Solutions for In-Class Problems 21 for Wednesday, April 18

These problems are from Pre-Class Problems 21.

1. For the following hyperbolas, identify the center, the vertices, the foci, and the asymptotes. Sketch the graph of the hyperbola in a, b, and d.

a. $\dfrac{x^2}{64} - \dfrac{y^2}{49} = 1$ b. $9y^2 - 25x^2 = 225$

c. $\dfrac{4x^2}{5} - \dfrac{9y^2}{16} = 1$ d. $9(y - 7)^2 - 4(x + 2)^2 = 144$

2. Write the standard form of the equation of the hyperbola with vertices of $(-\sqrt{15}, 0)$ and $(\sqrt{15}, 0)$ and foci of $(-8, 0)$ and $(8, 0)$.

3. For the following parabolas, identify the vertex, the focus, the directrix, and the axis of symmetry. Sketch the graph of the parabola.

a. $x^2 = 12y$ b. $(x - 3)^2 = -7(y + 5)$

c. $y^2 = -9x$ d. $(y + 4)^2 = 8(x - 2)$

SOLUTIONS:

1a. $\dfrac{x^2}{64} - \dfrac{y^2}{49} = 1$

Back to Problem 1.

Center: $(0, 0)$

Vertices: $(\pm a, 0) = (\pm 8, 0)$

NOTE: $a^2 = 64 \Rightarrow a = 8$ since $a > 0$

Foci: $(\pm c,\ 0) = (\pm\sqrt{113},\ 0)$

$$c^2 = a^2 + b^2 = 64 + 49 = 113 \Rightarrow c = \sqrt{113} \text{ since } c > 0$$

NOTE: $a^2 = 64$ and $b^2 = 49$

Asymptotes: $y = \pm \dfrac{7}{8}x$

NOTE: The slope of a line is the change in *y* divided by the change in *x*. That is, $m = \dfrac{\Delta y}{\Delta x}$. The numerical slope of the asymptotes is the square root of the number under the y^2 term divided by the square root of the number under the x^2 term. Use the slope formula to help you remember *y* over *x*.

Sketch: I will owe you the sketch.

1b. $9y^2 - 25x^2 = 225$ Back to Problem 1.

The equation of this hyperbola is not in standard form. To obtain the standard form of the hyperbola, we need to divide both sides of the equation by 225 in order to get 1 on the right side of the equation.

$$9y^2 - 25x^2 = 225 \Rightarrow \frac{9y^2}{225} - \frac{25x^2}{225} = \frac{225}{225} \Rightarrow \frac{y^2}{25} - \frac{x^2}{9} = 1$$

Center: $(0, 0)$

Vertices: $(0, \pm a) = (0, \pm 5)$

NOTE: $a^2 = 25 \Rightarrow a = 5$ since $a > 0$

Foci: $(0, \pm c) = (0, \pm \sqrt{34})$

$c^2 = a^2 + b^2 = 25 + 9 = 34 \Rightarrow c = \sqrt{34}$ since $c > 0$

NOTE: $a^2 = 25$ and $b^2 = 9$

Asymptotes: $y = \pm \dfrac{5}{3} x$

NOTE: The slope of a line is the change in *y* divided by the change in *x*. That is, $m = \dfrac{\Delta y}{\Delta x}$. The numerical slope of the asymptotes is the square root of the number under the y^2 term divided by the square root of the number under the x^2 term. Use the slope formula to help you remember *y* over *x*.

Sketch: I will owe you the sketch.

1c. $\dfrac{4x^2}{5} - \dfrac{9y^2}{16} = 1$ Back to Problem 1.

The equation of this hyperbola is not in standard form. In the first fraction, x^2 is being multiplied by $\dfrac{4}{5}$. Multiplication by $\dfrac{4}{5}$ is the same as division by $\dfrac{5}{4}$. Thus, we will write the first fraction of $\dfrac{4x^2}{5}$ as $\dfrac{x^2}{\frac{5}{4}}$. In the second fraction, y^2 is being multiplied by $\frac{9}{16}$. Multiplication by $\frac{9}{16}$ is the same as division by $\frac{16}{9}$. Thus, we will write the second fraction of $\frac{9y^2}{16}$ as $\frac{y^2}{\frac{16}{9}}$.

Thus,

$$\frac{4x^2}{5} - \frac{9y^2}{16} = 1 \Rightarrow \frac{x^2}{\frac{5}{4}} - \frac{y^2}{\frac{16}{9}} = 1$$

Center: $(0, 0)$

Vertices: $(\pm a,\ 0) = \left(\pm \frac{\sqrt{5}}{2},\ 0\right)$

NOTE: $a^2 = \frac{5}{4} \Rightarrow a = \frac{\sqrt{5}}{2}$ since $a > 0$

Foci: $(\pm c,\ 0) = \left(\pm \frac{\sqrt{109}}{6},\ 0\right)$

$$c^2 = a^2 + b^2 = \frac{5}{4} + \frac{16}{9} = \frac{45}{36} + \frac{64}{36} = \frac{109}{36} \Rightarrow c = \frac{\sqrt{109}}{6} \text{ since } c > 0$$

NOTE: $a^2 = \frac{5}{4}$ and $b^2 = \frac{16}{9}$

Asymptotes: $y = \pm \frac{8\sqrt{5}}{15} x$

$$y = \pm \frac{\frac{4}{3}}{\frac{\sqrt{5}}{2}} x = \pm \frac{8}{3\sqrt{5}} x = \pm \frac{8\sqrt{5}}{15} x$$

NOTE: The slope of a line is the change in *y* divided by the change in *x*. That is, $m = \dfrac{\Delta y}{\Delta x}$. The numerical slope of the asymptotes is the square root of the number under the y^2 term divided by the square root of the number under the x^2 term. Use the slope formula to help you remember *y* over *x*.

Sketch: I will owe you the sketch.

1d. $9(y - 7)^2 - 4(x + 2)^2 = 144$ Back to Problem 1.

The equation of this hyperbola is not in standard form. To obtain the standard form of the hyperbola, we need to divide both sides of the equation by 144 in order to get 1 on the right side of the equation.

$$9(y - 7)^2 - 4(x + 2)^2 = 144 \Rightarrow \frac{9(y - 7)^2}{144} - \frac{4(x + 2)^2}{144} = \frac{144}{144} \Rightarrow$$

$$\frac{(y - 7)^2}{16} - \frac{(x + 2)^2}{36} = 1$$

Center: $(-2, 7)$

Vertices: $(-2, 3)$, $(-2, 11)$

$$(0, \pm a) + (h, k) = (0, \pm 4) + (-2, 7) = (-2, 7 \pm 4)$$

NOTE: $a^2 = 16 \Rightarrow a = 4$ since $a > 0$

Foci: $(-2, 7 \pm 2\sqrt{13})$

$$(0, \pm c) + (h, k) = (0, \pm 2\sqrt{13}) + (-2, 7) = (-2, 7 \pm 2\sqrt{13})$$

$c^2 = a^2 + b^2 = 16 + 36 = 52 \Rightarrow c = \sqrt{52} = 2\sqrt{13}$ since $c > 0$

NOTE: $a^2 = 16$ and $b^2 = 36$

Asymptotes: $y - 7 = \pm \dfrac{2}{3}(x + 2)$

NOTE: The slope of a line is the change in *y* divided by the change in *x*. That is, $m = \dfrac{\Delta y}{\Delta x}$. The numerical slope of the asymptotes is the square root of the number under the y^2 term divided by the square root of the number under the x^2 term. Use the slope formula to help you remember *y* over *x*.

Sketch: I will owe you the sketch.

2. Vertices: $(-\sqrt{15}, 0)$ and $(\sqrt{15}, 0)$ Back to Problem 2.

Foci: $(-8, 0)$ and $(8, 0)$

Since the vertices are given by the points $(\pm a, 0)$, then $a = \sqrt{15}$.

Since the foci are given by the points $(\pm c, 0)$, then $c = 8$.

Since $c^2 = a^2 + b^2$ and $a = \sqrt{15}$ and $c = 8$, then

$c^2 = a^2 + b^2 \Rightarrow 64 = 15 + b^2 \Rightarrow b^2 = 49$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{15} - \frac{y^2}{49} = 1$$

Answer: $\frac{x^2}{15} - \frac{y^2}{49} = 1$ Back to Problem 2.

3a. $x^2 = 12y$ Back to Problem 3.

Vertex: $(0, 0)$

Opens: Up

NOTE: $4p = 12 \Rightarrow p = 3 > 0$

Axis of Symmetry: $x = 0$ (y-axis)

Focus: $(0, p) = (0, 3)$

NOTE: $4p = 12 \Rightarrow p = 3$

Directrix: $y = -3$

NOTE: $y = -p = -3$

Sketch: I owe you the sketch.

3b. $(x - 3)^2 = -7(y + 5)$ Back to Problem 3.

Vertex: $(3, -5)$

Opens: Down

NOTE: $4p = -7 \Rightarrow p = -\frac{7}{4} < 0$

Axis of Symmetry: $x = 3$

$x - 3 = 0 \Rightarrow x = 3$

Focus: $\left(3, -\frac{27}{4}\right)$

$(0, p) + (h, k) = \left(0, -\frac{7}{4}\right) + (3, -5) = \left(3, -\frac{7}{4} - \frac{20}{4}\right)$

NOTE: $4p = -7 \Rightarrow p = -\frac{7}{4}$

Directrix: $y = -\frac{13}{4}$

$$y + 5 = -p = \frac{7}{4} \Rightarrow y = \frac{7}{4} - \frac{20}{4}$$

NOTE: $p = -\frac{7}{4}$

Sketch: I owe you the sketch.

3c. $y^2 = -9x$ Back to Problem 3.

Vertex: $(0, 0)$

Opens: Left

NOTE: $4p = -9 \Rightarrow p = -\frac{9}{4} < 0$

Axis of Symmetry: $y = 0$ (x-axis)

Focus: $(p, 0) = \left(-\frac{9}{4}, 0\right)$

NOTE: $4p = -9 \Rightarrow p = -\frac{9}{4}$

Directrix: $x = \frac{9}{4}$

NOTE: $x = -p = \frac{9}{4}$

Sketch: I owe you the sketch.

3d. $(y + 4)^2 = 8(x - 2)$ Back to Problem 3.

Vertex: $(2, -4)$

Opens: Right

NOTE: $4p = 8 \Rightarrow p = 2 > 0$

Axis of Symmetry: $y = -4$

$y + 4 = 0 \Rightarrow y = -4$

Focus: $(4, -4)$

$(p, 0) + (h, k) = (2, 0) + (2, -4) = (4, -4)$

NOTE: $4p = 8 \Rightarrow p = 2$

Directrix: $x = 0$ (y-axis)

$x - 2 = -p = -2 \Rightarrow x = 0$

NOTE: $p = 2$

Sketch: I owe you the sketch.

Back to Problem 3.