Solutions for In-Class Problems 3 for Monday, January 29

These problems are from Pre-Class Problems 3.

You can go to the solution for each problem by clicking on the problem letter or problem number.

1. Solve the following equations using the quadratic formula.

   a. $x^2 - 10x + 7 = 0$ b. $\frac{1}{2}y^2 + \frac{2}{3}y = \frac{3}{4}$

2. Determine the type of solutions for the quadratic equation $12 - 5y - 3y^2 = 0$ by calculating the discriminant.

3. Solve the following problems.

   a. The length of a rectangular garden is twice the width. If the area of the garden is 500 square yards, then find the dimensions of the garden.

   b. Find two consecutive integers whose sum of their squares is 145.

   c. The base of a triangle is eight feet less than three times the height of the triangle. If the area of the triangle is 40 square feet, then find the base and height of the triangle.

4. Solve the following equations.

   a. $4(t^2 + 9)(t^2 - 7) = 0$ b. $12y^3 - 20y^2 - 27y + 45 = 0$

   c. $2w^5 = 128w^2$

5. Solve $\frac{3x}{x - 4} - \frac{5}{x + 6} = \frac{x^2 + 26x}{x^2 + 2x - 24}$

SOLUTIONS:

1a. $x^2 - 10x + 7 = 0$ Back to Problem 1.

$a = 1,\ b = -10,\ c = 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10 \pm \sqrt{100 - 4(1)(7)}}{2} = \frac{10 \pm \sqrt{100 - 28}}{2} =$$

$$\frac{10 \pm \sqrt{72}}{2} = \frac{10 \pm \sqrt{36 \cdot 2}}{2} = \frac{10 \pm 6\sqrt{2}}{2} = \frac{2(5 \pm 3\sqrt{2})}{2} = 5 \pm 3\sqrt{2}$$

Answer: $x = 5 \pm 3\sqrt{2}$ **or** $\{5 \pm 3\sqrt{2}\}$

1b. $\frac{1}{2}y^2 + \frac{2}{3}y = \frac{3}{4}$ Back to Problem 1.

LCD(2, 3, 4) = 12

$$\frac{1}{2}y^2 + \frac{2}{3}y = \frac{3}{4} \Rightarrow 12\left(\frac{1}{2}y^2 + \frac{2}{3}y\right) = \left(\frac{3}{4}\right)12 \Rightarrow 6y^2 + 8y = 9 \Rightarrow$$

$$6y^2 + 8y - 9 = 0$$

$a = 6,\ b = 8,\ c = -9$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{64 - 4(6)(-9)}}{12} = \frac{-8 \pm \sqrt{4(16 + 54)}}{12}$$

$$= \frac{-8 \pm \sqrt{4(70)}}{12} = \frac{-8 \pm 2\sqrt{70}}{12} = \frac{2(-4 \pm \sqrt{70}\,)}{12} = \frac{-4 \pm \sqrt{70}}{6}$$

Answer: $y = \dfrac{-4 \pm \sqrt{70}}{6}$ **or** $\left\{ \dfrac{-4 \pm \sqrt{70}}{6} \right\}$

2. Determine the type of solutions for the quadratic equation $12 - 5y - 3y^2 = 0$ by calculating the discriminant.

$a = -3,\ b = -5,\ c = 12$

$b^2 - 4ac = 25 - 4(-3)(12) = 25 + 144 > 0$

Answer: Two real solutions

Back to Problem 2.

3a. The length of a rectangular garden is twice the width. If the area of the garden is 500 square yards, then find the dimensions of the garden.

Let w = the width of the garden
Then $2w$ = the length of the garden

The area of the rectangle is length times width = $2w \cdot w = 2w^2$. Since the area of the rectangular garden is given as 500 square yard, then $2w^2 = 500$.

$$2w^2 = 500 \Rightarrow w^2 = 250 \Rightarrow w = \pm 5\sqrt{10}$$

NOTE: $w^2 = 250 \Rightarrow \sqrt{w^2} = \sqrt{250} \Rightarrow |w| = \sqrt{25 \cdot 10} \Rightarrow w = \pm 5\sqrt{10}$

Since the width is a dimension, it is a positive number. Thus, $w = 5\sqrt{10}$.

Length = $2w = 10\sqrt{10}$

Answer: Length = $10\sqrt{10}$ yards
Width = $5\sqrt{10}$ yards

Back to Problem 3.

3b. Find two consecutive integers whose sum of their squares is 145.

Let n = the first integer
Then $n + 1$ = the second integer

The square of the first integer is n^2. The square of the second integer is $(n + 1)^2$. The sum of these squares is $n^2 + (n + 1)^2$. This sum is to be 145. Thus, $n^2 + (n + 1)^2 = 145$.

$$n^2 + (n + 1)^2 = 145 \Rightarrow n^2 + n^2 + 2n + 1 = 145 \Rightarrow 2n^2 + 2n - 144 = 0$$

$$\Rightarrow 2(n^2 + n - 72) = 0 \Rightarrow 2(n + 9)(n - 8) = 0$$

$$n + 9 = 0 \Rightarrow n = -9$$
$$n - 8 = 0 \Rightarrow n = 8$$

There are two answers to this problem.

Answer 1: $-9, -8$

Answer 2: 8, 9

Back to Problem 3.

3c. The base of a triangle is eight feet less than three times the height of the triangle. If the area of the triangle is 40 square feet, then find the base and height of the triangle.

Let h = the height of the triangle
Then $3h - 8$ = the base of the triangle

The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where b is the base of the triangle and h is the height.

Thus, the area of our triangle is $A = \frac{1}{2}(3h - 8)h$. Since the area of the triangle is to be 40 square feet, then we have that $\frac{1}{2}(3h - 8)h = 40$.

$$\frac{1}{2}(3h - 8)h = 40 \Rightarrow (3h - 8)h = 80 \Rightarrow 3h^2 - 8h - 80 = 0 \Rightarrow$$

$$(h + 4)(3h - 20) = 0$$

$$h + 4 = 0 \Rightarrow h = -4$$

$$3h - 20 = 0 \Rightarrow 3h = 20 \Rightarrow h = \frac{20}{3}$$

Since h is a dimension, it is a positive number. Thus, h is $\frac{20}{3}$ feet.

The base is equal to $3h - 8$. Thus, when $h = \frac{20}{3}$, $3h - 8 = 20 - 8 = 12$ feet.

Answer: Base = 12 feet

Height = $\frac{20}{3}$ feet

Back to Problem 3.

4a. $4(t^2 + 9)(t^2 - 7) = 0$

Back to Problem 4.

$t^2 + 9 = 0 \Rightarrow t^2 = -9 \Rightarrow t = \pm 3i$

$t^2 - 7 = 0 \Rightarrow t^2 = 7 \Rightarrow t = \pm\sqrt{7}$

Answer: $t = \pm\sqrt{7}, \pm 3i$ **or** $\{\pm\sqrt{7}, \pm 3i\}$

4b. $12y^3 - 20y^2 - 27y + 45 = 0$

Back to Problem 4.

We can factor the expression $12y^3 - 20y^2 - 27y + 45$ by grouping:

$$12y^3 - 20y^2 - 27y + 45 = 4y^2(3y - 5) - 9(3y - 5) = (3y - 5)(4y^2 - 9)$$

$$12y^3 - 20y^2 - 27y + 45 = 0 \Rightarrow 4y^2(3y - 5) - 9(3y - 5) = 0 \Rightarrow$$

$$(3y - 5)(4y^2 - 9) = 0$$

$$3y - 5 = 0 \Rightarrow y = \frac{5}{3}$$

$$4y^2 - 9 = 0 \Rightarrow y^2 = \frac{9}{4} \Rightarrow y = \pm\frac{3}{2}$$

Answer: $y = \pm\frac{3}{2}, \frac{5}{3}$ **or** $\left\{\pm\frac{3}{2}, \frac{5}{3}\right\}$

4c. $2w^5 = 128w^2$

Back to Problem 4.

$$2w^4 = 128w^2 \Rightarrow 2w^4 - 128w^2 = 0 \Rightarrow 2w^2(w^3 - 64) = 0$$

NOTE: In order to factor the expression $w^3 - 64$, which is a difference of cubes, you will need the following factoring formula:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Thus, $w^3 - 64 = w^3 - 4^3 = (w - 4)(w^2 + 4w + 16)$

$$2w^4 = 128w \Rightarrow 2w^4 - 128w = 0 \Rightarrow 2w^2(w^3 - 64) = 0 \Rightarrow$$

$$2w^2(w - 4)(w^2 + 4w + 16) = 0$$

$$w^2 = 0 \Rightarrow w = 0$$

$$w - 4 = 0 \Rightarrow w = 4$$

$$w^2 + 4w + 16 = 0$$

$$a = 1,\ b = 4,\ c = 16$$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(1)(16)}}{2} = \frac{-4 \pm \sqrt{16(1-4)}}{2} =$$

$$\frac{-4 \pm 4\sqrt{-3}}{2} = \frac{-4 \pm 4i\sqrt{3}}{2} = -2 \pm 2i\sqrt{3}$$

Answer: $w = 0,\ 4,\ -2 \pm 2i\sqrt{3}$ **or** $\left\{0,\ 4,\ -2 \pm 2i\sqrt{3}\right\}$

Earn one bonus point because you checked the solution for this problem. Send me an email with the following in the Subject line: IPS3-4c.

5. Solve $\dfrac{3x}{x-4} - \dfrac{5}{x+6} = \dfrac{x^2 + 26x}{x^2 + 2x - 24}$

Back to Problem 5.

$$x^2 + 2x - 24 = (x+6)(x-4)$$

$$\frac{3x}{x-4} - \frac{5}{x+6} = \frac{x^2 + 26x}{x^2 + 2x - 24} \Rightarrow \frac{3x}{x-4} - \frac{5}{x+6} = \frac{x^2 + 26x}{(x+6)(x-4)}$$

NOTE: $x \neq -6,\ x \neq 4$

LCD $= (x+6)(x-4)$

$$\frac{3x}{x-4} - \frac{5}{x+6} = \frac{x^2 + 26x}{(x+6)(x-4)} \Rightarrow$$

$$(x+6)(x-4)\left(\frac{3x}{x-4}-\frac{5}{x+6}\right)=\left[\frac{x^2+26x}{(x+6)(x-4)}\right](x+6)(x-4)$$

$$\Rightarrow\ 3x(x+6)-5(x-4)=x^2+26x\ \Rightarrow$$

$$3x^2+18x-5x+20=x^2+26x\ \Rightarrow\ 3x^2+13x+20=x^2+26x\ \Rightarrow$$

$$2x^2-13x+20=0\ \Rightarrow\ (x-4)(2x-5)=0\ \Rightarrow\ x=4,\ \frac{5}{2}$$

If $x=4$, then two of the fractions in the equation are undefined because you would have division by zero. Thus, $x=\frac{5}{2}$ is the only solution.

Answer: $x=\frac{5}{2}$ **or** $\left\{\frac{5}{2}\right\}$

Back to Problem 5.