Solutions for In-Class Problems 23 for Wednesday, April 25

These problems are from Pre-Class Problems 23.

You can go to the solution for each problem by clicking on the problem letter.

1. Find the sum of the following arithmetic sequences.

a. $\sum_{i=1}^{25} (3i - 7)$ b. $\sum_{j=1}^{101} (j + 4)$

2. Determine if the following sequences are geometric. If the sequence is geometric, then find the common ratio.

a. $3, -6, 12, -24, 48, \ldots$ b. $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, 1 \ldots$

3. Write the first five terms of the geometric sequence $\{a_n\}$ with the given first term and common ratio.

a. $a_1 = -4$ and $r = 3$ b. $a_1 = 1$ and $r = -\frac{1}{4}$

4. Find the sum of the following geometric sequences, if possible.

a. $\sum_{n=1}^{5} (-3)2^{n-1}$ b. $\sum_{n=1}^{5} \left(-\frac{1}{5}\right)^{n-1}$ c. $\sum_{n=1}^{\infty} 5\left(\frac{3}{4}\right)^{n-1}$

d. $\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^{n-1}$ e. $\sum_{n=1}^{\infty} 12\left(\frac{5}{4}\right)^{n-1}$

SOLUTIONS:

1a. $\sum_{i=1}^{25}(3i - 7)$ Back to Problem 1.

$a_i = 3i - 7 \Rightarrow a_1 = -4$ and $a_{25} = 68$

Thus, $S_n = \frac{n(a_1 + a_n)}{2} \Rightarrow S_{25} = \frac{25(-4 + 68)}{2} = \frac{25(64)}{2} = 25(32) =$

$25 \cdot 4 \cdot 8 = 100 \cdot 8 = 800$

Answer: 800

1b. $\sum_{j=1}^{101}(j + 4)$ Back to Problem 1.

$a_j = j + 4 \Rightarrow a_1 = 5$ and $a_{101} = 105$

Thus, $S_n = \frac{n(a_1 + a_n)}{2} \Rightarrow S_{101} = \frac{101(5 + 105)}{2} = \frac{101(110)}{2} = 101(55) =$

$55(101) = 55(100 + 1) = 5500 + 55 = 5555$

Answer: 5555

2a. $3, -6, 12, -24, 48, \ldots$ Back to Problem 2.

$$\frac{a_2}{a_1} = \frac{-6}{3} = -2 \qquad \frac{a_3}{a_2} = \frac{12}{-6} = -2$$

$$\frac{a_4}{a_3} = \frac{-24}{12} = -2 \qquad \frac{a_5}{a_4} = \frac{48}{-24} = -2$$

NOTE: The ratio between each term and its preceding term is -2.

Answer: Yes. The common ratio is -2.

2b. $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, 1 \ldots.$ Back to Problem 2.

$$\frac{a_2}{a_1} = \frac{5}{2} \div 5 = \frac{5}{2} \cdot \frac{1}{5} = \frac{1}{2} \qquad \frac{a_3}{a_2} = \frac{5}{3} \div \frac{5}{2} = \frac{5}{3} \cdot \frac{2}{5} = \frac{2}{3}$$

This sequence is not geometric. The ratio between the second term and the first term is $\frac{1}{2}$. However, the ratio between the third term and the second term is $\frac{2}{3}$.

Answer: No

3a. $a_1 = -4$ and $r = 3$ Back to Problem 3.

Answer: -4, -12, -36, -108, -324

3b. $a_1 = 1$ and $r = -\frac{1}{4}$ Back to Problem 3.

Answer: $1, -\frac{1}{4}, \frac{1}{16}, -\frac{1}{64}, \frac{1}{256}$

4a. $\sum_{n=1}^{5} (-3)2^{n-1}$ Back to Problem 4.

$a_1 = -3$, $r = 2$, and $n = 5$

Thus, $S_n = \frac{a_1(r^n - 1)}{r - 1} \Rightarrow S_5 = \frac{-3(2^5 - 1)}{2 - 1} = \frac{-3(32 - 1)}{1} =$

$-3(31) = -93$

Answer: -93

4b. $\sum_{n=1}^{5} \left(-\frac{1}{5}\right)^{n-1}$ Back to Problem 4.

$a_1 = 1$, $r = -\frac{1}{5}$, and $n = 5$

NOTE: $S_n = \frac{a_1(r^n - 1)}{r - 1} \Rightarrow S_n = \frac{a_1(1 - r^n)}{1 - r}$

Thus, $S_n = \dfrac{a_1(1 - r^n)}{1 - r} \Rightarrow S_5 = \dfrac{1\left[1 - \left(-\frac{1}{5}\right)^5\right]}{1 - \left(-\frac{1}{5}\right)} = \dfrac{1 - \left(-\frac{1}{3125}\right)}{1 + \frac{1}{5}} =$

$$\frac{1 + \frac{1}{3125}}{1 + \frac{1}{5}} \cdot \frac{3125}{3125} = \frac{3125 + 1}{3125 + 625} = \frac{3126}{3750} = \frac{1563}{1875} = \frac{521}{625}$$

Answer: $\dfrac{521}{625}$

4c. $\displaystyle\sum_{n=1}^{\infty} 5\left(\frac{3}{4}\right)^{n-1}$

Back to Problem 4.

$r = \dfrac{3}{4} \Rightarrow |r| = \dfrac{3}{4} < 1 \Rightarrow$ the series is summable; $a_1 = 5$

Thus, $S = \dfrac{a_1}{1 - r} \Rightarrow S = \dfrac{5}{1 - \frac{3}{4}} = \dfrac{5}{1 - \frac{3}{4}} \cdot \dfrac{4}{4} = \dfrac{20}{4 - 3} = \dfrac{20}{1} = 20$

Answer: 20

4d. $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^{n-1}$

Back to Problem 4.

$r = -\frac{2}{3} \Rightarrow |r| = \frac{2}{3} < 1 \Rightarrow$ the series is summable; $a_1 = 1$

Thus, $S = \frac{a_1}{1 - r} \Rightarrow S = \frac{1}{1 + \frac{2}{3}} = \frac{1}{1 + \frac{2}{3}} \cdot \frac{3}{3} = \frac{3}{3 + 2} = \frac{3}{5}$

Answer: $\frac{3}{5}$

4e. $\sum_{n=1}^{\infty} 12\left(\frac{5}{4}\right)^{n-1}$

Back to Problem 4.

$r = \frac{5}{4} \Rightarrow |r| = \frac{5}{4} > 1 \Rightarrow$ the series is not summable

Answer: Sum does not exist