Solutions for In-Class Problems 20 for Wednesday, April 11

These problems are from Pre-Class Problems 20.

1. For the following ellipses, identify the center, the major axis, the vertices, the length of major axis, the foci, the minor axis, the endpoints of minor axis, and the length of the minor axis. Sketch the graph of the ellipse in a, b and d.

a. $\dfrac{x^2}{49} + \dfrac{y^2}{64} = 1$

b. $9x^2 + 25y^2 = 225$

c. $\dfrac{9x^2}{16} + \dfrac{36y^2}{5} = 1$

d. $9(x - 2)^2 + 4(y + 7)^2 = 144$

2. Write the standard form of the equation of the ellipse with vertices of $(0, -8)$ and $(0, 8)$ and foci of $(0, -6)$ and $(0, 6)$.

SOLUTIONS:

1a. $\dfrac{x^2}{49} + \dfrac{y^2}{64} = 1$

Back to Problem 1.

Center: $(0, 0)$

Major Axis: y-axis

Vertices: $(0, \pm 8)$

NOTE: $a^2 = 64 \Rightarrow a = 8$ since $a > 0$

Length of the Major Axis: 16

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = 16$ since $a = 8$.

Foci: $(0, \pm\sqrt{15})$

$c^2 = a^2 - b^2 = 64 - 49 = 15 \Rightarrow c = \sqrt{15}$ since $c > 0$

NOTE: $a^2 = 64$ and $b^2 = 49$

Minor Axis: *x*-axis

Endpoints of the Minor Axis: $(\pm 7, 0)$

NOTE: $b^2 = 49 \Rightarrow b = 7$ since $b > 0$

Length of the Minor Axis: 14

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = 14$ since $b = 7$.

Sketch: I will owe you the sketch.

1b. $9x^2 + 25y^2 = 225$ Back to Problem 1.

The equation of this ellipse is not in standard form. To obtain the standard form of the ellipse, we need to divide both sides of the equation by 225 in order to get 1 on the right side of the equation.

$$9x^2 + 25y^2 = 225 \Rightarrow \frac{9x^2}{225} + \frac{25y^2}{225} = \frac{225}{225} \Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Center: $(0, 0)$

Major Axis: *x*-axis

Vertices: $(\pm 5,\ 0)$

NOTE: $a^2 = 25 \Rightarrow a = 5$ since $a > 0$

Length of the Major Axis: 10

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = 10$ since $a = 5$.

Foci: $(\pm 4,\ 0)$

$c^2 = a^2 - b^2 = 25 - 9 = 16 \Rightarrow c = 4$ since $c > 0$

NOTE: $a^2 = 25$ and $b^2 = 9$

Minor Axis: *y*-axis

Endpoints of the Minor Axis: $(0,\ \pm 3)$

NOTE: $b^2 = 9 \Rightarrow b = 3$ since $b > 0$

Length of the Minor Axis: 6

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = 6$ since $b = 3$.

Sketch: I will owe you the sketch.

1c. $\dfrac{9x^2}{16} + \dfrac{36y^2}{5} = 1$ Back to Problem 1.

The equation of this ellipse is not in standard form. In the first fraction, x^2 is being multiplied by $\dfrac{9}{16}$. Multiplication by $\dfrac{9}{16}$ is the same as division by $\dfrac{16}{9}$. Thus, we will write the first fraction of $\dfrac{9x^2}{16}$ as $\dfrac{x^2}{\frac{16}{9}}$. In the second fraction, y^2 is being multiplied by $\dfrac{36}{5}$. Multiplication by $\dfrac{36}{5}$ is the same as division by $\dfrac{5}{36}$. Thus, we will write the second fraction of $\dfrac{36y^2}{5}$ as $\dfrac{y^2}{\frac{5}{36}}$. Thus,

$$\frac{9x^2}{16} + \frac{36y^2}{5} = 1 \Rightarrow \frac{x^2}{\frac{16}{9}} + \frac{y^2}{\frac{5}{36}} = 1$$

Center: $(0, 0)$

Major Axis: x-axis

Vertices: $\left(\pm \frac{4}{3},\ 0 \right)$

NOTE: $a^2 = \frac{16}{9} \Rightarrow a = \frac{4}{3}$ since $a > 0$

Length of the Major Axis: $\frac{8}{3}$

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = \frac{8}{3}$ since $a = \frac{4}{3}$.

Foci: $\left(\pm \frac{\sqrt{59}}{6},\ 0 \right)$

$$c^2 = a^2 - b^2 = \frac{16}{9} - \frac{5}{36} = \frac{64}{36} - \frac{5}{36} = \frac{59}{36} \Rightarrow c = \frac{\sqrt{59}}{6} \text{ since } c > 0$$

NOTE: $a^2 = \frac{16}{9}$ and $b^2 = \frac{5}{36}$

Minor Axis: y-axis

Endpoints of the Minor Axis: $\left(0,\ \pm \frac{\sqrt{5}}{6} \right)$

NOTE: $b^2 = \frac{5}{36} \Rightarrow b = \frac{\sqrt{5}}{6}$ since $b > 0$

Length of the Minor Axis: $\dfrac{\sqrt{5}}{3}$

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = \dfrac{\sqrt{5}}{3}$ since $b = \dfrac{\sqrt{5}}{6}$.

Sketch: I will owe you the sketch.

1d. $9(x - 2)^2 + 4(y + 7)^2 = 144$

Back to Problem 1.

The equation of this ellipse is not in standard form. To obtain the standard form of the ellipse, we need to divide both sides of the equation by 144 in order to get 1 on the right side of the equation.

$$9(x - 2)^2 + 4(y + 7)^2 = 144 \Rightarrow \frac{9(x - 2)^2}{144} + \frac{4(y + 7)^2}{144} = \frac{144}{144} \Rightarrow$$

$$\frac{(x - 2)^2}{16} + \frac{(y + 7)^2}{36} = 1$$

Center: $(2, -7)$

Major Axis: Vertical: $x = 0 + 2 \Rightarrow x = 2$

NOTE: The equation $x = 0$ is the equation of the *y*-axis.

Vertices: $(2, -13)$, $(2, -1)$

$$(0, \pm 6) + (2, -7) = (2, -7 \pm 6)$$

NOTE: $a^2 = 36 \Rightarrow a = 6$ since $a > 0$

Length of the Major Axis: 12

NOTE: The length of the major axis is the distance between the two vertices of the ellipse, which is $2a = 12$ since $a = 6$.

Foci: $(2, -7 - 2\sqrt{5})$, $(2, -7 + 2\sqrt{5})$

$$(0, \pm 2\sqrt{5}) + (2, -7) = (2, -7 \pm 2\sqrt{5})$$

$c^2 = a^2 - b^2 = 36 - 16 = 20 \Rightarrow c = \sqrt{20} = 2\sqrt{5}$ since $c > 0$

NOTE: $a^2 = 36$ and $b^2 = 16$

Minor Axis: Horizontal: $y = 0 + (-7) \Rightarrow y = -7$

NOTE: The equation $y = 0$ is the equation of the *x*-axis.

Endpoints of the Minor Axis: $(-2, -7)$, $(6, -7)$

$$(\pm 4, 0) + (2, -7) = (2 \pm 4, -7)$$

NOTE: $b^2 = 16 \Rightarrow b = 4$ since $b > 0$

Length of the Minor Axis: 8

NOTE: The length of the minor axis is the distance between the two endpoints of the minor axis of the ellipse, which is $2b = 8$ since $b = 4$.

Sketch: I will owe you the sketch.

2. Vertices: $(0, -8)$ and $(0, 8)$

Back to Problem 2.

Foci: $(0, -6)$ and $(0, 6)$

Since the vertices of an ellipse lie on the major axis and the vertices are $(0, \pm 8)$, then the major axis is the *y*-axis. Also, since the vertices are given by the points $(0, \pm a)$, then $a = 8$.

The foci of an ellipse, where the major axis is the *y*-axis, are given by $(0, \pm c)$. Since the foci are the points $(0, \pm 6)$, then $c = 6$.

Since $c^2 = a^2 - b^2$ and $a = 8$ and $c = 6$, then

$$c^2 = a^2 - b^2 \Rightarrow 36 = 64 - b^2 \Rightarrow b^2 = 28$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \Rightarrow \frac{x^2}{28} + \frac{y^2}{64} = 1$$

Answer: $\frac{x^2}{28} + \frac{y^2}{64} = 1$