3y \leq -13$$
$$y \geq -1 \qquad y \leq -\frac{13}{3}$$

Answer:



Answer: $\left(-\infty, -\frac{13}{3}\right] \cup [-1, \infty)$