Solutions for In-Class Problems 22 for Monday, April 23

These problems are from Pre-Class Problems 22.

1. Find the first four terms of the sequence.

   a. $a_n = \dfrac{n+2}{4n-1}$ b. $b_n = \sqrt{n^2+9}$ c. $c_n = \left(-\dfrac{3}{4}\right)^n$

   d. $b_n = \dfrac{7}{10^n}$ e. $a_n = (-1)^{n+1}\dfrac{2^n}{n!}$

2. Find the following sums.

   a. $\displaystyle\sum_{i=1}^{5}(3i+5)$ b. $\displaystyle\sum_{j=2}^{4}(-3)^j$ c. $\displaystyle\sum_{k=3}^{8}(k+1)(k-3)$

3. Determine if the following sequences are arithmetic. If the sequence is arithmetic, then find the common difference.

   a. $9, 5, 1, -3, -7, \ldots.$ b. $1, 4, 7, 12, 17, \ldots.$

4. Write the first five terms of the arithmetic sequence $\{a_n\}$ with the given first term and common difference.

   a. $a_1 = -5$ and $d = 8$ b. $a_1 = 6$ and $d = -3$

5. Find the *n*th term of the arithmetic sequence $\{b_n\}$ with $b_1 = 14$ and $d = 6$. Then find b_{25}.

SOLUTIONS:

1a. $a_n = \dfrac{n+2}{4n-1}$ Back to Problem 1.

$a_1 = \dfrac{3}{3} = 1$, $a_2 = \dfrac{4}{7}$, $a_3 = \dfrac{5}{11}$, $a_4 = \dfrac{6}{15} = \dfrac{2}{5}$

Answer: $1, \dfrac{4}{7}, \dfrac{5}{11}, \dfrac{2}{5}$

1b. $b_n = \sqrt{n^2+9}$ Back to Problem 1.

$b_1 = \sqrt{10}$, $b_2 = \sqrt{13}$, $b_3 = \sqrt{18} = 3\sqrt{2}$, $b_4 = 5$

Answer: $\sqrt{10}$, $\sqrt{13}$, $3\sqrt{2}$, 5

1c. $c_n = \left(-\dfrac{3}{4}\right)^n$ Back to Problem 1.

$c_1 = -\dfrac{3}{4}$, $c_2 = \dfrac{9}{16}$, $c_3 = -\dfrac{27}{64}$, $c_4 = \dfrac{81}{256}$

Answer: $-\dfrac{3}{4}, \dfrac{9}{16}, -\dfrac{27}{64}, \dfrac{81}{256}$

1d. $b_n = \dfrac{7}{10^n}$ Back to Problem 1.

$$b_1 = \frac{7}{10} = 0.7\ ,\ b_2 = \frac{7}{100} = 0.07\ ,\ b_3 = \frac{7}{1000} = 0.007\ ,$$

$$b_4 = \frac{7}{10000} = 0.0007$$

Answer: $\frac{7}{10}, \frac{7}{100}, \frac{7}{1000}, \frac{7}{10000}$

or $0.7,\ 0.07,\ 0.007,\ 0.0007$

1e. $a_n = (-1)^{n+1}\frac{2^n}{n!}$ Back to Problem 1.

$$a_1 = \frac{2}{1} = 2,\ a_2 = -\frac{4}{2} = -2,\ a_3 = \frac{8}{6} = \frac{4}{3},\ a_4 = -\frac{16}{24} = -\frac{2}{3}$$

Answer: $2,\ -2,\ \frac{4}{3},\ -\frac{2}{3}$

2a. $\sum_{i=1}^{5}(3i+5)$ Back to Problem 2.

$$\sum_{i=1}^{5}(3i+5) = 8 + 11 + 14 + 17 + 20 = 25 + 25 + 20 = 70$$

Answer: 70

2b. $\sum_{j=2}^{4} (-3)^j$ Back to Problem 2.

$$\sum_{j=2}^{4} (-3)^j = 9 + (-27) + 81 = 90 + (-27) = 63$$

Answer: 63

2c. $\sum_{k=3}^{8} (k+1)(k-3)$ Back to Problem 2.

$$\sum_{k=3}^{8} (k+1)(k-3) = 4(0) + 5(1) + 6(2) + 7(3) + 8(4) + 9(5) =$$

$$0 + 5 + 12 + 21 + 32 + 45 = 50 + 33 + 32 = 50 + 65 = 115$$

Answer: 115

3a. $9, 5, 1, -3, -7, \ldots$. Back to Problem 3.

$a_2 - a_1 = 5 - 9 = -4$ $\quad$ $a_3 - a_2 = 1 - 5 = -4$

$a_4 - a_3 = -3 - 1 = -4$ $\quad$ $a_5 - a_4 = -7 - (-3) = -4$

NOTE: The difference between each term and its preceding term is -4.

Answer: Yes. The common difference is -4.

3b. 1, 4, 7, 12, 17, ... Back to Problem 3.

$a_2 - a_1 = 4 - 1 = 3$ $\qquad$ $a_3 - a_2 = 7 - 4 = 3$

$a_4 - a_3 = 12 - 7 = 5$

This sequence is not arithmetic. The difference between the second term and the first term is 3. However, the difference between the fourth term and the third term is 5.

Answer: No

4a. $a_1 = -5$ and $d = 8$ Back to Problem 4.

Answer: -5, 3, 11, 19, 27

4b. $a_1 = 6$ and $d = -3$ Back to Problem 4.

Answer: 6, 3, 0, -3, -6

5. $b_1 = 14$ and $d = 6$. Find b_{25}. Back to Problem 5.

$b_n = b_1 + (n - 1)d \Rightarrow b_n = 14 + (n - 1)6$

Simplifying, we have that

$b_n = 14 + (n - 1)6 = 14 + 6n - 6 = 6n + 8.$

Thus, $b_{25} = 6(25) + 8 = 150 + 8 = 158$

Answer: $b_n = 6n + 8,\ b_{25} = 158$